Notes

A little more about Bessel function:

properties:

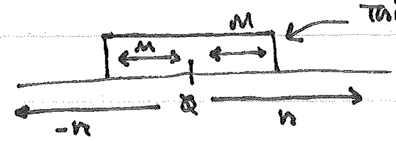
$$J_n(\beta) = J_{-n}(\beta) \qquad n \text{ even}$$

$$J_n(\beta) = -J_{-n}(\beta) \qquad n \text{ odd}$$

$$\sum_{-\infty}^{\infty} J_n^2(\beta) = 1 \qquad \forall \beta$$

In general $J_n(\beta) \approx \dfrac{\beta^n}{2^n n!}$ => $n \to \infty$ then $J_n(\beta) \to \phi$

=> signal power is significant only over specified BW.
what is n then; that is for what values of n power is still significant?



This is where power is significant!

we know:

$$X_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta_f \sin(2\pi f_m t)\right]$$

or using Fourier Coef.

$$X_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} C_n \cos\left[(2\pi f_c + n f_m)t\right]$$

but $C_n = J_n(\beta)$

mod. index

$$X_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[(2\pi f_c + n f_m)t\right]$$

so what is $\underline{\underline{M}}$; that is, for what value of (M) power is still significant. (we call this in-band power)

$$P_{X_{FM}}(\text{in-band}) = \frac{A_c^2}{2} \sum_{n=-M}^{M} J_n^2(\beta)$$

$$P_{X_{FM}}(\text{Total Avg. Power}) = A^2/2$$

The inband power ratio $\equiv \dfrac{P_{x_{FM}}(\text{in-band})}{P_{x_{FM}}(\text{total})} = \sum_{n=-M}^{M} J_n^2(\beta)$
$= P_{IBR}$

$$= J_0^2(\beta) + 2\sum_{n=1}^{M} J_n^2(\beta)$$

M is selected such that $P_{IBR} \geq 98\%$

we call this 98% power BW

$$\underset{98\%}{B_T} = 2M f_m$$

This is equivalent to saying

$$B_T(FM) = 2(\beta_f + 1) f_m = 2(\Delta F + f_m)$$

$$B_T(PM) = 2(\beta_p + 1) f_m = 2(\Delta\phi + 1) f_m$$

nch.

$\Delta F = D_f \cdot V_m$

$\Delta\phi = D_p \cdot V_m$

Another way to calculate BW of X_{FM} is using Carson's rule

$$\underset{\text{carson}}{B_T} = 2(\beta + 1) B_m$$

BW of m(t)

modulation index diff. for FM & PM!

Example A

$m(t) = V_m \sin(2\pi \cdot 1000t)$ $f_m = 1000$

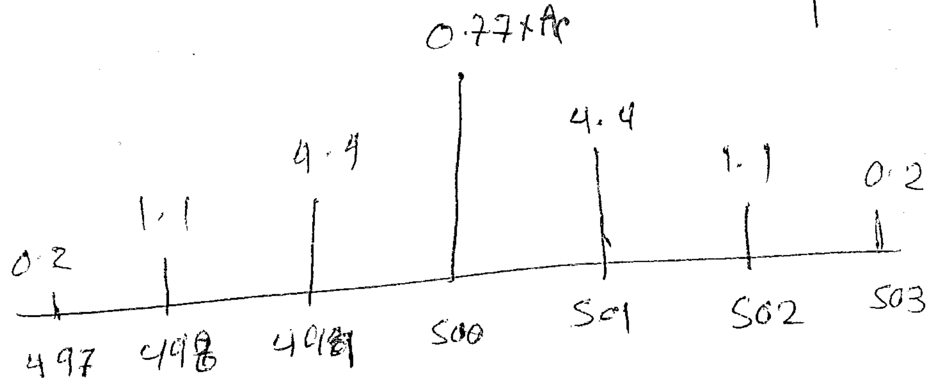
$V_c(t) = 10 \sin(2\pi \times 500 \times 10^3 t)$ $f_c = 500\text{kHz}$

$$G(f) = A_c \sum_{n=0}^{\infty} J_n(\beta)\, \delta(f - n f_m)$$

f_c

$\beta_f = 1 =$

	$J_n(1)$	$G(f)$ & $G($	$f_c - n f_m$	$f_c + n f_m$
$n=0$	0.77	$(0.77)\cdot 10 \cdot \delta(f_c) = 7.7\,\delta(f_c)$	500 KHz	500
$n=1$	0.44	$(4.4)\,\delta(f_c - f_m)$ $\delta(f_c + f_m)$	$500-1 = 499$	501
$n=2$	0.11	$1.1\,\delta(f_c - 2f_m)$	498	502
$n=3$	0.02	$0.2\,\delta(f_c - 3f_m)$	497	503



Example 3

a) $m(t) = 2\cos(2\pi \cdot 2000t)$ FM

$D_f = 5\,kHz/V$

1) Find peak freq. deviation
2) Find modulation index β_f

For FM: $\Delta F_{rad} = \frac{D_f V_m}{2\pi}$

$\Delta F_{Hz} = D_f \cdot V_m = 5(2) = \boxed{10\,kHz}$

$\beta_f = \frac{\Delta F}{B} = \frac{10\,kHz}{2000} = 5$

b) $D_p = 2.5\,rad/V$

Find Peak phase deviation = (modulation index)

$\Delta\theta = D_p V_m = (2.5)(2) = 5\,rad \;\therefore\; \beta_p = 5$

Note β_p is unitless

note $\boxed{\beta_f = \beta_p}$

c) what happens if V_m ~~increase~~ changes:

β_f & β_p change

d) what happen if f_m changes?

* β_f changes $(\Delta F/B)$
* β_p stays the same

How to Tell

Example: Assom β is Given to be $\underline{2}$. Find the value of M such that 98% of power is captured.

using the table: $\beta = 2 \rightarrow n = 6$; $\underline{M \leq n}$

we want

$P_{IBR} \geq 98\%$

$$P_{IBR} = \sum_{n=-M}^{M} J_n^2(\beta) \geq 98\%$$

$$= J_0^2(2) + 2\sum_{n=1}^{M} J_n^2(2)$$

From table ($\beta = 2$)

$$(0.224)^2 + 2(0.577)^2 + 2(0.353)^2 + (0.129)^2 2 \approx \underline{0.99\%}$$

meets 98% rule!

$\rightarrow \underline{M = 3}$

Note: $\sum_{n=-\infty}^{+\infty} J_n^2(\beta) = 1$ $\rightarrow$ $(0.224)^2 +$
$\beta = 2$

$2(0.577)^2 +$

$\vdots$

$2(0.001)^2$

≈ 1

From table

Good Example

Assume

$$x_{FM}(t) = 100\cos\Big[\overbrace{2\pi(100\times10^{6})\,t + \underbrace{5\sin(10^{4}\times\pi t) + 3\sin(2\times10^{4}\times\pi t)}_{\theta(t)\,=\,\text{Excess phase of angle mod.}}}^{\phi_i(t)\,=\,\text{inst. phase}}\Big]$$

Answer the following :

1) what is inst. phase of $X_{FM}(t)$

2) what is the excess phase of X_{FM}

3) Find f_m (modulating freq.) (Hz)

4) Find BW of the modulating signal B_m (Hz)

5) Find modulation index for X_{FM} (β_f)

6) Find Total BW of X_{FM} using Carson's rule (Hz)

7) Find Total Average power of X_{FM} (W)

8) Assuming β_f increases what will be the max B_T (BW)

9) calculate the BW of X_{FM} using 98% Power BW rule

(3b) Find max freq. deviation (Hz)

(3c) find max phase deviation (radian)

10) plot freq. spectrum for 98% Power BW

3) note that $f_m \equiv \{5KHz \ \& \ 10KHz\}$

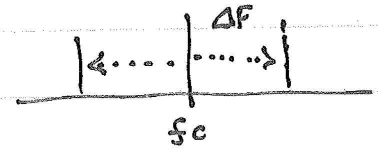
$\rightarrow BW \equiv B_m = \max\{f_m\} = \underline{\underline{10\,KHz}}$

modolation index of FM $= \underset{(Hz)}{\beta_{fM}} = \dfrac{D_f \cdot V_m}{B_m} = \dfrac{\Delta F}{B_m}$

but we don't know D_f.

Thus, let's find $\Delta F \equiv$ max freq. deviation

$\Delta F = \max\{f_i(t)\}$ — inst. freq. or max change of carrier



$f_i(t) = \dfrac{1}{2\pi} \dfrac{d\phi(t)}{dt}$ — inst. phase

$= \frac{1}{2\pi}\left[2\pi(100\times10^6) + 5\cos(\quad)10^4\pi + 3\cos(\quad)2\times10^4\times\pi \right]$

$= 100\times10^6 + 2.5\times10^4\cos(\quad) + 3\times10^4\cos(\quad)\big]$

(3b) $\max\{f_i(t)\} = \underbrace{100\times10^6}_{\text{carrier}} \pm \underbrace{5.5\times10^4}_{\text{max freq. deviation}}$

$\boxed{\Delta F = 5.5\times10^4\ Hz}$

(3c) max phase deviation $\max\{\theta(t)\} = (\underline{\underline{8}}\ rad)$

(5) $\beta_f = \dfrac{\Delta F}{B_m} = \dfrac{5.5\times10^4}{10\,KHz} = \boxed{5.5 \ \text{unitless}}$

→

(6) Total BW using Carson's rule :

$$B_T = 2(\beta_f + 1) B_m$$

$$= 2(5.5 + 1) 10^4 = 130 KHz$$

(8) note that if β_f increases $\beta_f \gg 1$

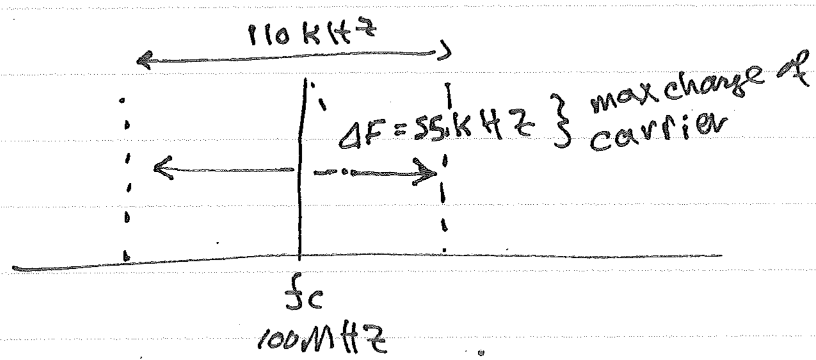
$$B_T \approx 2(\beta_f) B_m \Rightarrow$$

(7) Total Average power $P_{x_{FM}} = \frac{A_c^2}{2} = \frac{(100)^2}{2} = \boxed{5 KW}$

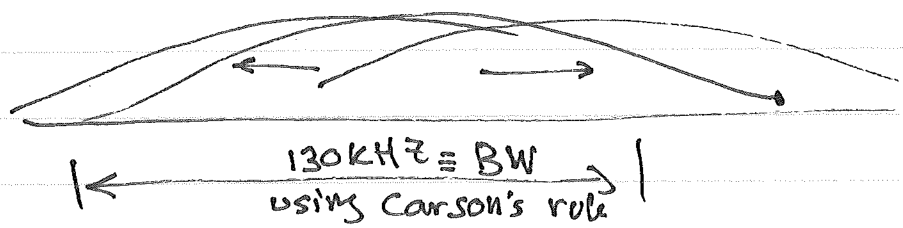
(9) $2(\Delta F + f_m)$

$= 2(65 \times 10^3)$

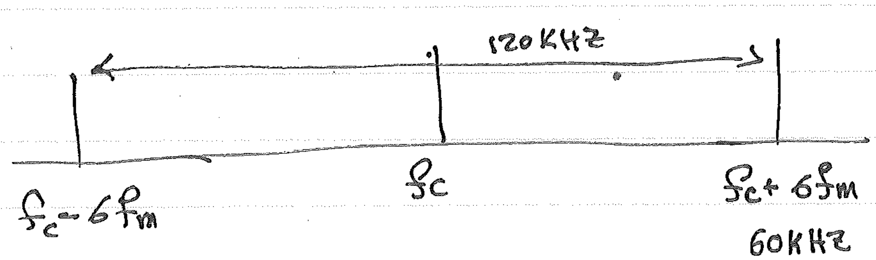
$= \underline{130 KHz}$



Using 98% Power BW



10) For 98% In Band power $n = M = 6$ from table



sufficiently close to Carson's rule