System Examples

Filters, Cables, and Transmission Lines

Updated:2/25/15

Outline

Remember: Linear Time Invariant System

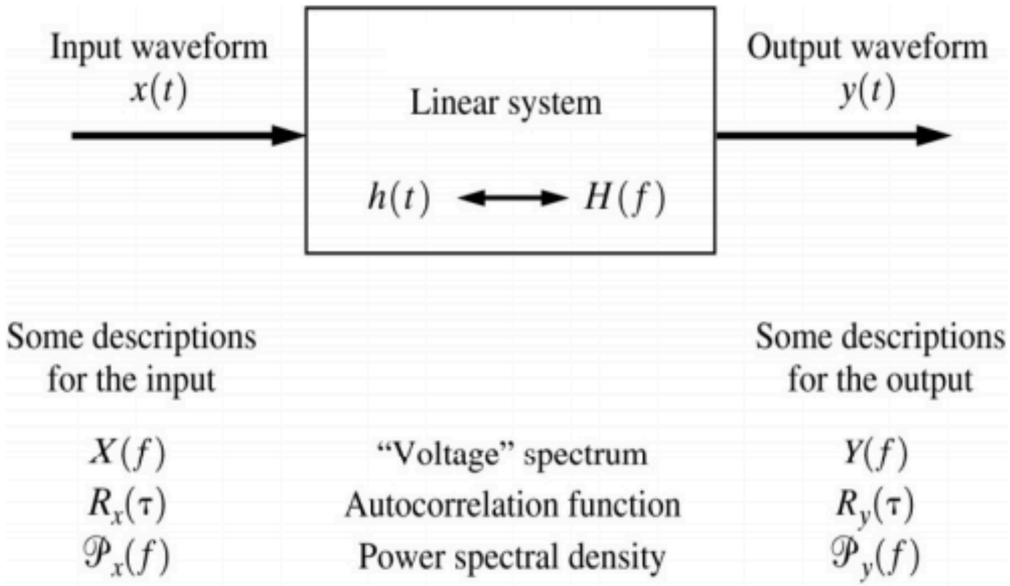
- An electronic filter or system is **linear** when superposition holds

$$y(t) = L[a_1 x_1(t) + a_2 x_2(t)] = a_1 L[x_1(t)] + a_2 L[x_2(t)]$$ L is a Linear Operation

where $a_1 x_1(t) + a_2 x_2(t)$ is the input & y is output.

- Note that the above satisfies both scaling & superposition properties.
- The system is **time invariant** if, for any delayed input $x(t - t_0)$, the output is delayed by the same amount $y(t - t_0)$.
  - Shape of response stays the same.

Example:
$y(t) = t – 3$
Is a linear time invariant system



Linear Time Invariant System (LTIS)

- **Transfer Function:** The spectrum of output is obtained by taking FT of both sides of $y(t) = x(t) * h(t)$, i.e.,

$$Y(f) = X(f)H(f), \text{ or } H(f) = \frac{Y(f)}{X(f)}$$

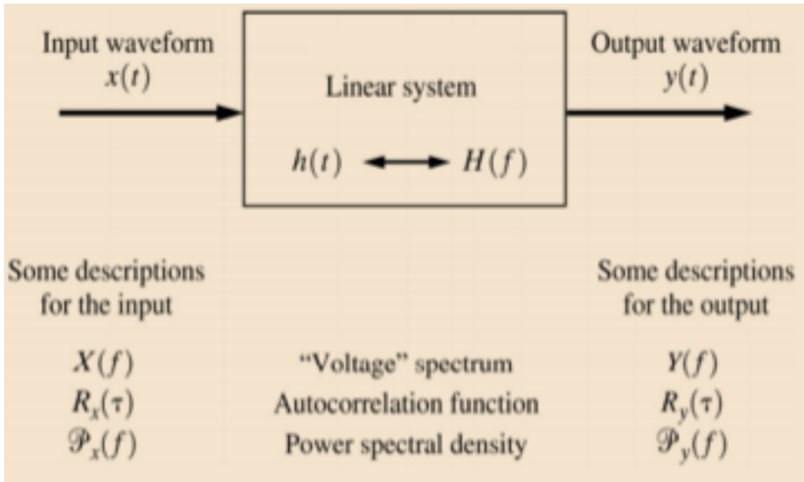
H(f) = F[h(t)] = Transfer Function *or* system frequency response

$$H(f) = |H(f)|\, e^{j\angle H(f)}, \angle H(f) = \tan^{-1}\left[\frac{\text{Im } [H(f)]}{\text{Re } [H(f)]}\right]$$

- Since h(t) is real,

$|H(f)|$ is an even function &

$\angle H(f)$ is an odd function of frequency.



Example: RC Low-Pass Filter Characterization

- In the RC LP filter,

$x(t) = Ri(t) + y(t)$, where

$$i(t) = C\frac{dy}{dt}, \text{ or } \; x(t) = RC\frac{dy}{dt} + y(t)$$

- Taking FT of both sides

$$X(f) = RC\,(j2\pi f)Y(f) + Y(f), \; or$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{1}{1 + j2\pi fRC}, \; and$$

$$h(t) = \begin{cases} \frac{1}{\tau_0} e^{-t/\tau_0}, t \geq 0, \tau_0 = RC \\ 0, t < 0 \end{cases}$$

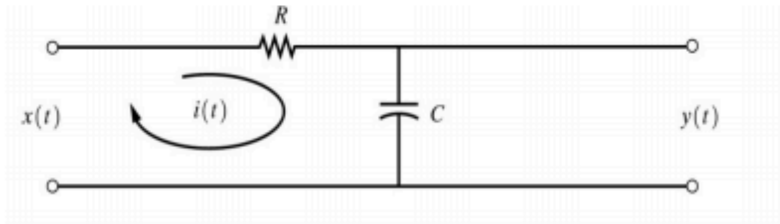
See Fourier Pair Table (Exponential one-sided)

RC=Time Constant , $f_0 = 1/(2\pi RC)$.

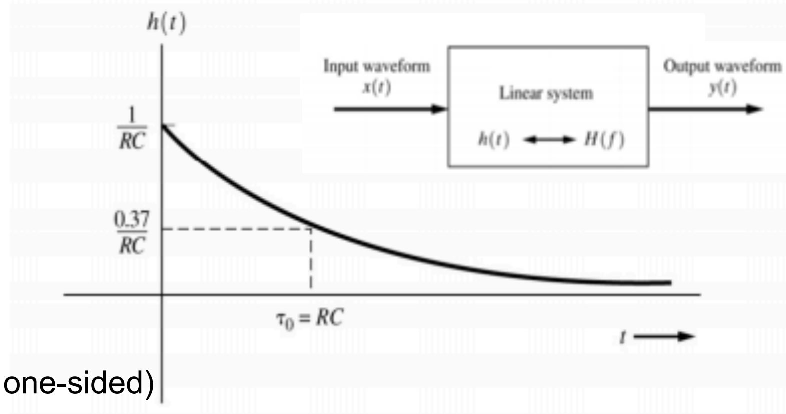
Power Transfer Function

$$G_h(f) = |H(f)|^2 = \frac{1}{1 + (f/f_0)^2}$$

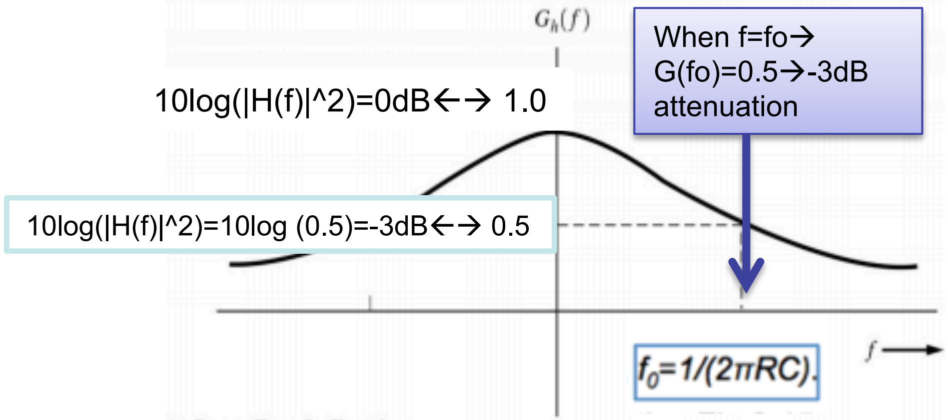
- Power gain at $f = f_0$ is $G_h(f_0) = 1/2$.
- Output is attenuated by 3-dB $f = f_0$.



(a) RC Low-Pass Filter



(b) Impulse Response



(c) Power Transfer Function

Is the Output of an RC Filter Distortionless?

Remember, for RC filter:

Power Transfer Function

$$G_h(f) = |H(f)|^2 = \frac{1}{1+(f/f_0)^2}$$

- Power gain at $f = f_0$ is $G_h(f_0) = 1/2$.
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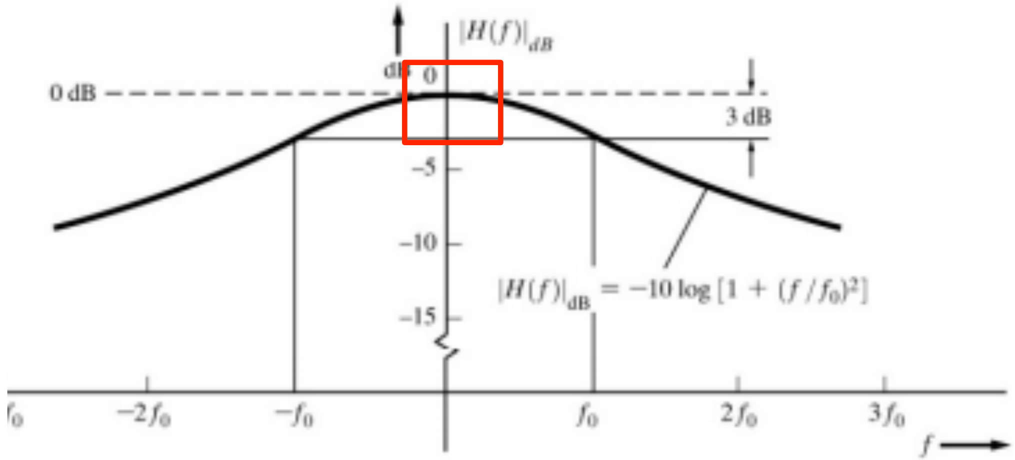


- What is the distortion effect caused by an *RC* low-pass filter?
- The amplitude & phase are

$$|H(f)| = \frac{1}{\sqrt{1+(f/f_0)^2}}, \; H(f)\Big|_{dB} = -10\log_{10}[1+(\frac{f}{f_0})^2]$$

$$\theta(f) = \angle H(f) = -\tan^{-1}(f/f_0)$$

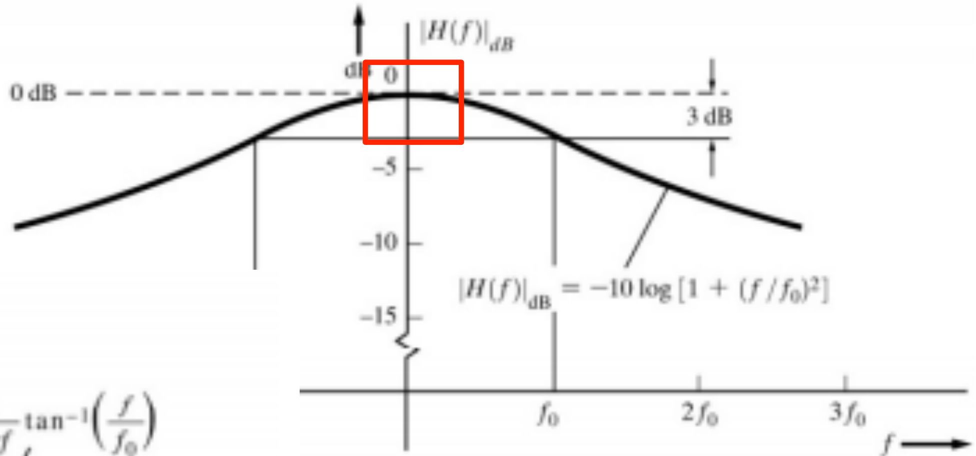
- The time delay function is

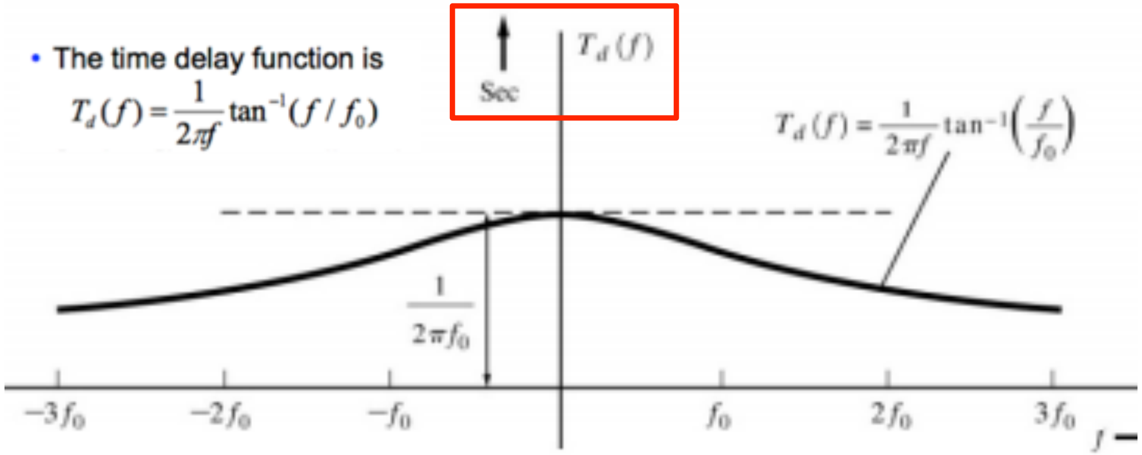
$$T_d(f) = \frac{1}{2\pi f}\tan^{-1}(f/f_0)$$

Introducing both amplitude and phase distortion!
…see next

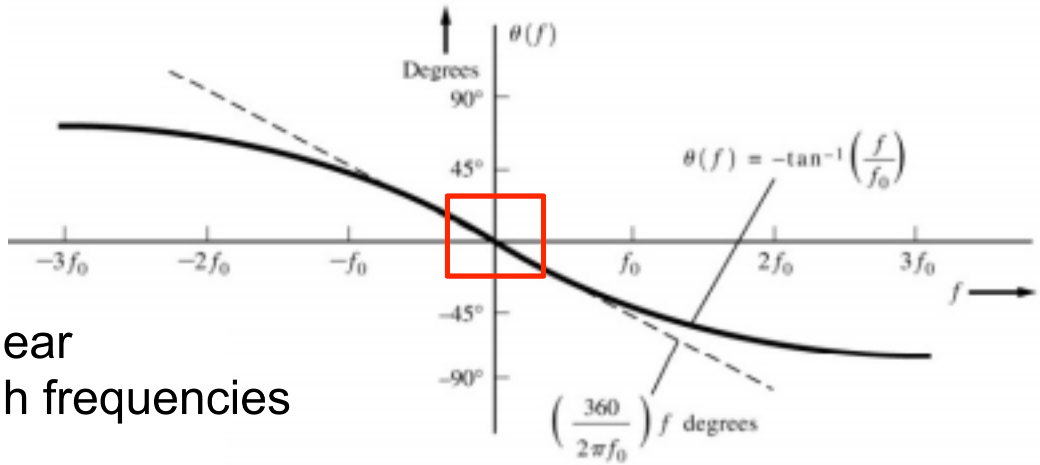
Is the Output of an RC Filter Distortionless?

Amplitude distortion if the amplitude response is not flat





Range of frequencies (<0.5fo) where (almost) no distortion occurs:
For example: If fo=10KHz, @ Td(1KHz)=1/2πfo=0.2 msec delay; producing small percentage of phase error.



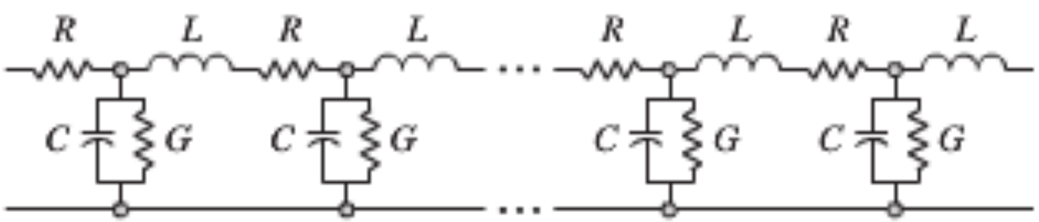
Phase response is not a linear function of frequency at high frequencies

Distortion Transmission

- An LTI system is termed **distortionless** if it introduces the same attenuation to all spectral components and offers linear phase response over the frequency band of interest
- Types of distortions
  - Amplitude
  - Group delay
  - Phase delay

Cable Characterization

- Find frequency domain characterization of twisted wire pairs (TWPs) and coaxial cables → use transmission line theory concepts
- Transmission line theory concepts:



- R = Series resistance per meter
- L = Series inductance per meter
- C = Shunt capacitance per meter
- G = Shunt conductance per meter

- The transmission line is lossless if R = G = 0
- At high frequencies $R(f) \approx c_o\sqrt{2\pi f}$
  - This is due to skin effect: tendency of high frequencies in a signal to travel near the surface of a conductor in a layer some tens of microns thick

Propagation Constant

- If voltage $x_i(t)$ is applied at the input to the transmission line at time $t=0$, the voltage along the line declines exponentially with distance over time t:

$$x(z, t) = Ae^{j2\pi ft}e^{-\gamma z} \quad \text{at time } t$$

- The propagation constant is a complex function of frequency and is given in terms the lumped-circuit model element values as

$$\gamma(f) = \alpha(f) + j\beta(f) = \sqrt{(R + j2\pi fL)(G + j2\pi fC)}$$

- where α= attenuation coefficient (it is 0 for lossless line), and β=phase shift coefficient $=2\pi/\lambda$ (radians/meter).

Characteristic Impedance

- Another important parameter of the transmission line is its characteristic impedance, Zo
- Zo is defined as the input impedance of an infinite line or that of a finite line terminated with a load impedance, $Z_L = Z_o$.
- Zo is given in terms the lumped-circuit model element values as

$$Z_o = \sqrt{\frac{R + j2\pi fL}{G + j2\pi fC}}$$

- Thus, the transfer function $H_{TWP}(f, \ell)$ is given by

ℓ is in km!!

$$H_{TWP}(f, \ell) = e^{-\gamma(f)\ell}$$

α is the real part of prop. const

- The attenuation or insertion loss is defined as the reduction or loss in signal power as it is transferred across the transmission medium & it is determined by the magnitude of its transfer function,

$$|H_{TWP}(f, \ell)| = e^{-\alpha(f)\ell}$$

TWP Attenuation

- For high frequencies: $f \geq 300\text{ kHz}, \alpha(f) \approx c_1\sqrt{f}$
- Thus, for $|H_{TWP}(f, \ell)| = e^{-\alpha(f)\ell}$
- For TWP $\alpha(f) = c_1\sqrt{f} + c_2 f$
- It turns out that $f \geq 300\text{ kHz}, \alpha(f) \approx c_1\sqrt{f}$

- c 1 and c 2 Parameters

For popular TWP Cables

Type	c_1	c_2
Cat 3	4.31×10^{-3}	4.26×10^{-7}
Cat 4	3.89×10^{-3}	4.82×10^{-7}
Cat 5	3.83×10^{-3}	2.41×10^{-8}
AWG 26	4.8×10^{-3}	-1.71×10^{-8}
AWG 24	3.8×10^{-3}	-0.54×10^{-8}
AWG 22	3.0×10^{-3}	0.035×10^{-8}

- Thus, the attenuation of TWP is usually expressed in dB as

$$\text{Insertion Loss} = -20\log_{10}|H_{TWP}(f, \ell)| = -20\log_{10}e^{-\alpha(f)\ell} = 8.686\alpha(f)\ell \text{ dB}$$

- Note: where f and ℓ , are specified in Hz and miles, respectively.

TWP Attenuation

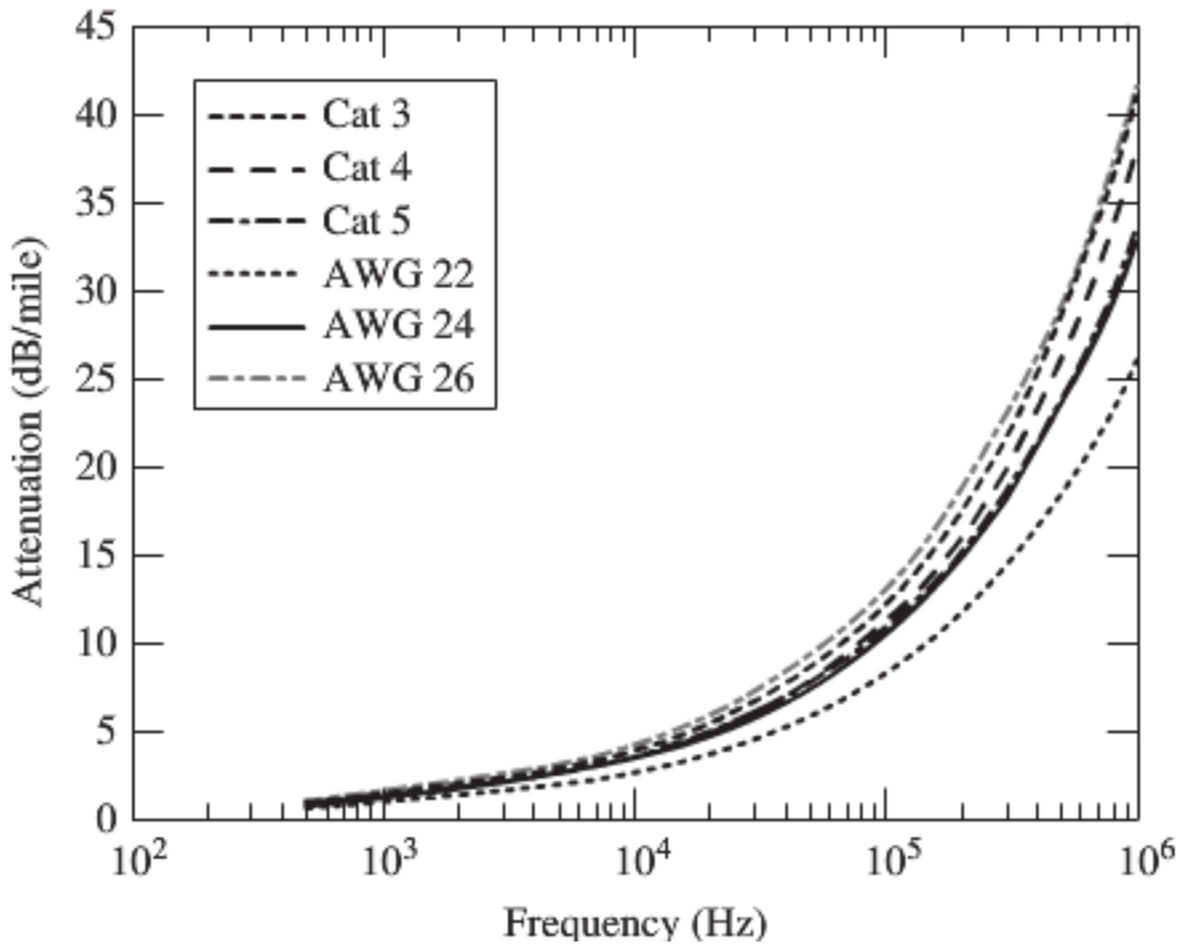
- Remember:

$$\text{Insertion Loss} = -20\log_{10}|H_{TWP}(f,\ell)| = -20\log_{10}e^{-\alpha(f)\ell} = 8.686\alpha(f)\ell \text{ dB}$$

- Note that larger ℓ

→ Results in more Loss

→ limiting the BW!



Example 1:

- Consider a 2-pair 24-AWG TWP
- We often use this cable to connect the subscribers & limited to18,000 feet drops (about 5.45 km).
- Determine the output of 6-km TWP for an input sinusoidal signal 5 5cos(6800πt), that is f=3200 Voice signal

$$C = 0.05\ \mu\text{F/km}$$
$$L = 0.673\ \text{mH/km}$$
$$R = 180\ \text{ohms/km}$$
$$G = 0$$

Prop. Constant

$$\gamma(f)\big|_{f=3.4\text{ kHz}} = \sqrt{(180 + j6.8\pi \times 0.673)(0 + j6.8\pi \times 0.05 \times 10^{-3})}$$
$$= 0.2979 + j0.3227 = 0.4392\measuredangle 47.3^\circ$$

Transfer Function

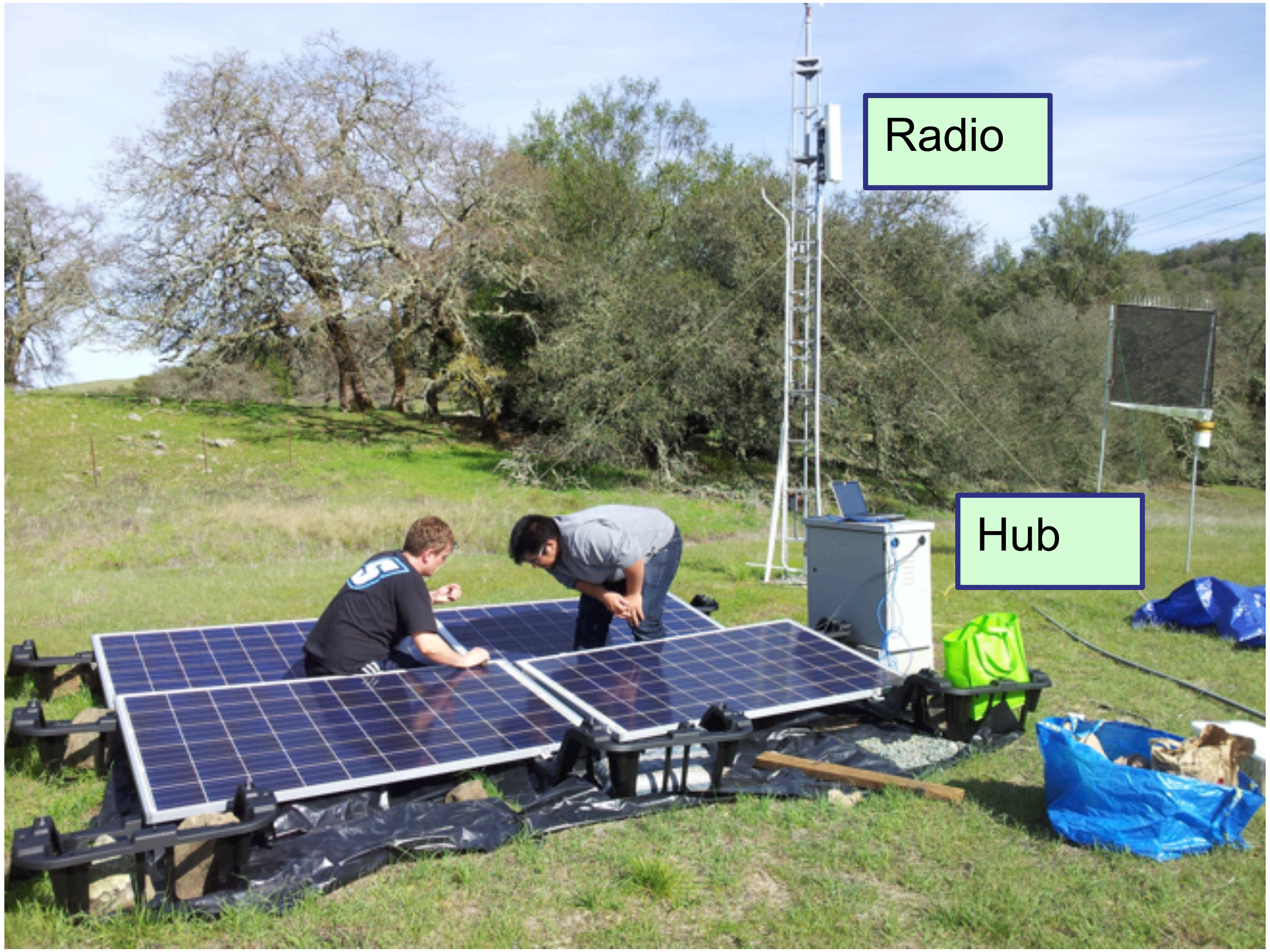
$$H_{TWP}(f)\big|_{f=3.4\text{ kHz}} = e^{-6\times\gamma(f)|_{f=3.4\times10^3}} = e^{-(0.2979+j0.3227)\times 6}$$
$$= -0.0598 - j0.1563 = 0.1674\measuredangle -110.9^\circ$$

Output Signal

$$y(t) = \left|H_{TWP}(f)\right|_{f=3.4\text{ kHz}} \times 5\cos\left[6800\pi t + \measuredangle H_{TWP}(f)\big|_{f=3.4\text{ kHz}}\right]$$
$$= 5 \times 0.1674\cos(6800\pi t - 110.9^\circ) = 0.837\cos(6800\pi t - 110.9^\circ)$$

Example 2:

If the maximum run length using Category (Cat) 5 TWP from the antenna to the hub is about 100 feet, what is the expected power level at the hub assuming the radio outputs 250 mW at 100 MHz?



Example 2:

If the maximum run length using Category (Cat) 5 TWP from the antenna to the hub is about 100 feet, what is the expected power level at the hub assuming the radio outputs 250 mW at 100 MHz?

- The attenuation of a Cat 5 TWP is given by

$$\text{Insertion Loss} = 8.686 \times 3.83 \times 10^{-3} \times \sqrt{10^8}\ \text{dB/mile} = 8.686 \times 38.3 = 332.67\ \text{dB/mile}$$

- Thus:

$$\ell = 100 \text{ foot drop of Cat 5 TWP cable} = 332.67 \times 100/5000 = 6.65 \text{ dB.}$$

- Power at the radio is 250mW → 24 dBm
- Thus at the hub, the power will be 24 – 6.65 = 17.35 dBm or

$$\text{dBm} = 10^{17.35/10} = 54 \text{ mW}$$

References

- M. Farooque Mesiya, Contemporary Communication Systems, 2012 – Chapter 2
- Leon W. Couch II, Digital and Analog Communication Systems, 8th edition, Pearson / Prentice, Chapter 1
- Electronic Communications System: Fundamentals Through Advanced, Fifth Edition by Wayne Tomasi – Chapter 2

  (https://www.goodreads.com/book/show/209442.Electronic_Communications_System)