

Fourier Transform Pairs

Function	Time Waveform $w(t)$	Spectrum $W(f)$
Rectangular	$\Pi\left(\frac{t}{T}\right)$	$T[\text{Sa}(\pi fT)]$
Triangular	$\Lambda\left(\frac{t}{T}\right)$	$T[\text{Sa}(\pi fT)]^2$
Unit step	$u(t) \triangleq \begin{cases} +1, & t > 0 \\ 0, & t < 0 \end{cases}$	$\frac{1}{2} \delta(f) + \frac{1}{j2\pi f}$
Signum	$\text{sgn}(t) \triangleq \begin{cases} +1, & t > 0 \\ -1, & t < 0 \end{cases}$	$\frac{1}{j\pi f}$
Constant	1	$\delta(f)$
Impulse at $t = t_0$	$\delta(t - t_0)$	$e^{-j2\pi f t_0}$
Sin c	$\text{Sa}(2\pi Wt)$	$\frac{1}{2W} \Pi\left(\frac{f}{2W}\right)$
Phasor	$e^{j(\omega_0 t + \varphi)}$	$e^{j\varphi} \delta(f - f_0)$
Sinusoid	$\cos(\omega_c t + \varphi)$	$\frac{1}{2} e^{j\varphi} \delta(f - f_c) + \frac{1}{2} e^{-j\varphi} \delta(f + f_c)$
Gaussian	$e^{-\pi(t/t_0)^2}$	$t_0 e^{-\pi(f t_0)^2}$
Exponential, one-sided	$\begin{cases} e^{-t/T}, & t > 0 \\ 0, & t < 0 \end{cases}$	$\frac{2T}{1 + j2\pi fT}$

Other FT Properties

Operation	Function	Fourier Transform
Linearity	$a_1 w_1(t) + a_2 w_2(t)$	$a_1 W_1(f) + a_2 W_2(f)$
Time delay	$w(t - T_d)$	$W(f) e^{-j\omega T_d}$
<u>Scale change</u>	$w(at)$	$\frac{1}{ a } W\left(\frac{f}{a}\right)$
Conjugation	$w^*(t)$	$W^*(-f)$
Duality	$W(t)$	$w(-f)$
Real signal frequency translation [$w(t)$ is real]	$w(t) \cos(\omega_c t + \theta)$	$\frac{1}{2} [e^{j\theta} W(f - f_c) + e^{-j\theta} W(f + f_c)]$
Complex signal frequency translation	$w(t) e^{j\omega_c t}$	$W(f - f_c)$
Bandpass signal	$\text{Re}\{g(t) e^{j\omega_c t}\}$	$\frac{1}{2} [G(f - f_c) + G^*(-f - f_c)]$
Differentiation	$\frac{d^n w(t)}{dt^n}$	$(j2\pi f)^n W(f)$