Chapters 4 & 5

AM Modulation

3/25/15

Outline

- Complex Envelope Representation of Bandpass Waveforms,
- Representation of Modulated Signals,
- Spectrum of Bandpass Signals,
- Evaluation of Power,
- Bandpass Filtering and Linear Distortion,
- Bandpass Sampling Theorem,
- Received Signal Plus Noise,
- Classification of Filters and Amplifiers,
- Nonlinear Distortion, Limiters, Mixers, Up Converters, and Down Converters,
- Frequency Multipliers, Detector Circuits, Phase-Locked Loops and Frequency Synthesizers, Transmitters and Receivers,
- Software Radios?

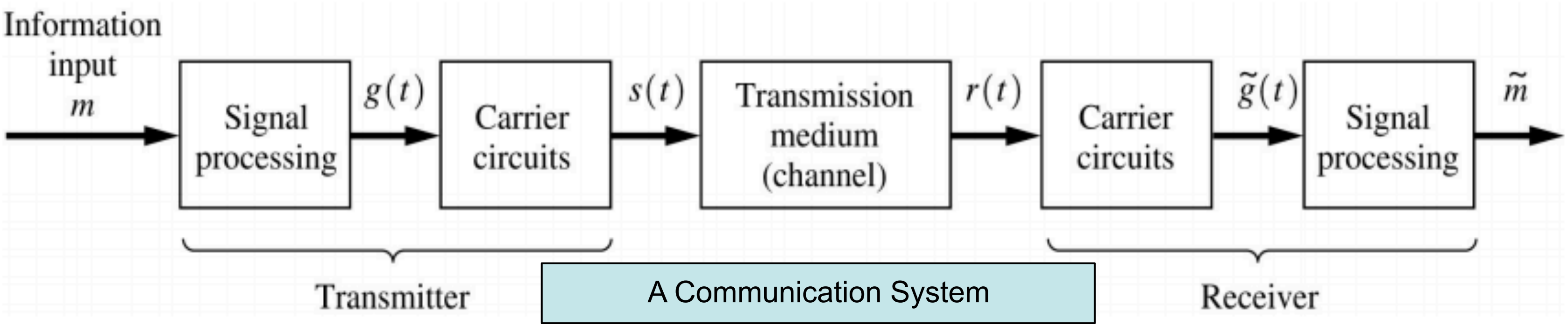
Baseband & Bandpass Waveforms

- A baseband waveform has a spectral magnitude that is nonzero for freq in the vicinity of the origin (f=0) and negligible elsewhere.
  - It is a signal whose range of freq is measured from zero to a maximum bandwidth
  - E.g., an audio signal from a microphone, a TTL signal from a digital circuit.
- A bandpass waveform has a spectral magnitude that is nonzero for freq in some band concentrated about a freq $f = \pm f_c$.
  - The spectral magnitude is negligible elsewhere.
  - f_c is called carrier freq.
  - E.g., An AM radio signal that broadcast news over f_c=850 kHz is a bandpass signal

Why Modulation?

- In order to transfer signals we need to transfer the frequency to higher level
- One approach is using modulation
- Modulation:
  - Changing the amplitude of the carrier
- AM modulation is one type of modulation
  - Easy, cheap, low-quality
  - Used for AM receiver and CBs (citizen bands)
  - Generally high carrier frequency is used to modulate the voice signal (300 – 3000 Hz)

Baseband & Bandpass Waveforms, Modulation



A Communication System

- Modulation is the process of imparting the source information onto a bandpass signal with carrier freq f_c using amplitude or phase perturbation (or both).
  - The bandpass signal is called modulated signal s(t).
  - The baseband signal is called modulating signal m(t).
- Bandpass communication signal is obtained by modulating a baseband analog or digital signal on a carrier.
  - Whereas baseband signal cannot go far, a bandpass signal goes a long distance.

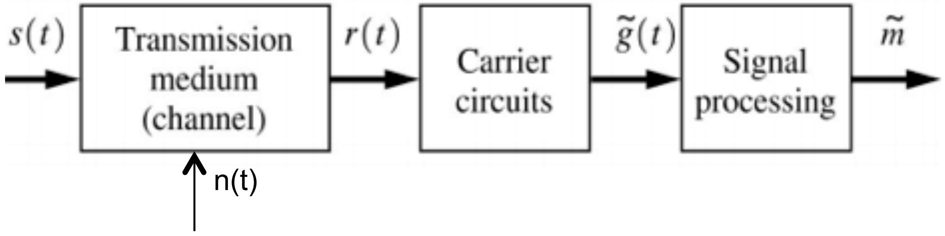
Complex Envelope Representation

- A physical bandpass waveform can be represented by $v(t) = \text{Re}\{g(t)e^{jw_c t}\}$
  - where g(t) is called the complex envelope of v(t), $\omega_c = 2\pi f_c$.

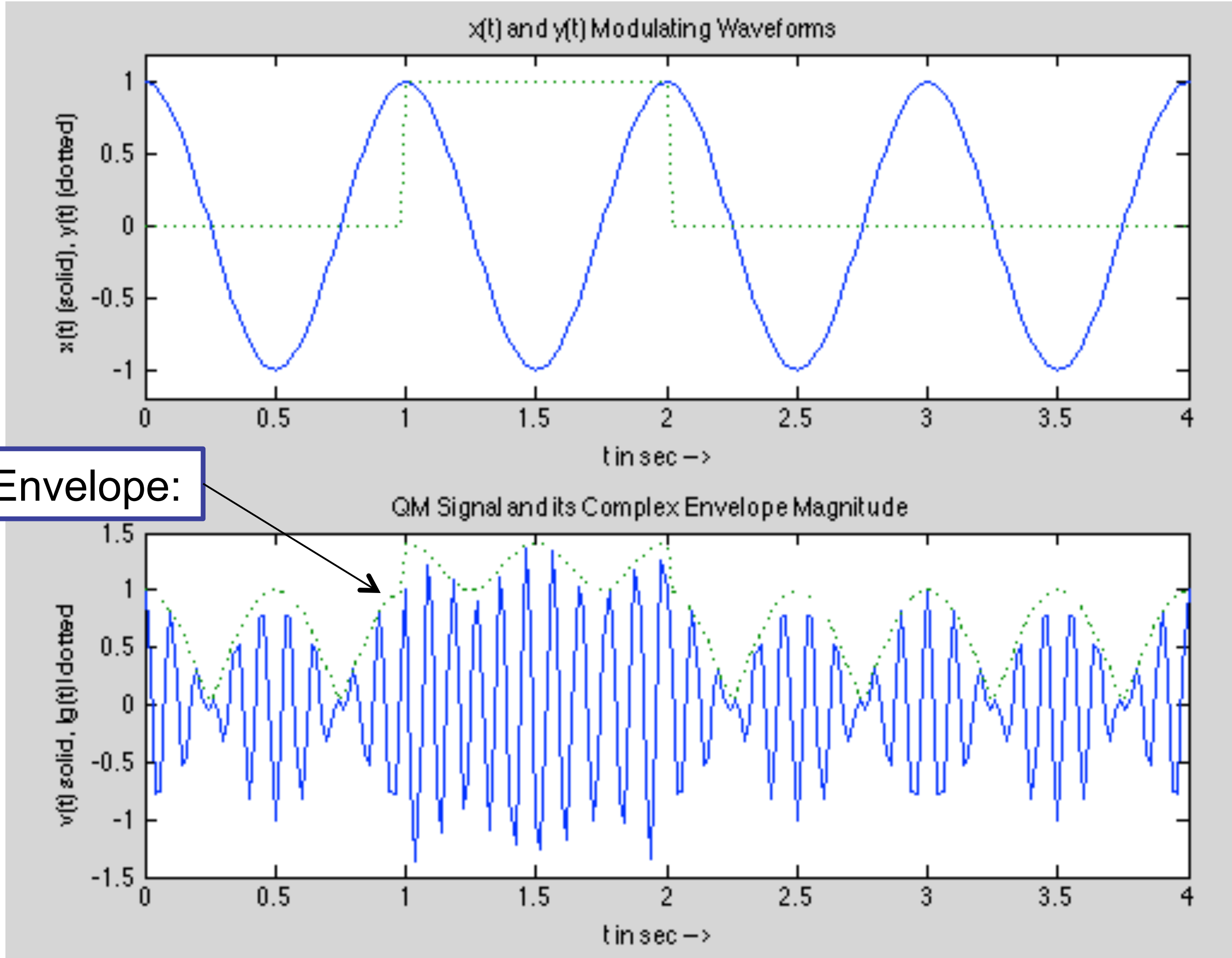
$$g(t) = x(t) + jy(t) = |g(t)|e^{j\angle g(t)} = R(t)e^{j\theta(t)}$$

  - $e^{jw_c t}$ factor shifts (translates) the spectrum of the baseband *g(t)* signal from baseband up to carrier freq f_c.
  - *R(t)* is said to be amplitude modulation (AM) on *v(t)*.
  - *θ(t)* is said to be phase modulation (PM) on *v(t)*.

s(t) = v(t)



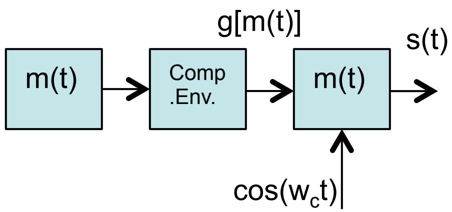
Modulating & Modulated Signals



Complex Envelope:

Representation of Modulated Signal

- Modulation is the process of encoding the source information *m(t)* into a bandpass signal *s(t)*.
- the modulated signal is an application of bandpass representation, i.e., $s(t) = \mathrm{Re}\left\{g(t)e^{jw_c t}\right\}$
- *The complex envelope g(t)* is a function of the modulating signal *m(t)*, i.e., $g(t) = g[m(t)]$
  - E.g., for AM modulation, $g[m(t)] = A_c[1 + m(t)]$



Let's find FT, PSD, and P_{v_norm} of v(t)!

Spectrum of Bandpass Signal

Theorem: If the bandpass waveform is represented by $v(t) = \mathrm{Re}\{g(t)e^{jw_c t}\}$ then the spectrum of the bandpass waveform is

$$V(f) = \frac{1}{2}[G(f - f_c) + G^*(-f - f_c)], \quad \& \quad PSD = P_v(f) = \frac{1}{4}[P_g(f - f_c) + P_g(-f - f_c)]$$

Proof for *V(f)*:

$$v(t) = \mathrm{Re}\{g(t)e^{jw_c t}\} = \frac{1}{2}g(t)e^{jw_c t} + \frac{1}{2}g^*(t)e^{-jw_c t}, \ \&$$

$$V(f) = F[v(t)] = \frac{1}{2}F[g(t)e^{jw_c t}] + \frac{1}{2}F[g^*(t)e^{-jw_c t}] = \frac{1}{2}[G(f - f_c) + G^*(-f - f_c)]$$

where we used the fact that $F[g^*(t)] = G^*(-f)$

s(t) = v(t)

Note: Re{a+jb}=(a+jb)/2 + (a-jb)/2 = a

Power Evaluation

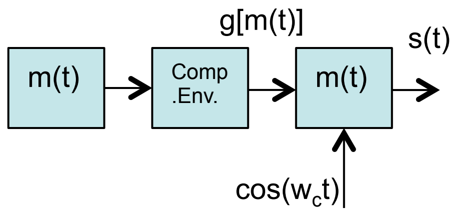
- The total average normalized power of bandpass waveform v(t) is

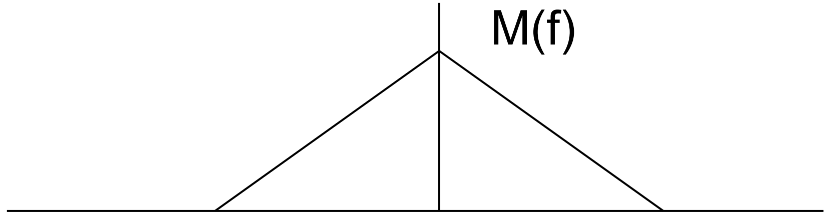
$$P_v = \langle v^2(t) \rangle = \int_{-\infty}^{\infty} \mathcal{P}_v(f)\, df = R_v(0) = \boxed{\frac{1}{2} \langle |g(t)|^2 \rangle}$$

Let's look at an example!

Example: Spectrum of Amplitude Modulated Signal

- Assume the complex envelop $g[m(t)] = A_o[1+m(t)]$
- Thus, $s(t) = A_o[1+m(t)]\cos(w_c t)$





- Find the mathematical expression for S(f) and |S(f)| for all f using the given M(f):
  - Find S(f)
  - Find |S(f)|
  - Normalized power $P_s = P_v$

Example: Spectrum of Amplitude Modulated Signal

AM Modulation

Evaluate the magnitude spectrum for an AM signal with the complex envelope $g[m(t)]=A_c[1+m(t)]$.

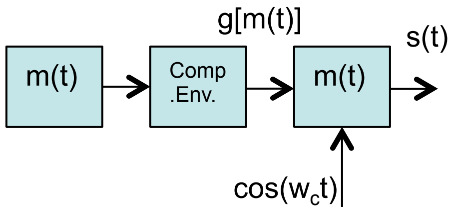
Solution: The spectrum of complex envelope is $G(f) = A_c\delta(f) + A_c M(f)$

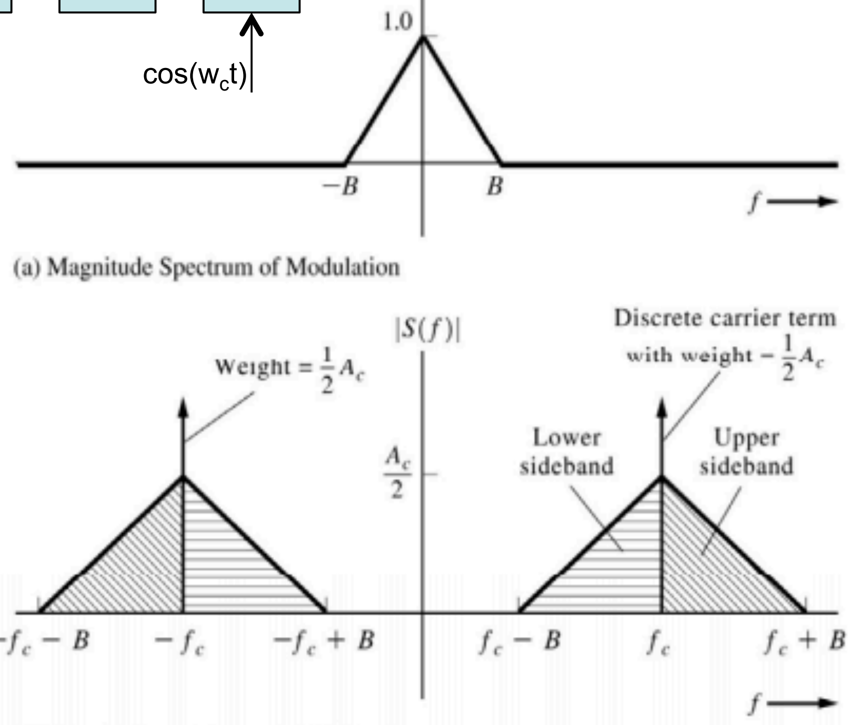
$$s(t) = \text{Re}\{g(t)e^{jw_c t}\} = A_c[1+m(t)]\cos\omega_c t$$

$$S(f) = \frac{A_c}{2}[\delta(f-f_c) + M(f-f_c) + \delta(f+f_c) + M(f+f_c)]$$

where because m(t) is real, $M^*(f)=M(-f)$ & $\delta(f)=\delta(-f)$ is even.

$$|S(f)| = \begin{cases} \frac{A_c}{2}\delta(f-f_c) + \frac{A_c}{2}|M(f-f_c)|, f>0 \\ \frac{A_c}{2}\delta(f+f_c) + \frac{A_c}{2}|M^*(-f-f_c)|, f<0 \end{cases}$$





(a) Magnitude Spectrum of Modulation

(b) Magnitude Spectrum of AM Signal

- The 1 in $g(t)= A_c[1+m(t)]$ causes extra delta functions to occur in spectrum at $f= \pm f_c$.

Example: Spectrum of Amplitude Modulated Signal

- Total average signal power

$$P_v = \frac{1}{2} A_c^2 \langle |1+m(t)|^2 \rangle = \frac{1}{2} A_c^2 \langle 1+2m(t)+m^2(t) \rangle$$

$$= \frac{1}{2} A_c^2 \left[1 + 2\langle m(t) \rangle + \langle m^2(t) \rangle \right]$$

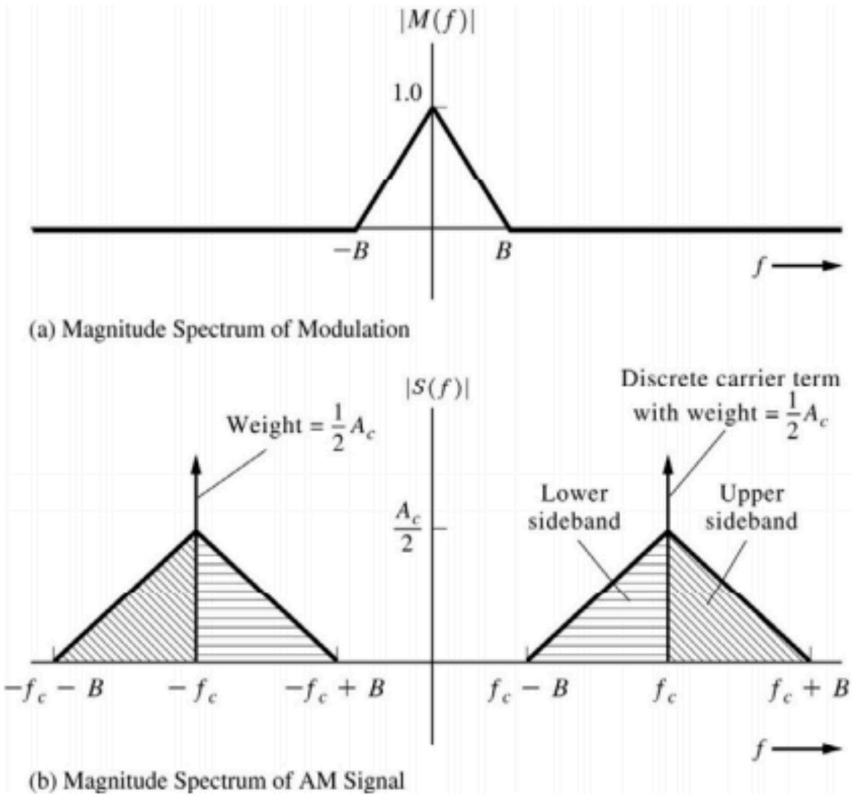
- If we assume that DC value of modulation is zero, then <m(t)>=0.

— Average signal Power = $P_v = \frac{1}{2} A_c^2 [1 + P_m]$

— Power in the modulation $m(t) = P_m = \langle m^2(t) \rangle$

— Carrier Power $= \frac{1}{2} A_c^2$

— Power in the sidebands of $s(t) = \frac{1}{2} A_c^2 P_m$



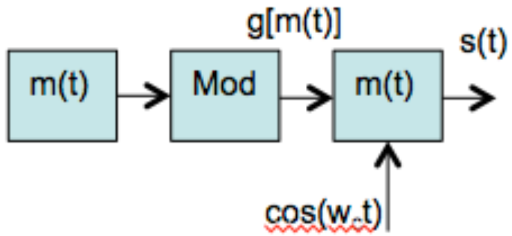
1) Note that $s(t) = \text{Re}\{g(t)e^{jw_c t}\} = A_c[1+m(t)]\cos\omega_c t$

2) $\langle v(t)^2 \rangle$ for periodic sinusoidal functions results in half power

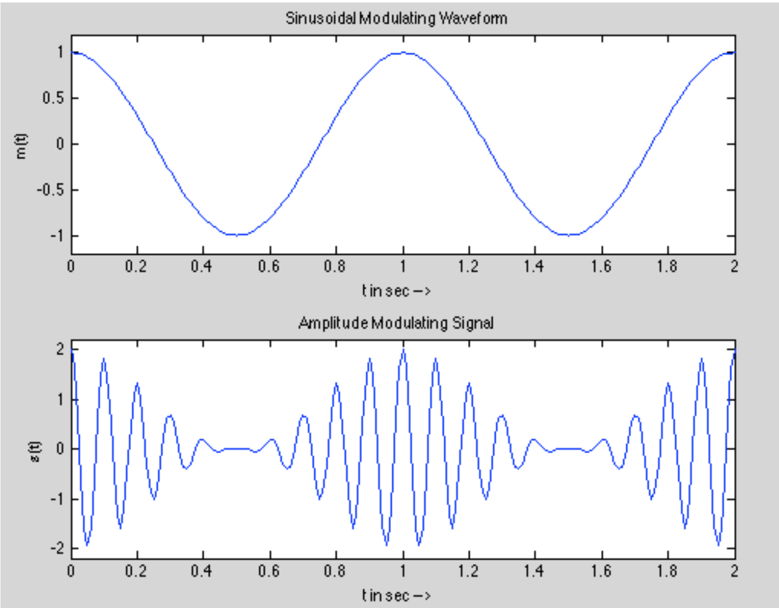
3) $m = \langle m(t)^2 \rangle$

Amplitude Modulation

Evaluate the magnitude spectrum for an AM signal with the complex envelope *g[m(t)]=A$_c$[1+m(t)]*.




```
m = cos(wa*t);   % Sinusoidal Modulating Waveform
m = m(:);
j = sqrt(-1);
g = 1 + m;
carrier = exp(j*wc*t);
g = g(:);
carrier = carrier(:);
s = real(g.*carrier);   % Amplitude Modulating Signal
```



Assume:

$m(t) = \cos(w_a t)$

$g[m(t)]=g(t)=1+m(t)$

$s(t) = Re\{g(t).e^{\wedge}jw_c t\}$

$= g(t).\cos(w_c t)$

$= [1+m(t)].\cos(w_c t)$

More About AM Modulation

- In AM modulation the carrier signal changes (almost) linearly according to the modulating signal - m(t)
- AM modulating has different schemes
  - Double-sideband Full Carrier (DSB-FC)
    - Also called the Ordinary AM Modulation (AM)
  - Double-sideband suppressed carrier (DSB-SC)
  - Single-sideband (SSB)
  - Vestigial Sideband (VSB) – Not covered here!

More About AM Modulation

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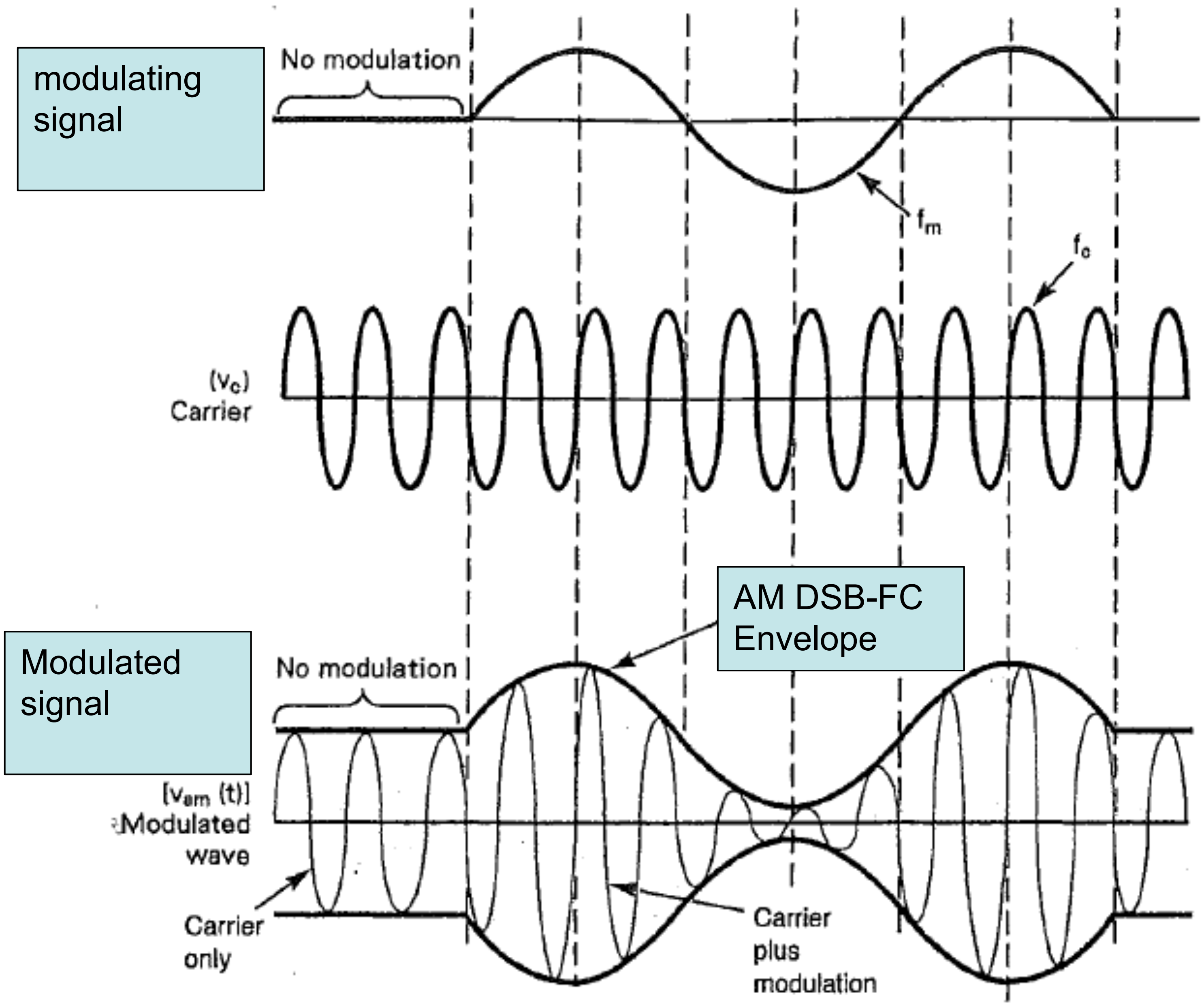
Let's focus on the simplest case!

We generally assume the Modulating Signal is Sinusoidal

AM Modulation (DSB-FC)



The result is a modulated signal!

Review: Bandpass Signal

- Remember for bandpass waveform we have

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\}$$

- The voltage (or current) spectrum of the bandpass signal is

$$S(f) = \tfrac{1}{2}[G(f-f_c) + G^*(-f-f_c)]$$

- The PSD will be

$$\mathcal{P}_s(f) = \frac{1}{4}[\mathcal{P}_g(f-f_c) + \mathcal{P}_g(-f-f_c)]$$

- In case of Ordinary AM (DSB – FC) modulation:

$$g(t) = A_c[1 + m(t)]$$

- In this case Ac is the power level of the carrier signal with no modulation;
- Therefore: $s(t) = A_c[1 + m(t)] \cos \omega_c t$

Make sure you know where these come from!

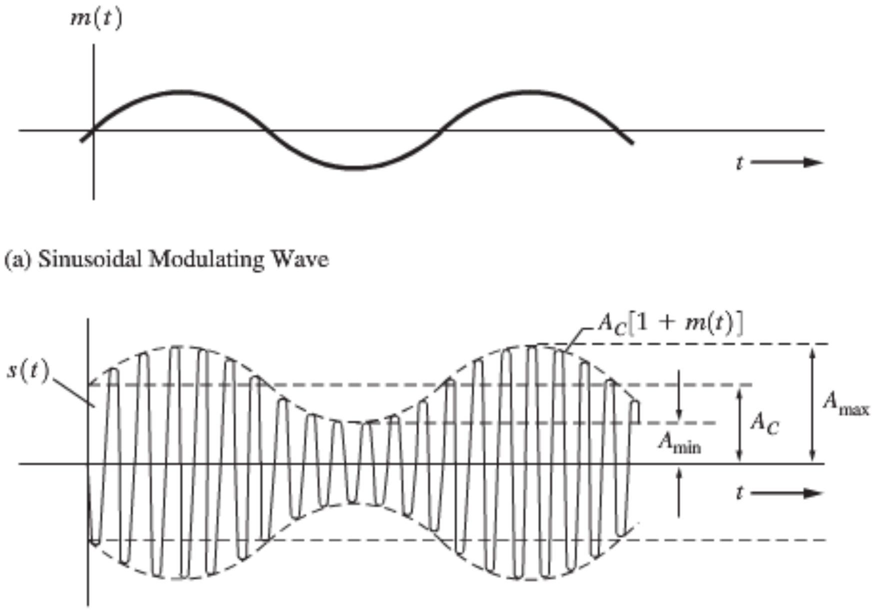
AM: Modulation Index

- Modulation Percentage (m)

$$\% \text{modulation} = \frac{A_{max} - A_{min}}{2A_c} \times 100 = \frac{\max[m(t)] - \min[m(t)]}{2} \times 100$$

- Note that m(t) has peak amplitude of $A_m = mE_m = mA_c$
- We note that for ordinary AM modulation,
  - if the modulation percentage > %100, implying $m(t) < -1$,
  - Therefore →

$$s(t) = \begin{cases} A_c[1 + m(t)] \cos w_c t, & \text{if } m(t) \geq -1 \\ 0, & \text{if } m(t) < -1 \end{cases}$$

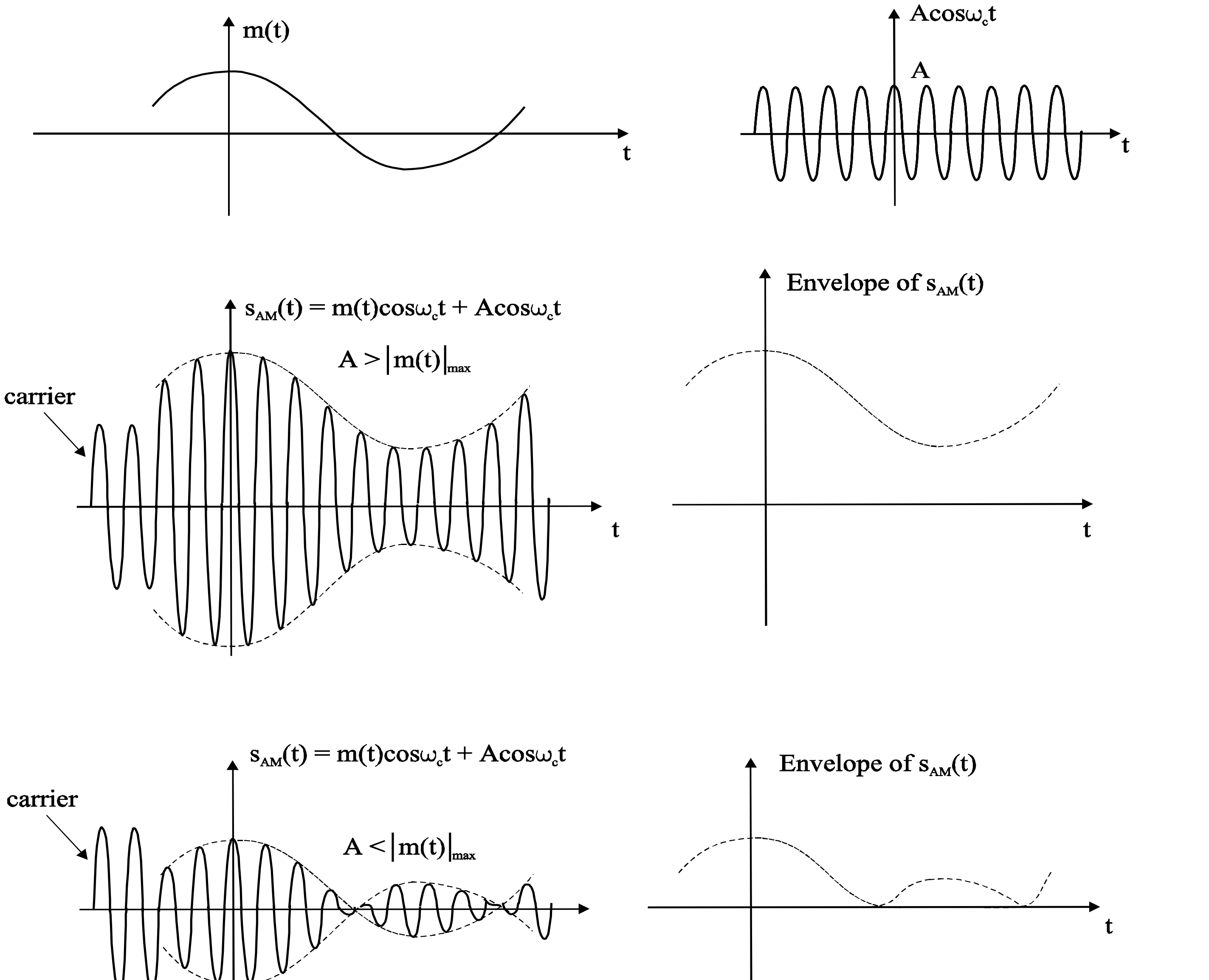


(a) Sinusoidal Modulating Wave

(b) Resulting AM Signal

$$m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

AM: Modulation Index



if the modulation percentage > %100

AM: MATLAB Model

- This is how we generate the ordinary AM using MATLAB

```
fc = 10;                % carrier frequency
fa = 1;                 % modulating frequency
N  = 200;               % number of samples
To = 4;                 % observation time: To x periods
MI = 1;                 % Modulation Index (0.0-2.0 or 0 to 200 percent)
Ec = 1;                 % Ec is the level of the AM envelope in the
                        % absence of modulation, when m(t) = 0;

Ta = 1/fa;
dt = To*Ta/N;
wc = 2*pi*fc;
wa = 2*pi*fa;

t = 0:dt:To*Ta;      % simulation time

m = MI*cos(wa*t);    % modulating signal: m(t)
m = m(:);

y = zeros(length(t),1); % In this part we force [1+m] = 0 if
for (i = 1:1:length(t)) %
  if (m(i) > -1)        % in other words, we ensure [1+m(t)]=0 if
    y(i) = 1;           % m(t) < -1
  end;
end;
```

AM: Normalized Average Power

- Normalized Average Power
- Note that

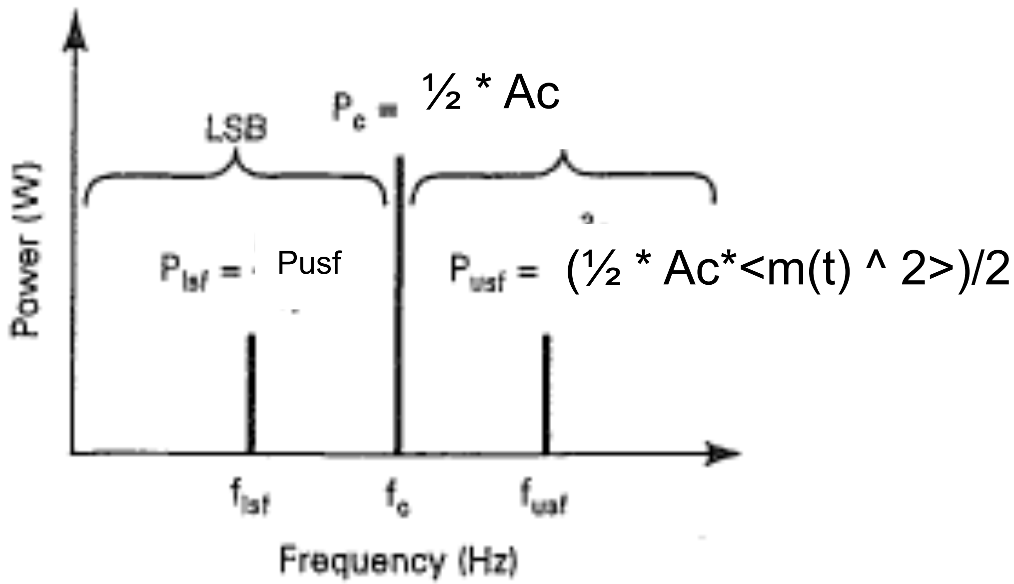
$$\langle s^2(t)\rangle = \tfrac{1}{2}\langle |g(t)|^2\rangle = \tfrac{1}{2}A_c^2\langle [1+m(t)]^2\rangle$$
$$= \tfrac{1}{2}A_c^2\langle 1 + 2m(t) + m^2(t)\rangle$$
$$= \tfrac{1}{2}A_c^2 + A_c^2\langle m(t)\rangle + \tfrac{1}{2}A_c^2\langle m^2(t)\rangle$$

$$\langle s^2(t)\rangle = \underbrace{\tfrac{1}{2}A_c^2}_{\text{discrete carrier power}} + \underbrace{\tfrac{1}{2}A_c^2\langle m^2(t)\rangle}_{\text{sideband power}}$$

(Total)→ each sideband will have half the power!

- Pc is the normalized carrier power(1/2)Ac^2
- The rest is the power of each side band (lower sideband or LSB & USB)
- Thus:

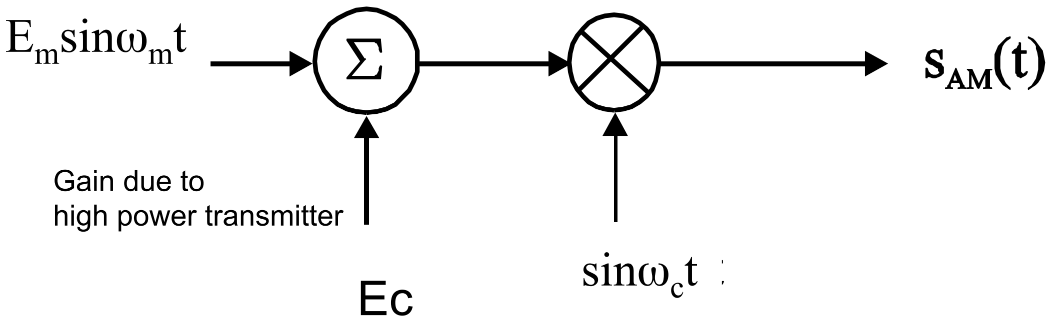
Note that we are assuming Load Resistance, R=1.



A Practical Example:

Ordinary AM Mathematical Expression

- In this case:
  - $Vc(t) = Ec \sin\omega_c t$; Carrier signal
  - $Vm(t) = Em \sin\omega_m t$; Modulating signal
  - $V_{AM}(t) = S_{AM}(t) = Ec \sin\omega_c t + Em \sin\omega_m t \cdot \sin\omega_c t$; AM modulated signal



Modulation Index!

  - $V_{AM}(t) = [Ec + Em \sin\omega_m t\ ]. \sin\omega_c t = [1 + m.\sin\omega_m t\ ]. Ec. \sin\omega_c t$

Amplitude of the modulated Wave

Constant + Modulated Signal

Modulated Carrier

Assume Em = mEc; where 0<m<1 → m is called the modulation index, or percentage modulation!

AM Modulation and Modulation Index

- Rearranging the relationship:

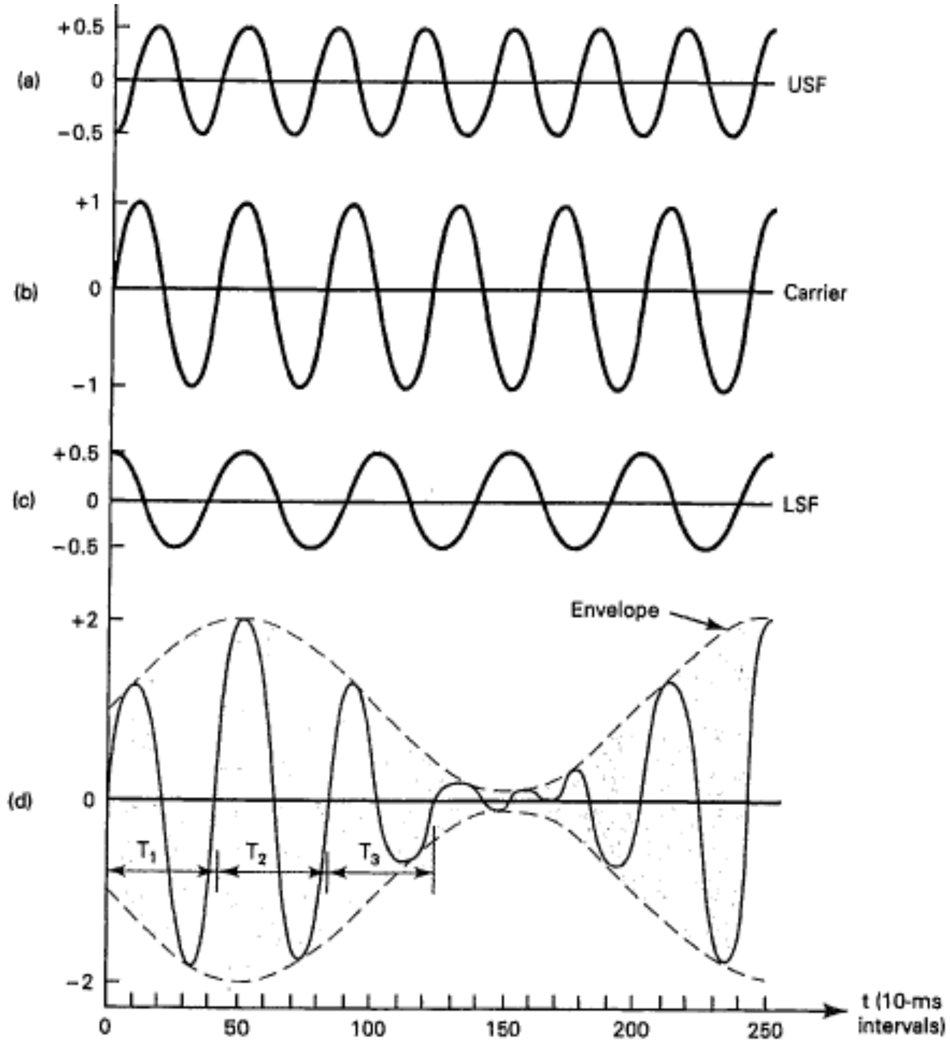
$$v_{am}(t) = E_c \sin(2\pi f_c t) + [mE_c \sin(2\pi f_m t)][\sin(2\pi f_c t)]$$

$$v_{am}(t) = E_c \sin(2\pi f_c t) - \frac{mE_c}{2}\cos[2\pi(f_c + f_m)t] + \frac{mE_c}{2}\cos[2\pi(f_c - f_m)t]$$

- In this case we have:
  - | Carrier | + | LSB | + | USB |
- Note that for m=1 (modulation percentage of 100 percent)
  - V_{am_max} = Ec + mEc = 2Ec
  - V_{am_min} = 0 ;

AM Modulation and Modulation Index

$$v_{am}(t) = E_c \sin(2\pi f_c t) - \frac{mE_c}{2}\cos[2\pi(f_c + f_m)t] + \frac{mE_c}{2}\cos[2\pi(f_c - f_m)t]$$



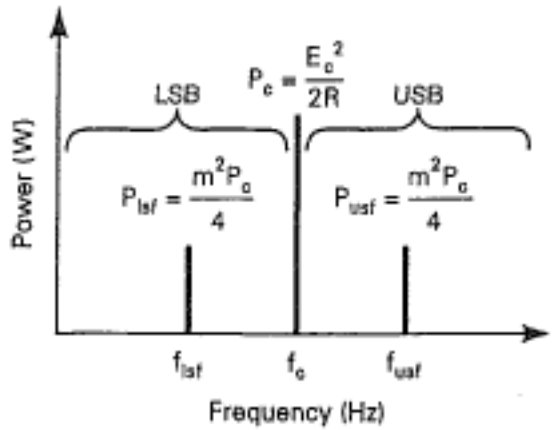
AM Power Distribution

- $P = E^2/2R = Vp^2/2R$; R = load resistance
- Remember: Pavg $Vrms^2/R$; where Vrms for sinusoidal is Vp/sqrt(2)

$$v_{am}(t) = E_c \sin(2\pi f_c t) - \frac{mE_c}{2}\cos[2\pi(f_c + f_m)t] + \frac{mE_c}{2}\cos[2\pi(f_c - f_m)t]$$

- $P_{carrier\_average} = Ec^2/2R$
- $P_{usb\_average} = (mEc/2)^2/2R = (m^2/4)Pc$
- $P_{total} = P_{carrier\_average} + P_{usb\_average} + P_{lsb\_average}$

What happens as m increases?



Current Analysis

- Measuring output voltage may not be very practical, that is measuring Vp in **P = Vp²/2R** is difficult in across an antenna!
- However, measuring the current passing through an antenna may be more possible: Total Power is $P_T = I_T^2 R$

Total power → P_t, Carrier power → P_c

$$\frac{P_t}{P_c} = \frac{I_t^2 R}{I_c^2 R} = \frac{I_t^2}{I_c^2} = 1 + \frac{m^2}{2}$$

$$\frac{I_t}{I_c} = \sqrt{1 + \frac{m^2}{2}}$$

$$I_t = I_c\sqrt{1 + \frac{m^2}{2}}$$

Note that we can obtain m if we measure currents!

Examples (5A, 5C)

General Case: m(t) can be any bandpass

AM: Modulation Efficiency

- Defined as the percentage of the total power of the modulated signal that conveys information
  - Signal Power/Total Power

$$s(t) = A_c[1 + m(t)] \cos \omega_c t$$

- Defined as: $$E = \frac{\langle m^2(t) \rangle}{1 + \langle m^2(t) \rangle} \times 100\%$$

Note that $\langle m^2(t) \rangle = m^2 . P_{ns}$; P_{ns} is the normalized signal power

- Normalized Peak Envelop Power is defined as

$$P_{PEP} = (A_c^2 / 2) * (1 + A_{max})^2$$

(when load resistance R=1)

- We use P_{PEP} to express transmitter output power.
- In general, Normalized Peak Envelop Power, P_{PEP} ,can be expressed as follow: $$\frac{1}{2} \max \{|g(t)|^2\}$$

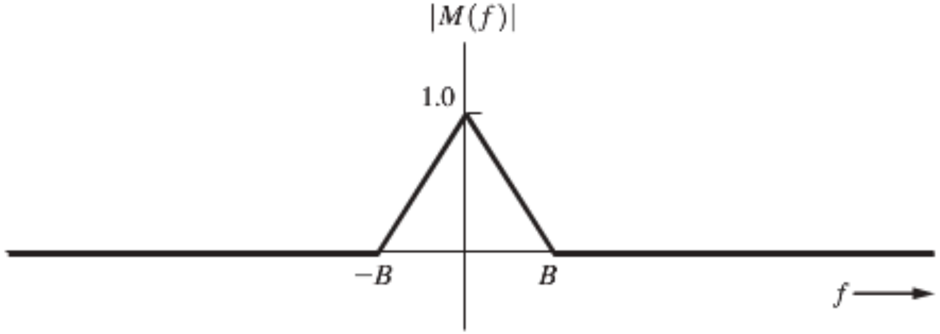
Example (5B)

- Assume Pc_avg = 5000 W for a radio station (un-modulated carrier signal); If m=1 (100 modulation index) with modulated frequency of 1KHz sinusoid find the following:
  - Peak Voltage across the load (Ac)
  - Total normalized power ($\langle s(t)^2 \rangle$)
  - Total Average (actual) Power
  - Normalized PEP
  - Average PEP
  - Modulation Efficiency – Is it good?

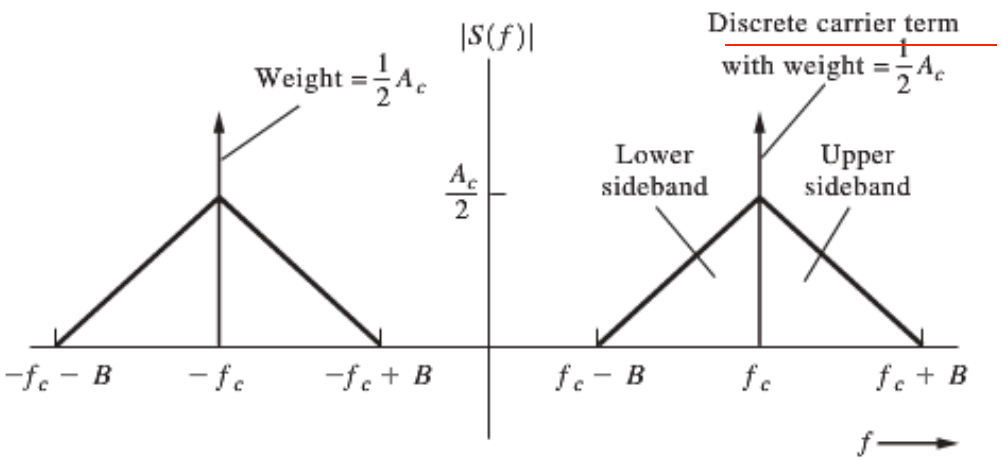
AM: Voltage and Current Spectrum Issues

- We know for AM: $s(t) = A_c[1 + m(t)] \cos \omega_c t$
- The voltage or Current Spectrum will be

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + M(f - f_c) + \delta(f + f_c) + M(f + f_c)]$$



(a) Magnitude Spectrum of Modulation



(b) Magnitude Spectrum of AM Signal

Note that BW is 2B – doubled compared to M(f) →

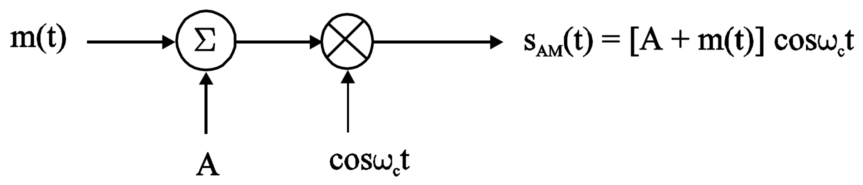
1- Large bandwidth requirement

2- Duplicated Information in Upper and Lower Sides

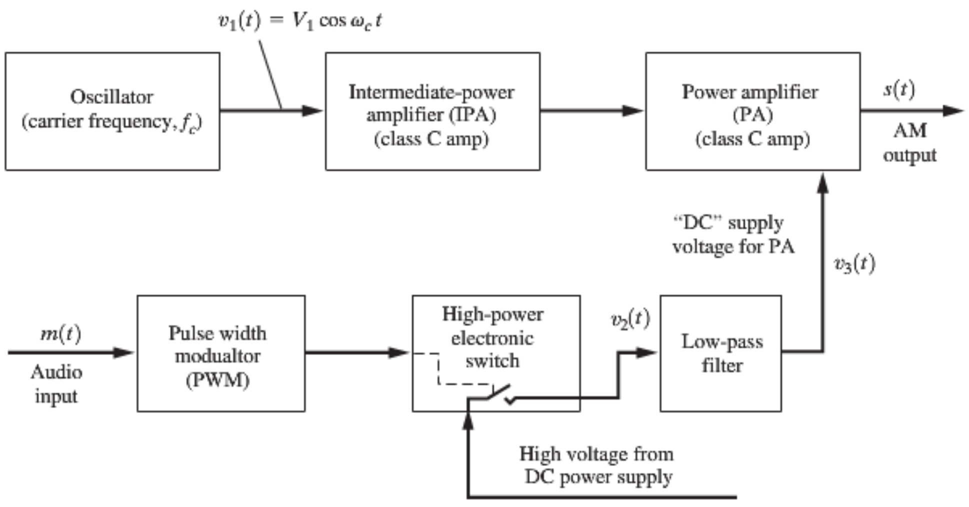
3- We are wasting power to send the discrete carrier power

Building an Ordinary AM Modulator

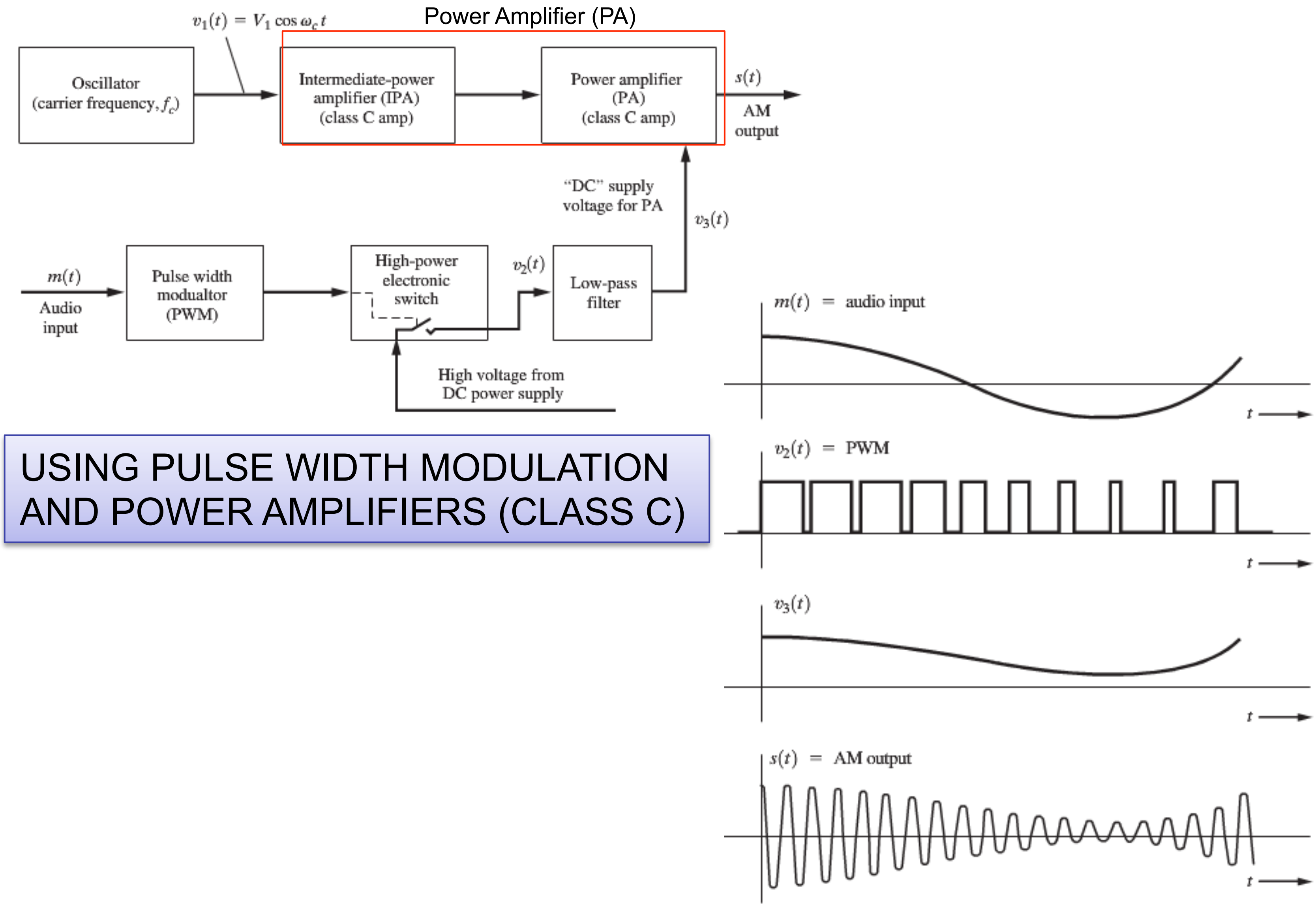
- Transferring AC power to RF power!
- Two general types
  - Low power modulators
  - High power modulators
- Low Power Modulators
  - Using multipliers and amplifiers
  - Issue: Linear amplifiers must be used; however not so efficient when it comes to high power transfer



- High Power Modulators
  - Using PWM



Building an Ordinary AM Modulator



USING PULSE WIDTH MODULATION AND POWER AMPLIFIERS (CLASS C)

Double Sideband Suppressed Carrier

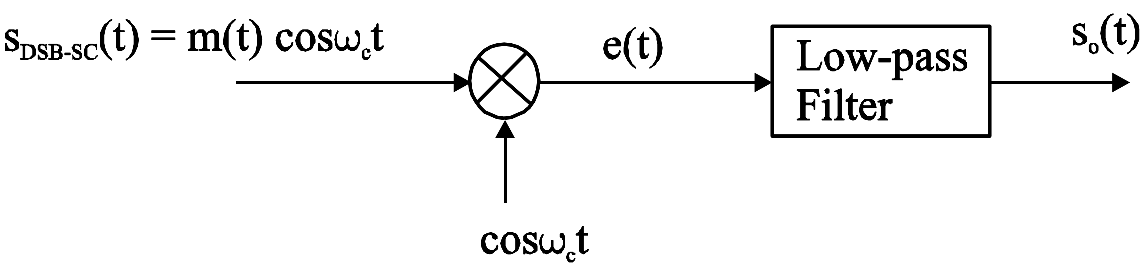
- DSB-SC is useful to ensure the discrete carrier signal is suppressed:

$$s(t) = A_c m(t) \cos \omega_c t$$

- The voltage or current spectrum of DSB-SC will be

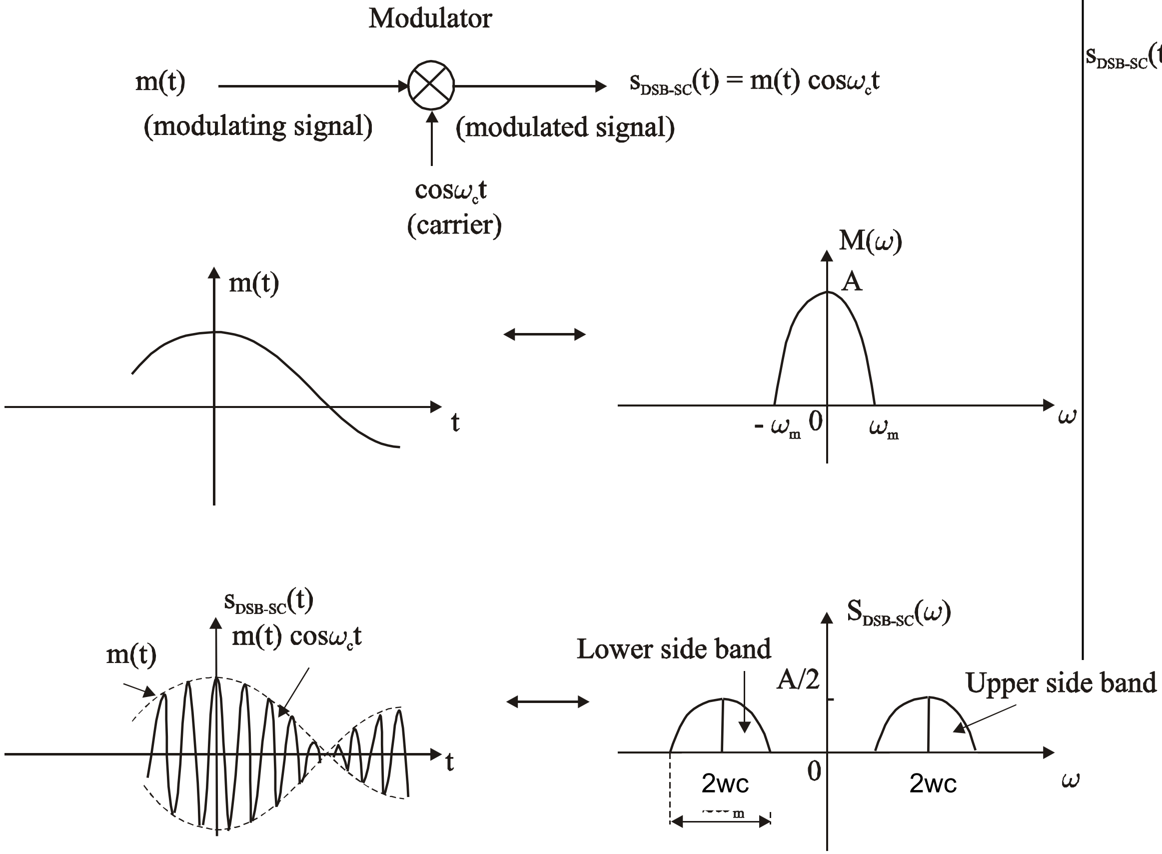
$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$

- Therefore no waste of power for discrete carrier component !
- What is the modulation efficiency? → 100 Percent!
  - Effic = $\langle m(t)^2 \rangle / \langle m(t)^2 \rangle$
  - percentage of the total power of the modulated signal that conveys information
- DSB-SC:

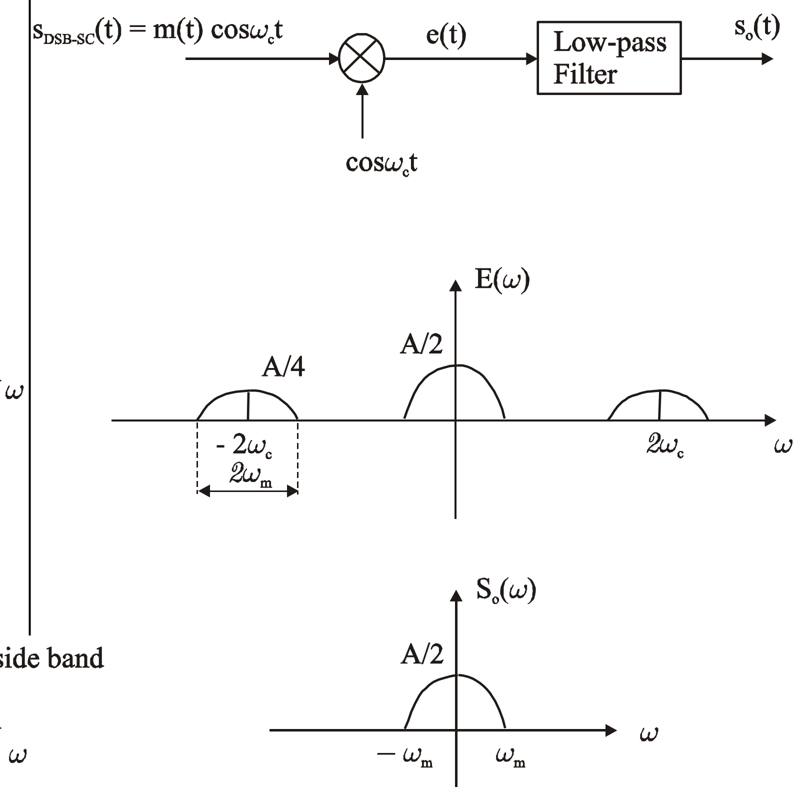


DSB-SC – Modulation & Coherent Demodulation

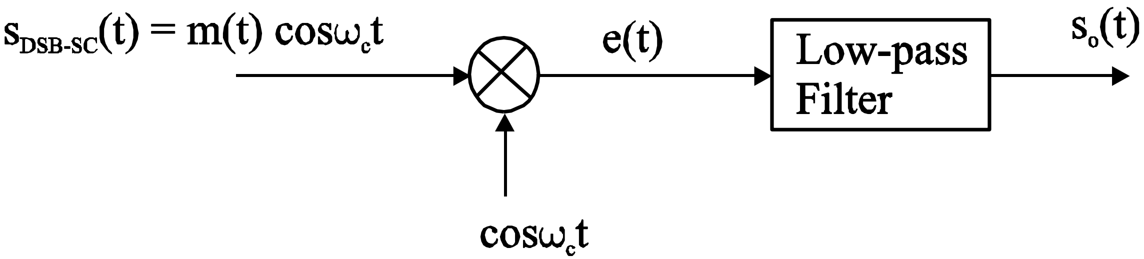
Modulation



Demodulation



DSB-SC – Coherent Demodulation



Multiplying the signal $m(t)\cos\omega_c t$ by a local carrier wave $\cos\omega_c t$

$$e(t) = m(t)\cos^2\omega_c t = (1/2)[m(t) + m(t)\cos 2\omega_c t]$$

$$E(\omega) = (1/2)M(\omega) + (1/4)[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$$

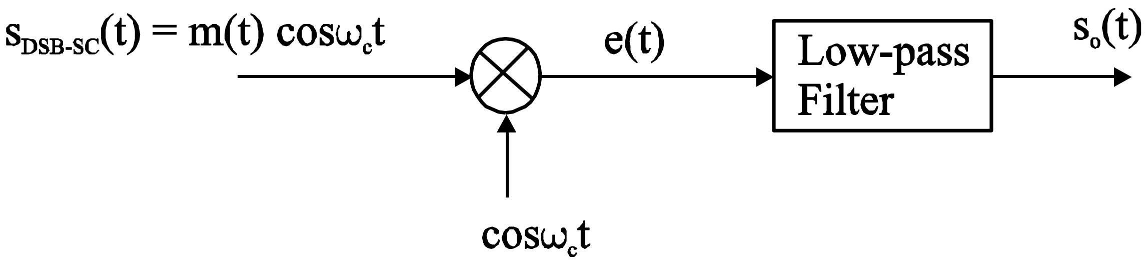
Passing through a low pass filter: $S_o(\omega) = (1/2)M(\omega)$

The output signal: $s_o(t) = (1/2)m(t)$

The issue is how to keep the same exact fc on modulator & demodulator!
→The coherent demodulator must be synchronized with the modulator both in frequency and phase!
BUT…what if it is not?

DSB-SC – Coherent Demodulation Issues

So, what if the Local Oscillator frequency is a bit off with the center frequency ($\Delta\omega$)?



Multiplying the signal $m(t)\cos\omega_c t$ by a local carrier wave $\cos[(\omega_c+\Delta\omega)t]$

$$e(t) = m(t)\cos\omega_c t \,.\, \cos[(\omega_c+\Delta\omega)t]$$
$$= (1/2)[m(t)] \,.\, \{\cos[\omega_c t - (\omega_c+\Delta\omega)t] + \cos[\omega_c t + (\omega_c+\Delta\omega)t]\}$$
$$= (1/2)[m(t)] \,.\, \{\cos(\Delta\omega t) + \cos(2\omega_c+\Delta\omega)t\}$$
$$= m(t)/2 \,.\, \cos(\Delta\omega t)$$ ← The beating factor (being distorted)

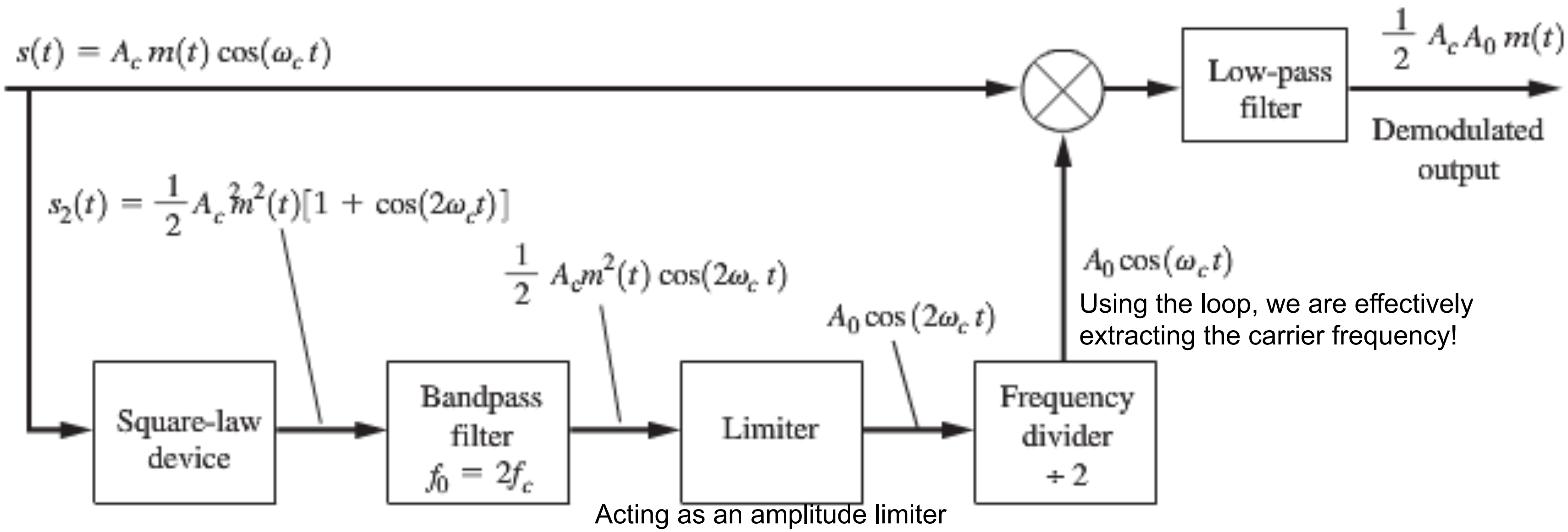
The coherent demodulator must be synchronized with the modulator both in frequency and phase!

Disadvantages:

1. **It transmits both sidebands which contain identical information and thus waste the channel bandwidth resources;**
2. **It requires a fairly complicated (expensive) circuitry at a remotely located receiver in order to avoid phase errors.**

Demodulation DSB-SC

- One common approach to eliminate phase error impact is using Squaring Loop:



Note that in this case the initial phase must be known!

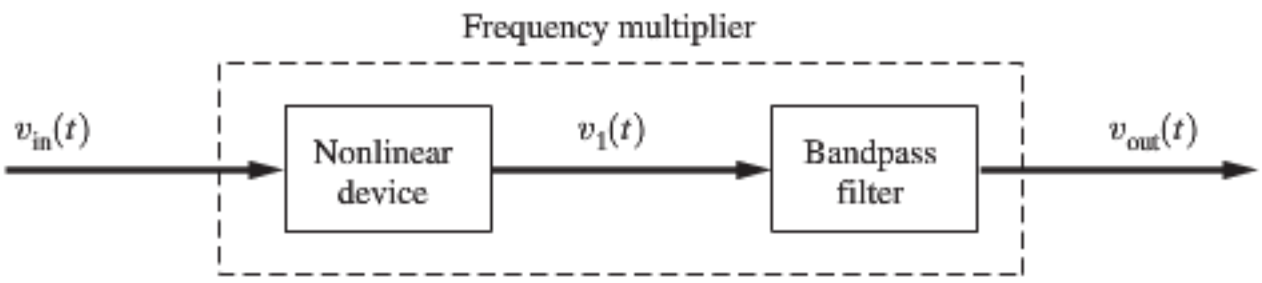
Building AM Modulators

- AM Modulating Circuits are categorized as
  - Low-level Transmitters
  - Medium-level Transmitters
  - High-level Transmitters

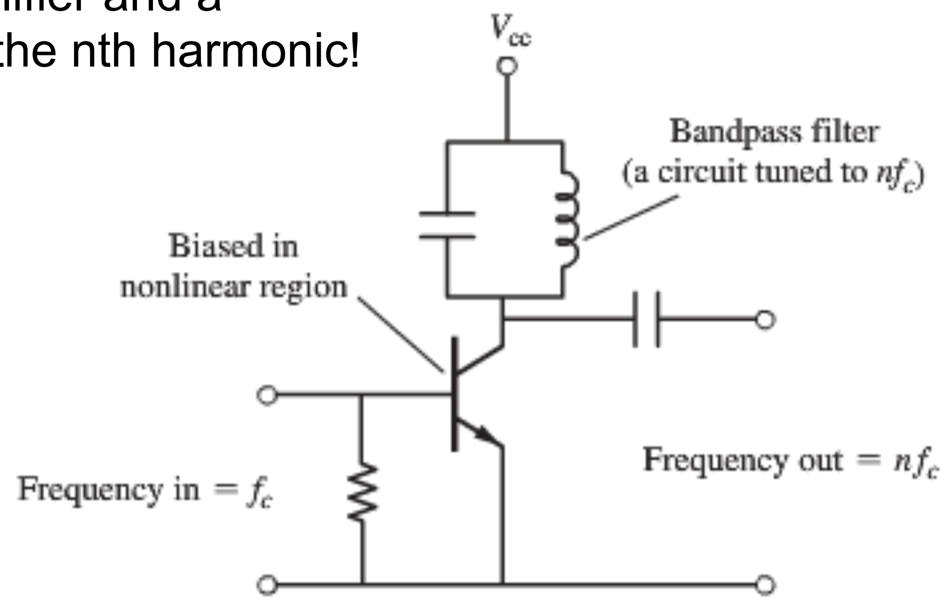
Other Key Components

- Mixers & Multipliers
- Phase shifter
  - RC
  - Inverters
- Amplifiers
  - Linear
  - Nonlinear

AM Modulators: Frequency Multiplier

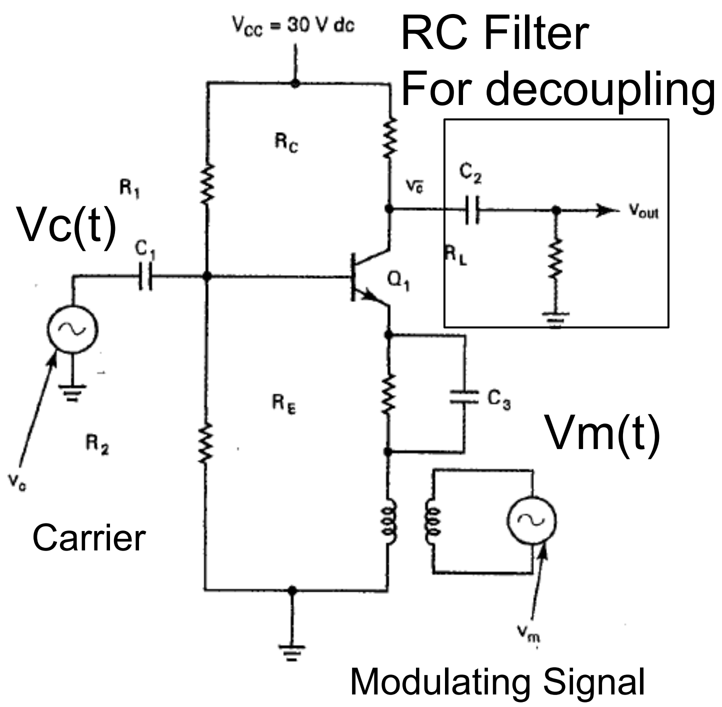


Nonlinear amplifier and a
filter to extract the nth harmonic!



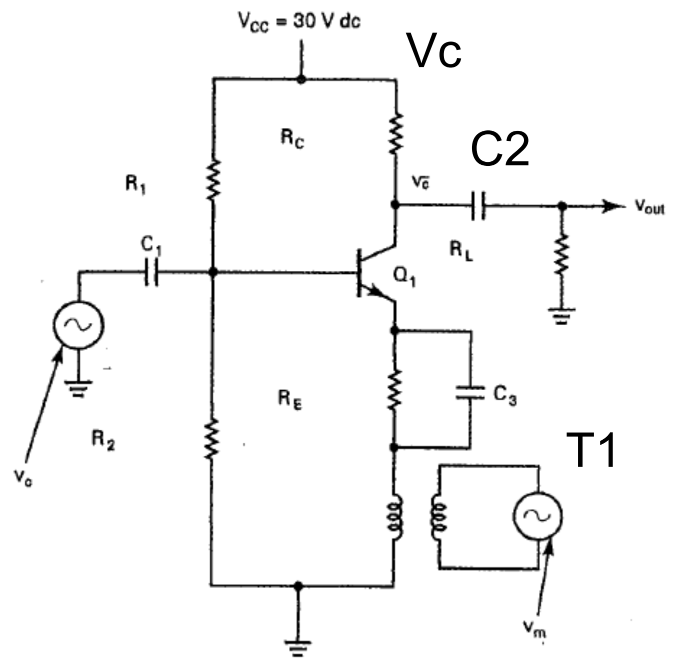
Low-Level AM Modulators

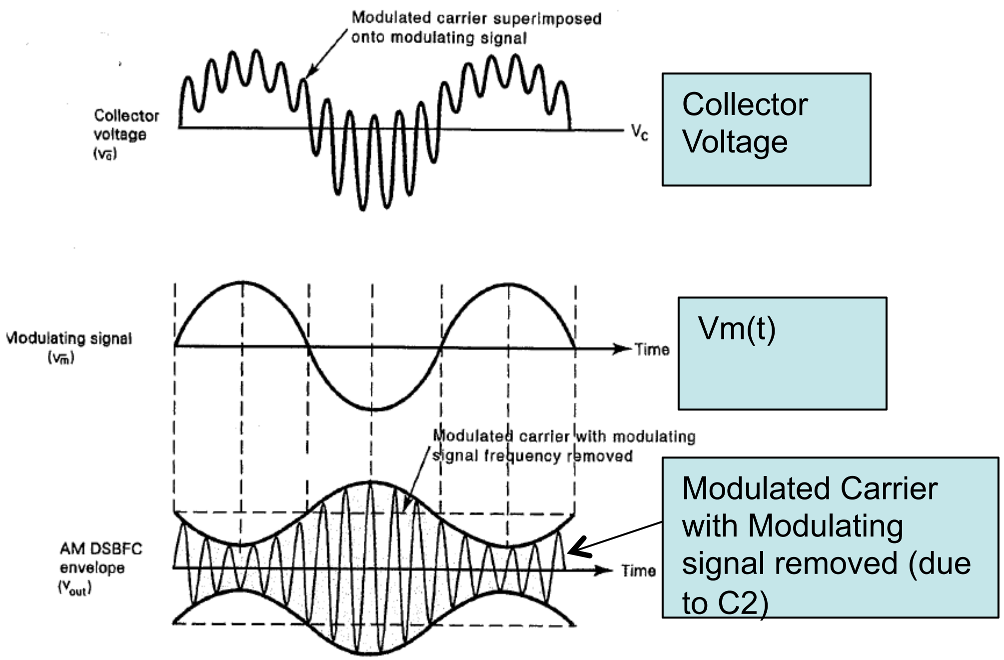
- Mainly for low-power applications
- Requires less modulating signal power to achieve high *m*
- Uses an Emitter Modulator (low power)
  - Incapable of providing high-power
- The amplifier has two inputs: *Vc(t)* and *Vm(t)*
- The amplifier operates in both linear and nonlinear modes –
  - HOW? See next slide!



Low-Level AM Modulators – Circuit Operation

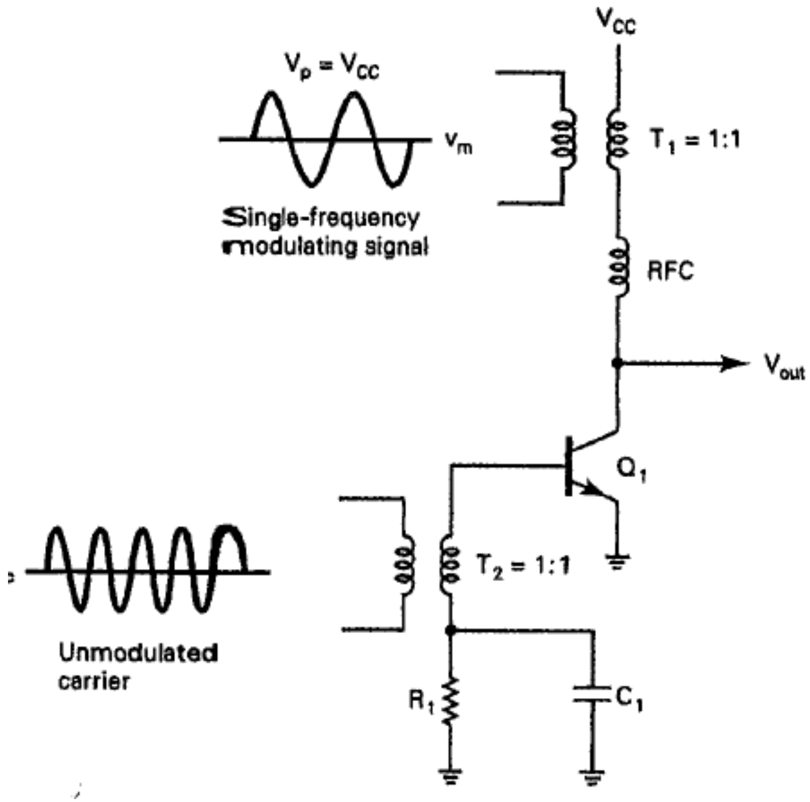
- If Vm(t) =0 → amplifier will be in **linear** mode
  - → Aout=$V_c$cos(w_ct); Vc is voltage gain – collector voltage (unit less)
- If Vm(t) >0 → amplifier will be in **nonlinear** mode
  - → Aout=[V_c + $V_m$cos(w_ct)] cos(w_ct)
- Vm(t) is isolated using T1
  - The value of Vm(t) results in Q1 to go into cutoff or saturation modes
- C2 is used for coupling
  - Removes modulating frequency from AM waveform





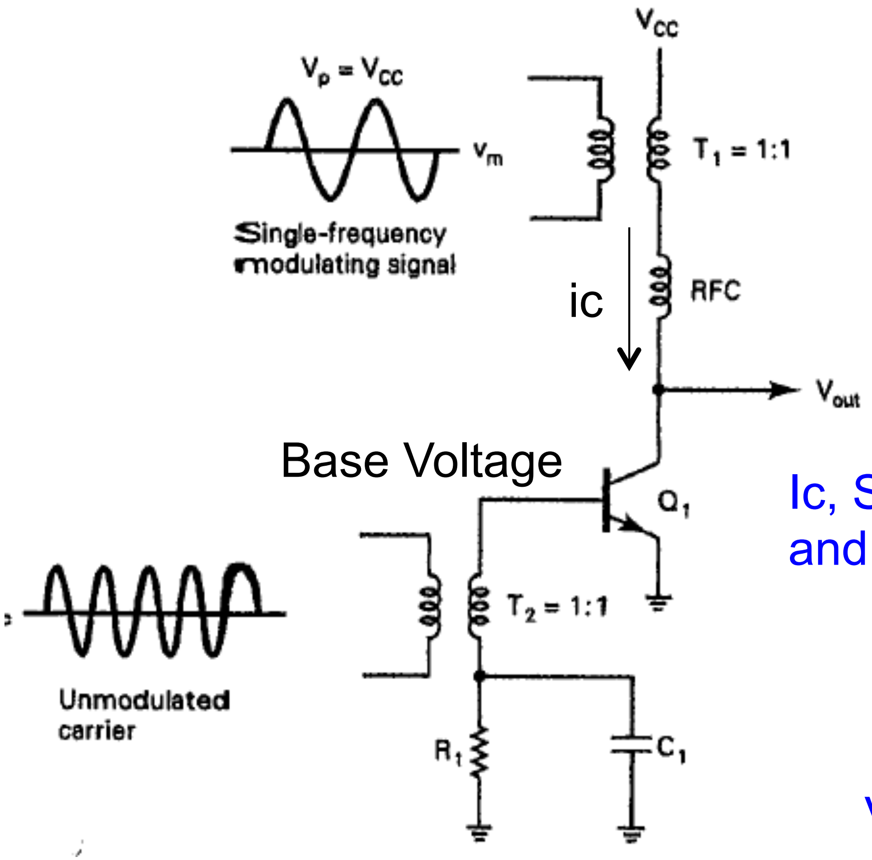
High-Level AM Modulators – Circuit Operation

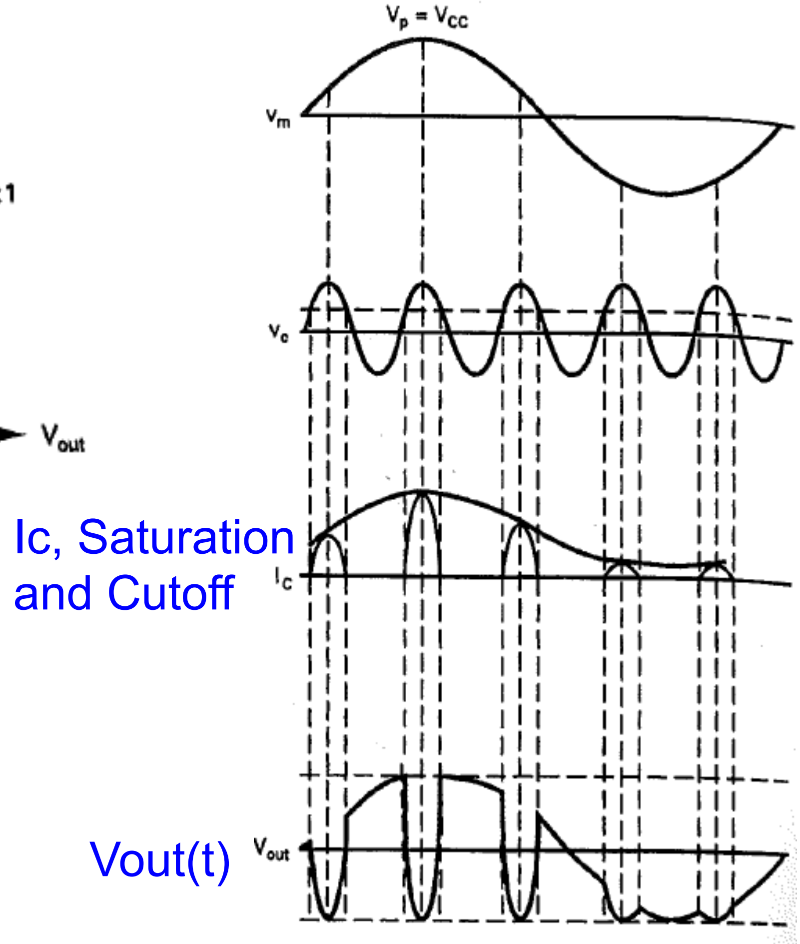
- Used for high-power transmission
- Uses an Collector Modulator (high power)
  - Nonlinear modulator
- The amplifier has two inputs: *Vc(t)* and *Vm(t)*
- RFC is radio frequency choke – blocks RF



High-Level AM Modulators – Circuit Operation

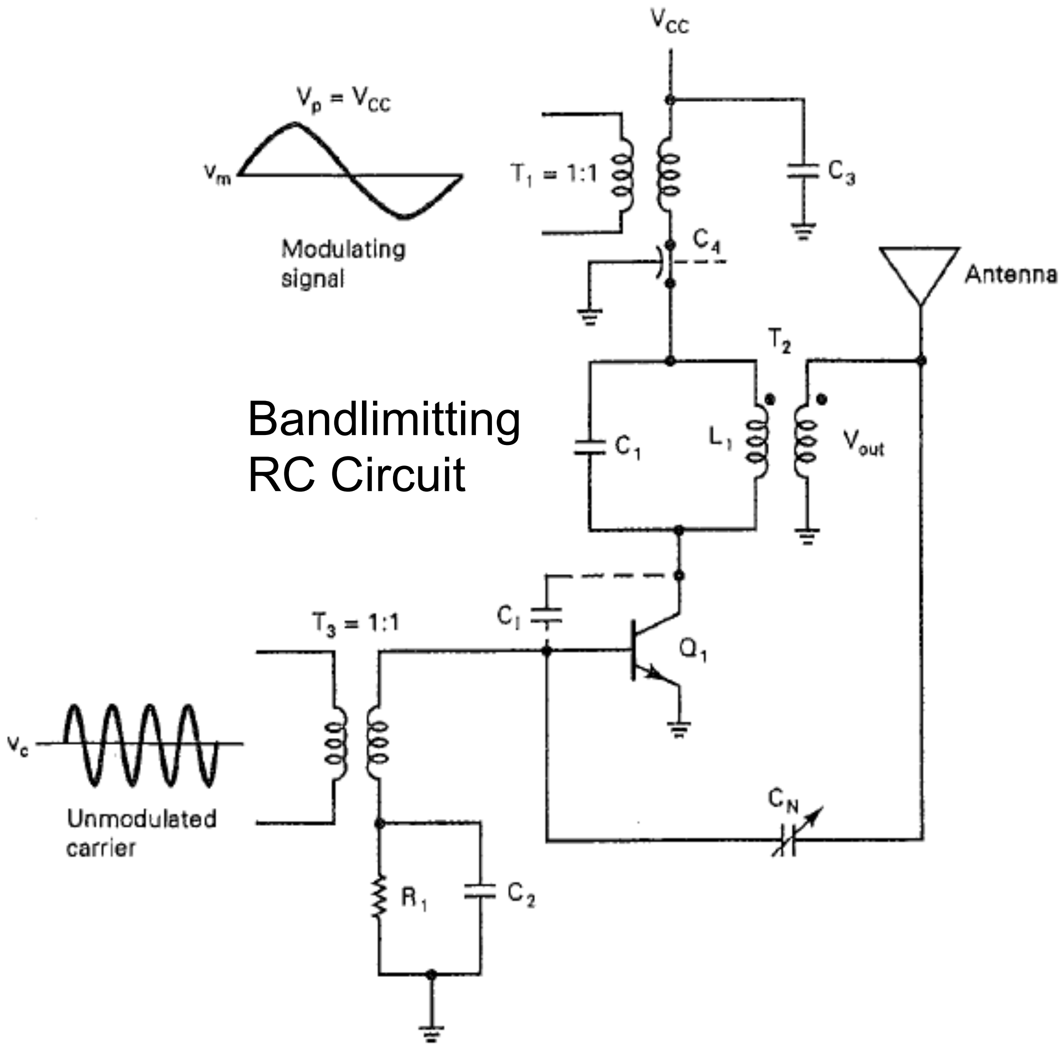
- General operation:
  - If Base Voltage > 0.7 → Q1 is ON → Ic != 0 → Saturation
  - If Base Voltage < 0.7 → Q1 is OFF → Ic = 0 → Cutoff
  - The Transistor changes between Saturation and Cutoff
- When in nonlinear → high harmonics are generated→ Vout must be bandlimited
  - See next slide…





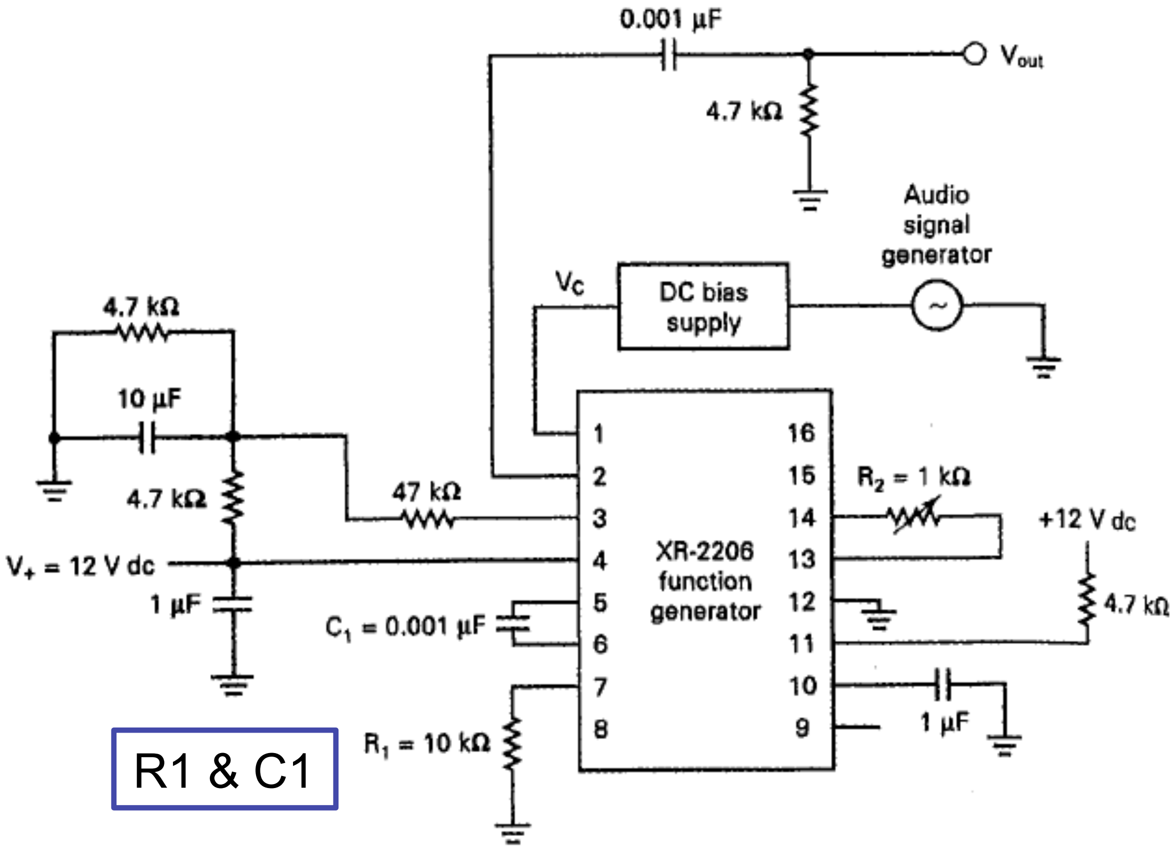
High-Level AM Modulators – Circuit Operation

- C_L and L_L tank can be added to act as Bandlimited
  - Only fc + fm and fc – fm can be transmitted



AM Modulators – Using Integrated Devices

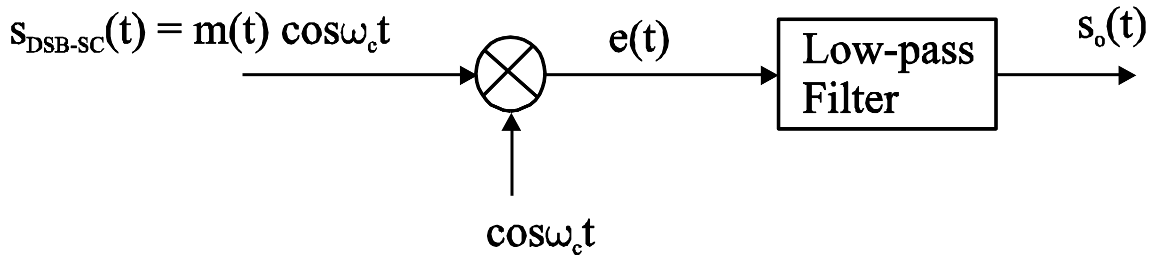
- XR-2206 is an integrated circuit function generator
- In this case $fc=1/R_1C_1$ Hz
- Assuming fm = 4kHz; fc = 100kHz we will have the following:



https://www.youtube.com/watch?v=5p5ZMXkaPW0

Building AM Demodulators

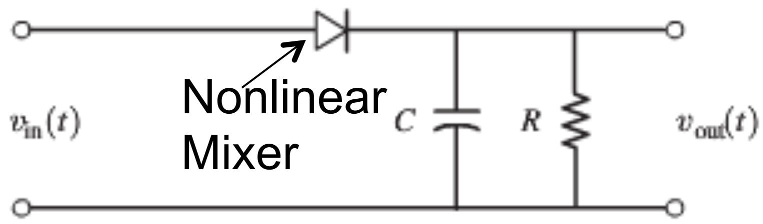
- Coherent
- Non-Coherent
  - Squaring Loop
  - Envelope Detectors



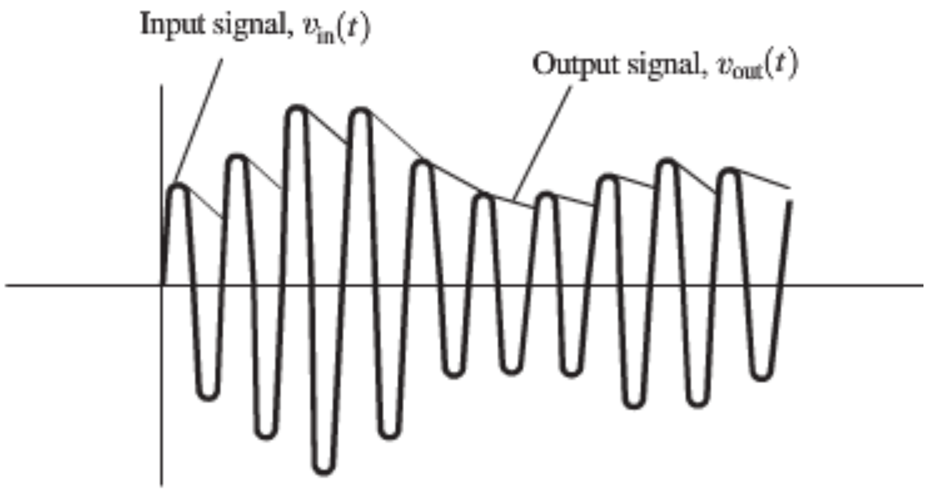
AM Demodulators: Envelope Detector

- It is considered as a **non-coherent** demodulator
- The diode acts as a **nonlinear** mixer
- Other **names**
  - Diode Detector
  - Peak Detector (Positive)
  - Envelope Detector
- Basic operation: Assume fc = 300 KHz and fm = 2KHz
  - Then there will be frequencies 298, 300, 302 KHz
  - The detector will detect many different frequencies (due to nonlinearity)
  - AM frequencies + AM harmonics + SUM of AM frequencies + DIFF of AM frequencies
  - The RC LPF is set to pass only DIFF frequencies

$$B \ll \frac{1}{2\pi RC} \ll f_c$$



(a) A Diode Envelope Detector

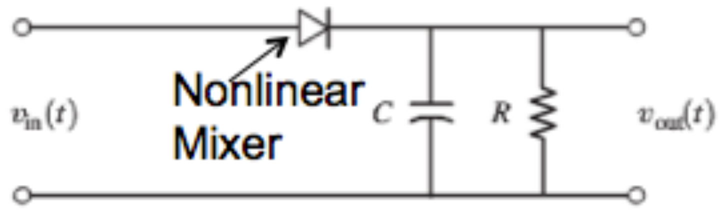


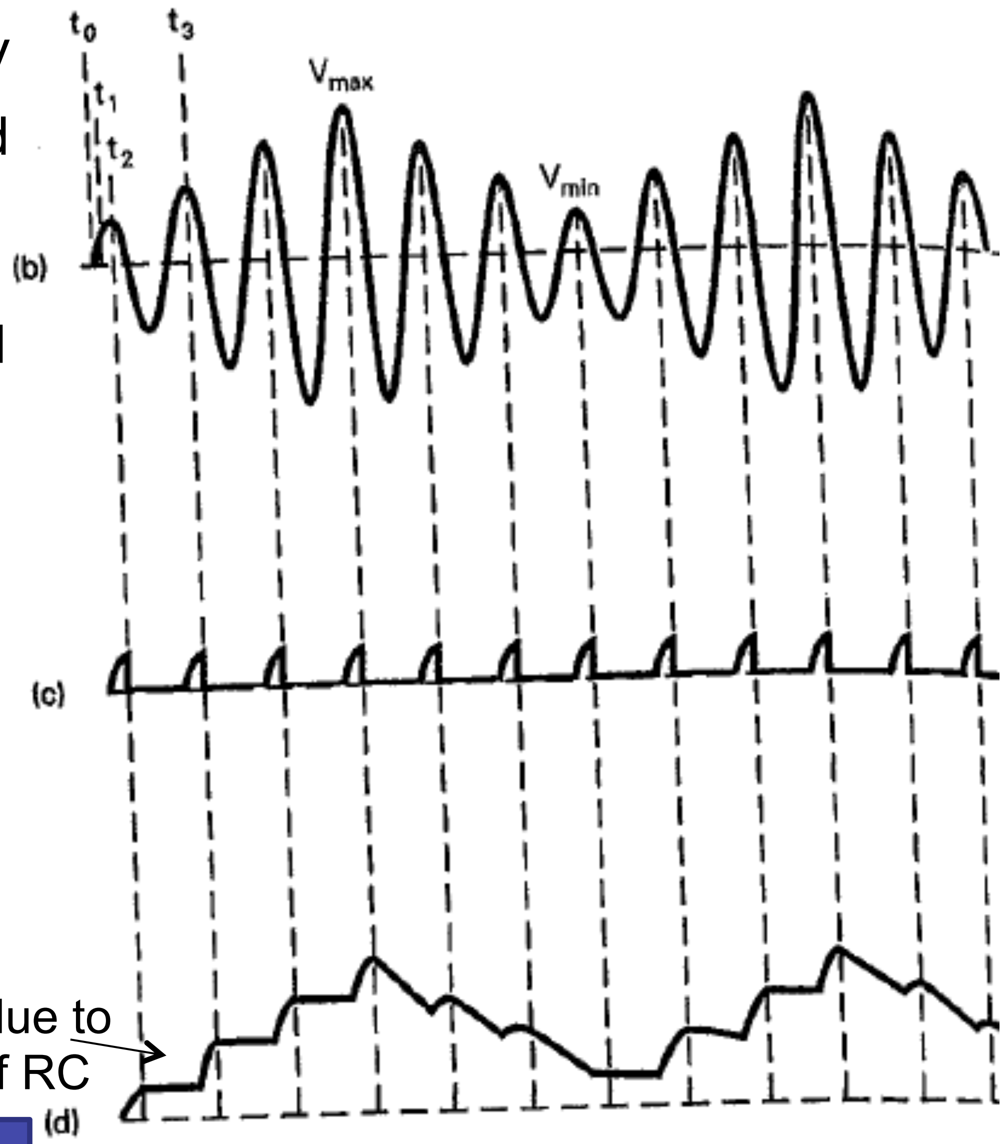
(b) Waveforms Associated with the Diode Envelope Detector

Envelope Detector – Basic Operation

- The diode has $V_{barrier} = V_b = 0.3V$
- When $V_{in} < V_b$ → Reverse Biased → DIODE is OFF
  - → $i_d = 0$ → $V_{cap} = 0$
- When $V_{in} > V_b$ → Forward Biased → DIODE is ON
  - → $i_d > 0$ → $V_{cap} = V_{in} - 0.3$

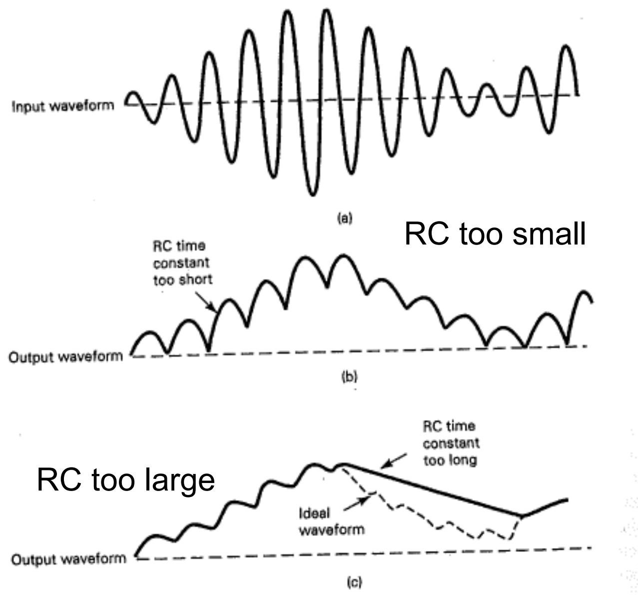
$$B \ll \frac{1}{2\pi RC} \ll f_c$$





What should be the value of RC, then?

Envelope Detector – Distortion



- What should be the value of **RC**?
  - If too low then discharges too fast
  - If too high the envelope will be distorted
  - The highest modulating signal:

$$f_{m(\max)} = \frac{\sqrt{(1/m^2)-1}}{2\pi RC}$$

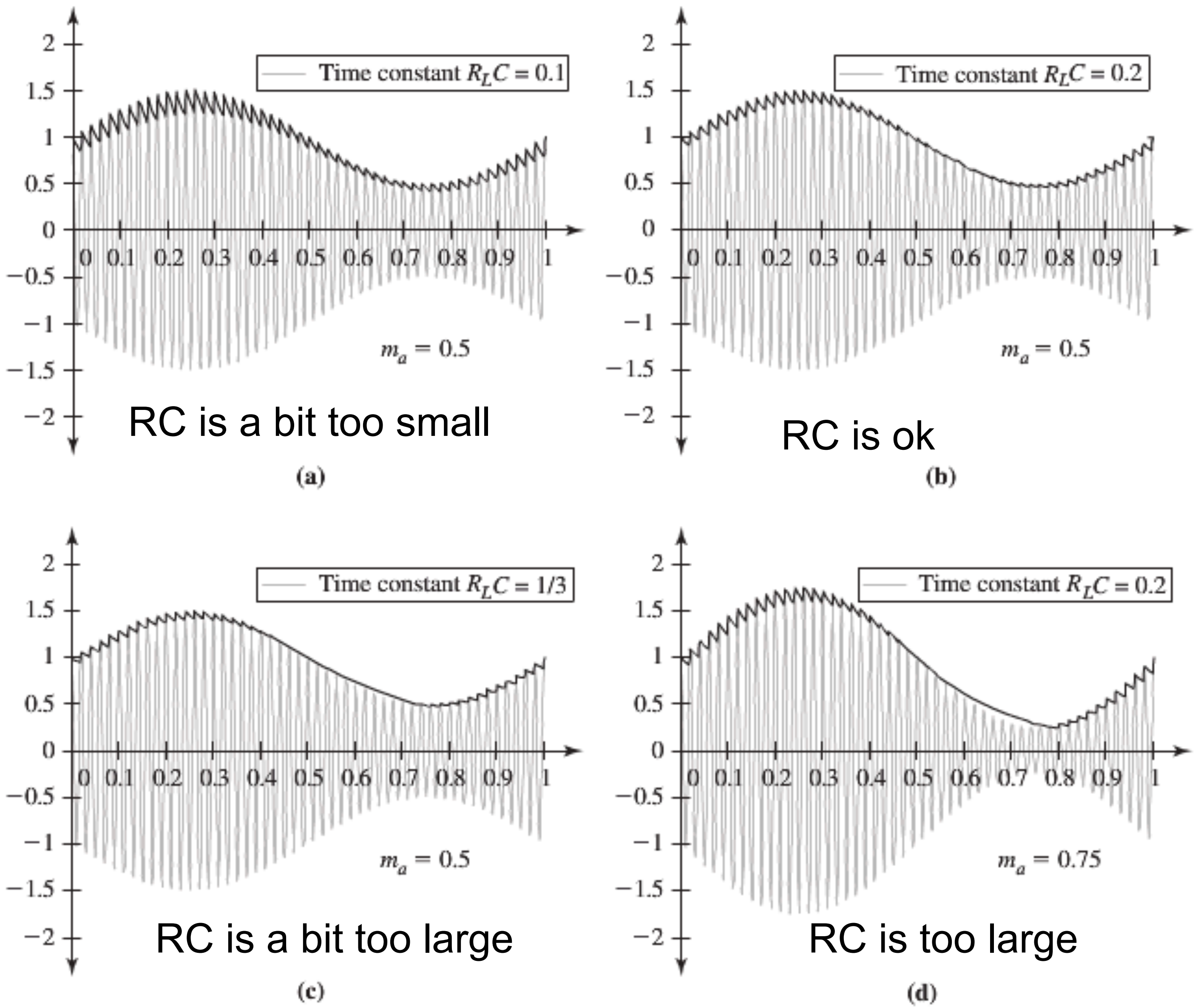
  - Note that in most cases m=0.70 or 70 percent of modulation →

$$f_{m(\max)} = \frac{1}{2\pi RC}$$

Therefore:

$$B \ll \frac{1}{2\pi RC} \ll f_c$$

Envelope Detection for Different RC



Applets

- Crystal Radio (receiver with no amplifier)
  - http://www.falstad.com/circuit/e-amdetect.html
- Amplitude clipper
  - http://www.falstad.com/circuit/e-diodeclip.html

Single Sideband AM (SSB)

- Is there anyway to reduce the bandwidth in ordinary AM?
- The complex envelop of SSB AM is defined by

$$g(t) = A_c[m(t) \pm j\hat{m}(t)]$$

Note that
(+) → USSB &
(-) → LSSB

- Thus, we will have

$$s(t) = A_c[m(t) \cos \omega_c t \mp \hat{m}(t) \sin \omega_c t]$$

- We define (m^(t) as the **Hilbert Transfer** of m(t)):

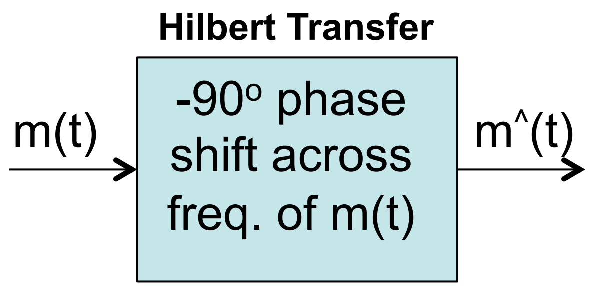
- Where:
- With impulse response of
- Thus:

$$\hat{m}(t) \triangleq m(t) * h(t)$$

$$h(t) = \frac{1}{\pi t}$$

$$H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \end{cases}$$

H(f)=0, f=0



$$G(f) = A_c\{M(f) \pm j\mathscr{F}[\hat{m}(t)]\} \longrightarrow G(f) = A_c M(f)[1 \pm jH(f)]$$

See Notes

Simple Example on Hilbert Transfer

- What is the H[x(t)] if x(t) is $s(t)\cos(2\pi f_c t+\phi)$:
  - Shifted by -90 degree → cos() → sin()
  - → $H[x(t)] = s(t)\sin(2\pi f_c t+\phi)$

Frequency Spectrum of SSB-AM - USSB

For Upper SSB use (+) $G(f) = A_c M(f)[1 \pm jH(f)]$

$$H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \end{cases} \longrightarrow G(f) = \begin{cases} 2A_c M(f), & f > 0 \\ 0, & f < 0 \end{cases}$$

$$s(t) = A_c[m(t) \cos \omega_c t \mp \hat{m}(t) \sin \omega_c t]$$

Therefore:

$$S(f) = \tfrac{1}{2}[G(f-f_c) + G^*(-f-f_c)]$$

(f-fc)>0
f>fc

$$S(f) = A_c \begin{cases} M(f-f_c), & f > f_c \\ 0, & f < f_c \end{cases} + A_c \begin{cases} 0, & f > -f_c \\ M(f+f_c), & f < -f_c \end{cases}$$

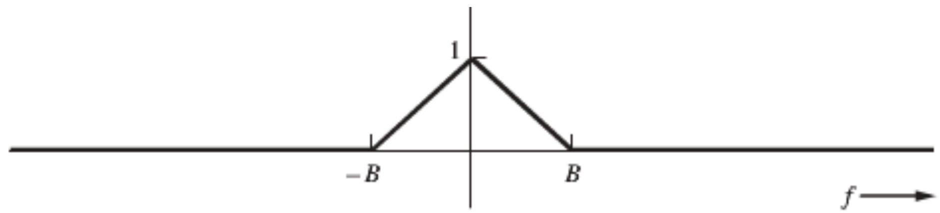
(-f-fc)>0
-f>fc
→f<-fc

Normalized Average Power:

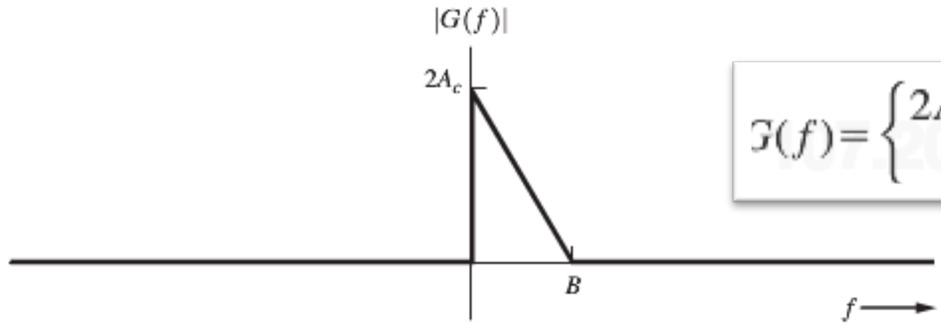
$$\langle s^2(t)\rangle = \tfrac{1}{2}\langle |g(t)|^2\rangle = \tfrac{1}{2}A_c^2\langle m^2(t) + [\hat{m}(t)]^2\rangle \qquad \langle \hat{m}(t)^2\rangle = \langle m^2(t)\rangle$$

$$\langle s^2(t)\rangle = A_c^2\langle m^2(t)\rangle$$

Frequency Spectrum of SSB-AM - USSB

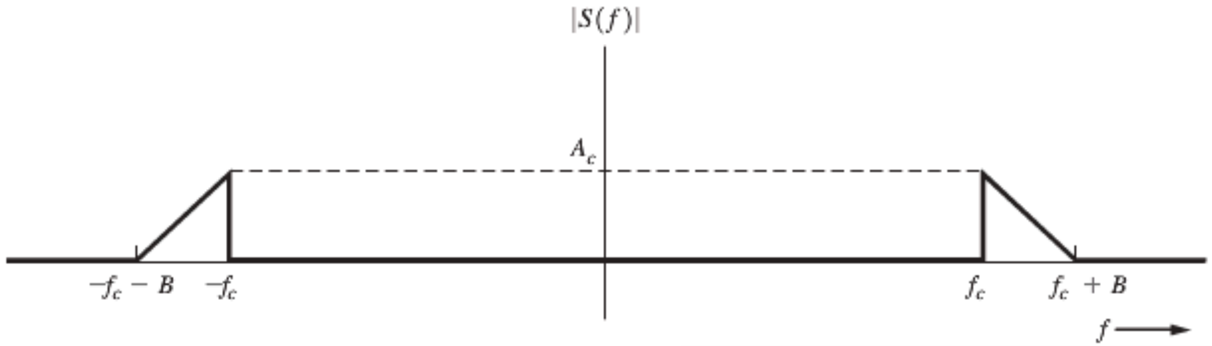


(a) Baseband Magnitude Spectrum



$$G(f)=\begin{cases}2A_cM(f), & f>0\\ 0, & f<0\end{cases}$$

(b) Magnitude of Corresponding Spectrum of the Complex Envelope for USSB

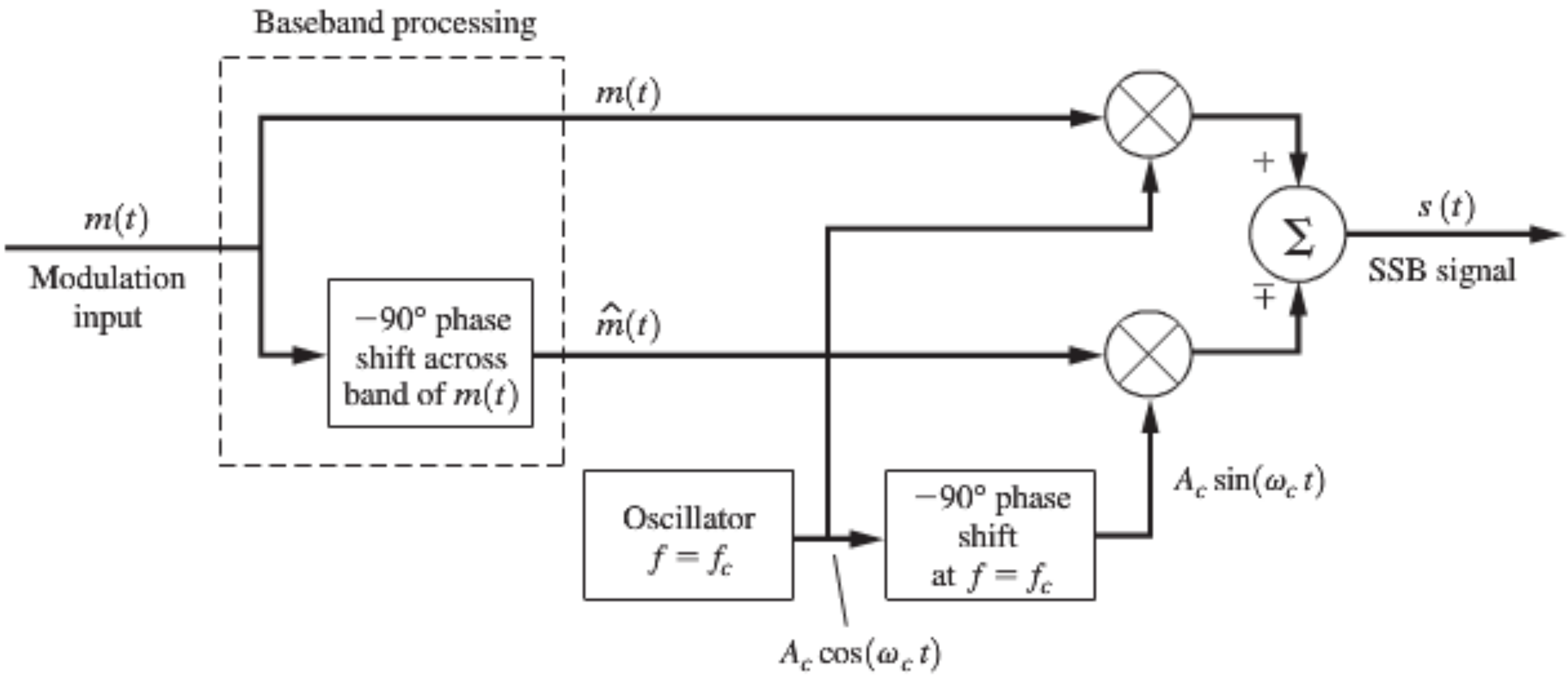


(c) Magnitude of Corresponding Spectrum of the USSB Signal

$$S(f)=A_c\begin{cases}M(f-f_c), & f>f_c\\ 0, & f<f_c\end{cases}+A_c\begin{cases}0, & f>-f_c\\ M(f+f_c), & f<-f_c\end{cases}$$

Basic Method

$$s(t) = A_c[m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t]$$



This is also called Quadrature AM (QAM) modulator
with **I** and **Q** channels
I refers to In phase; Q refers to Quadrature phase)

References

- Leon W. Couch II, Digital and Analog Communication Systems, 8th edition, Pearson / Prentice, Chapter 5
- Electronic Communications System: Fundamentals Through Advanced, Fifth Edition by Wayne Tomasi – Chapter 4 & 5

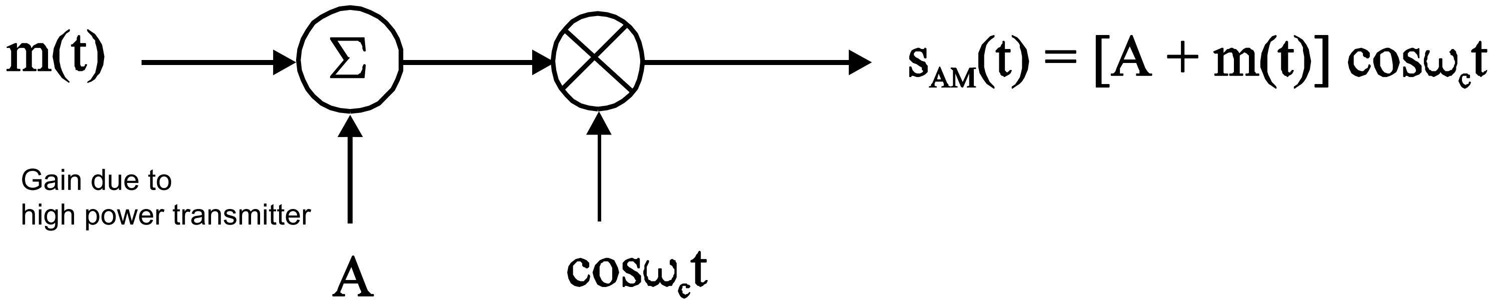
  (https://www.goodreads.com/book/show/209442.Electronic_Communications_System)

See Notes

Side Notes

Standard (Ordinary) AM

AM signal generation



Waveform :

$$s_{AM}(t) = A\cos\omega_c t + m(t)\cos\omega_c t = [A + m(t)]\cos\omega_c t$$

Spectrum :

$$S_{AM}(\omega) = (1/2)[M(\omega + \omega_c) + M(\omega - \omega_c)] + \pi A[\delta(\omega + \omega_m) + \delta(\omega - \omega_m)]$$

Standard (Ordinary) AM

- The disadvantage of high cost receiver circuit of the DSB-SC system can be solved by use of AM, but at the price of a less efficient transmitter

- An AM system transmits a **large power carrier** wave, $A\cos\omega_c t$, along with the modulated signal, $m(t)\cos\omega_c t$, so that there is no need to generate a carrier at the receiver.
  - Advantage : simple and low cost receiver

- *In a broadcast system, the transmitter is associated with a large number of low cost receivers. The AM system is therefore preferred for this type of application.*