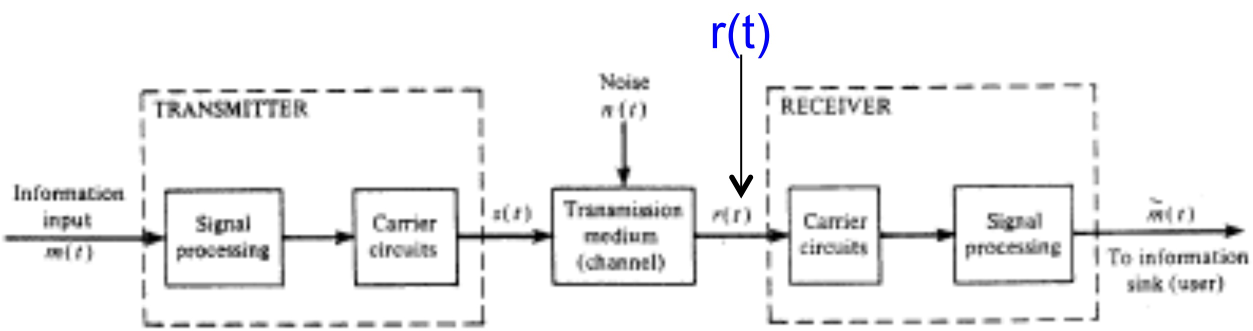
Chapter 2

Signals and Spectra

Updated:2/2/15

Waveform Properties

- In communications, the received waveform r(t) basically comprises two parts:
  - Desired signal or Information
  - Undesired signal or Noise
- Waveforms belong to many different categories
  - Physically realizable or non-physically realizable
  - Deterministic or stochastic
  - Analog or digital
  - Power or energy
  - Periodic or non-periodic



Waveform Characteristics (Definitions)

- Time average Operator

$$\langle[\cdot]\rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} [\cdot]dt$$

Note that [.] is the function!
< > is the operation!

- Periodic waveform

$\omega(t) = \omega(t + T_0)$ for all t

- Waveform DC (Direct Current) value

If w(t) is periodic with To, *lim*1/T→1/To

$$W_{dc} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \omega(t)dt = \text{Average value}$$

$w(t)$ can related to v(t) or i(t)
Note that in this expression [.] is $w(t)$

- For a physical waveform the DC value over a finite interval t_1 to t_2

$$W_{dc} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \omega(t)dt$$

Note: $W_{dc} = \langle w(t) \rangle = \overline{w(t)}$

Different expressions

Waveform Characteristics (Definitions)

- Instantaneous power

$$p(t) = power = \frac{work}{time} = \frac{work}{charge} \cdot \frac{charge}{time} = v(t).i(t)$$

- Average power $P = \langle p(t) \rangle = \langle v(t) \cdot i(t) \rangle$
- RMS Value $W_{rms} = \sqrt{\langle \omega^2(t) \rangle}$
- Average power for resistive load is

$$P_{av} = \frac{\langle v^2(t) \rangle}{R} = \langle i^2(t) \rangle R = \frac{V_{rms}{}^2}{R} = I_{rms}{}^2 R = V_{rms} I_{rms}$$

- Average normalized power

Pnorm =Pav, when RLoad=1

$$P_{norm} = \langle \omega^2(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \omega^2(t)\, dt$$

Note : w(t) can be v(t) or i(t)

$$P_{av} = \langle p(t) \rangle = \langle v(t) \cdot i(t) \rangle$$

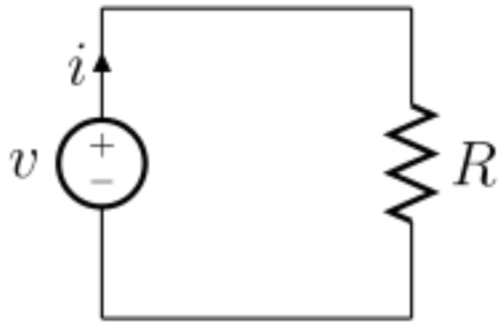
Note: $\langle w^2(t) \rangle = W_{rms}$

Pay attention: $<[.]^2>$ is different from $<[.]>^2$

Remember: $< >$ is the time average operation!

Average power is average of instantaneous power!

Note that rms is derived from time average ($<[.]^2>$)$^{1/2}$



Average Power At the load: Pav

V(t) across the load; with i(t) going through the load

Real Meaning of RMS

RMS for a set of n components

$$x_{\mathrm{rms}} = \sqrt{\frac{1}{n}\left(x_1^2 + x_2^2 + \cdots + x_n^2\right)}.$$

RMS for continuous function from T1 to T2

$$f_{\mathrm{rms}} = \sqrt{\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} [f(t)]^2\, dt},$$

RMS for a function over all the times

$$f_{\mathrm{rms}} = \lim_{T\to\infty} \sqrt{\frac{1}{T}\int_{0}^{T} [f(t)]^2\, dt}.$$

Waveform Characteristics (**Summary**)

- Time average Operator

$$\langle[\cdot]\rangle = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2}[\cdot]dt$$

- Periodic waveform

$\omega(t) = \omega(t+T_0)$ for all t

- /eform DC (Direct Current)

e

If w(t) is periodic with To, *lim*1/T→1/To

$$\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2}\omega(t)dt = \text{Average value}$$

re w(t) and W can be *v* or *i*.

- For a physical waveform the DC value over a finite interval t_1 to t_2

$$W_{dc} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2}\omega(t)dt$$

Note: $W_{dc} = \langle w(t)\rangle = \overline{w(t)}$

See Notes:2a

- Instantaneous power

$$p(t) = power = \frac{work}{time} = \frac{work}{charge}\cdot\frac{charge}{time} = v(t).i(t)$$

- Average power $P = \langle p(t)\rangle = \langle v(t)\cdot i(t)\rangle$
- RMS Value $W_{rms} = \sqrt{\langle\omega^2(t)\rangle}$
- Average power for resistive load is

$$P_{av} = \frac{\langle v^2(t)\rangle}{R} = \langle i^2(t)\rangle R = \frac{V_{rms}^2}{R} = I_{rms}^2 R = V_{rms} I_{rms}$$

- Average normalized power

Pnorm =Pav, when RLoad=1

$$P_{norm} = \langle\omega^2(t)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\omega^2(t)dt$$

Note : w(t) can be v(t) or i(t)

$$P_{av} = \langle p(t)\rangle = \langle v(t)\cdot i(t)\rangle$$

Note: $\langle w^2(t)\rangle = W_{rms}$

Energy & Power Waveforms

- Average normalized power

$$P = \langle \omega^2(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \omega^2(t)\,dt$$

- Total normalized energy is

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} \omega^2(t)\,dt \quad *$$

- w(t) is an <u>energy waveform</u> if & only if total normalized energy is finite & ≠0

Signal Definition:

$$Energy\_Signal \to 0 < E < \infty$$

$$Power\_Signal \to 0 < P < \infty$$

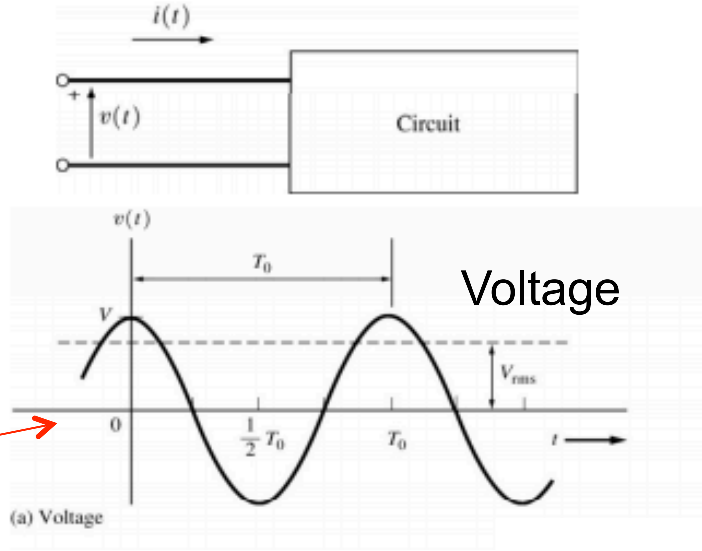
Note that a signal can either have Finite total normalized energy or Finite average normalized power

Note:
If w(t) is periodic with To, *lim*1/T→1/To

Remember: $p(t) = power = \frac{work}{time}$ *

Example

- The circuit contains a 120V, 60Hz lamp with in-phase voltage & current waveforms. Find the DC voltage value



(a) Voltage

- DC voltage value:

Periodic Signal!

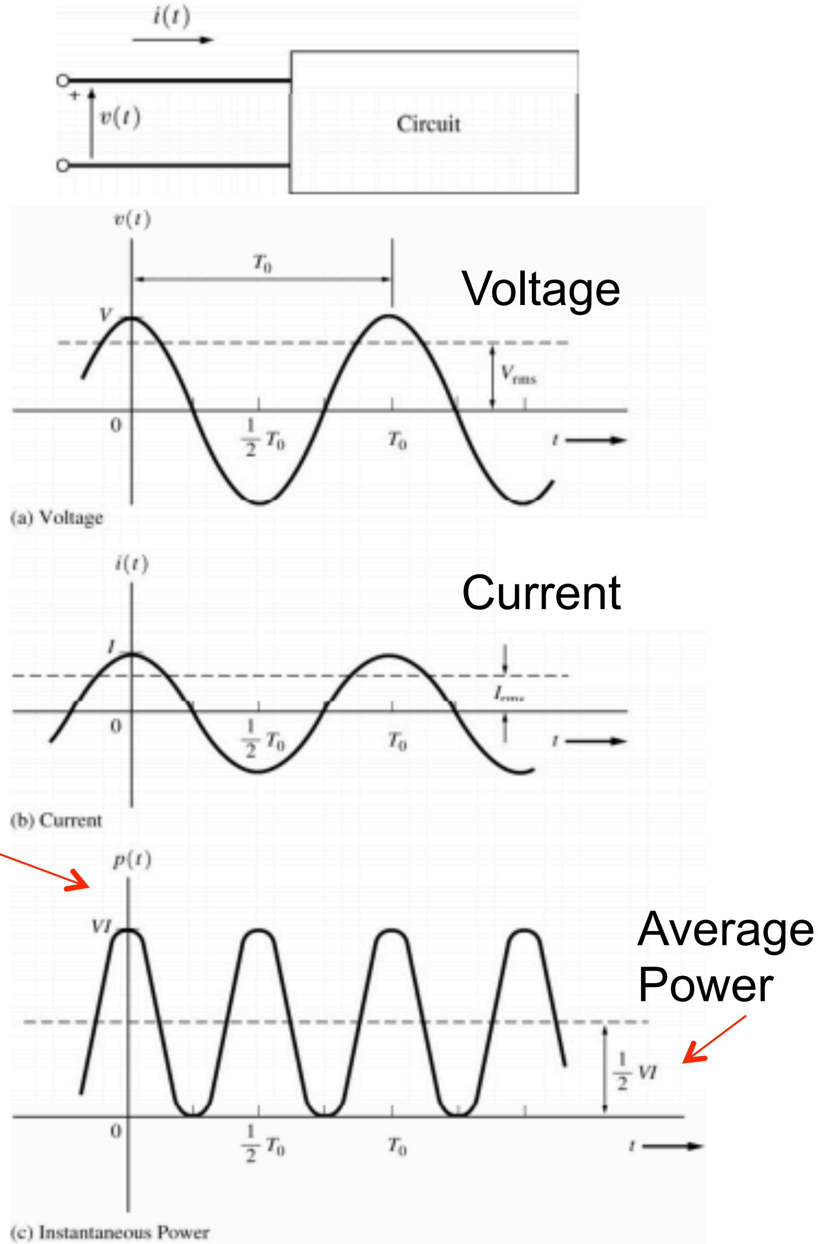
$$V_{dc} = \langle v(t) \rangle = \langle V\cos(\omega_0 t) \rangle = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} V\cos(\omega_0 t)dt = 0$$

where $\omega_0 = 2\pi / T_0$ & $f_0 = 1/T_0 = 60$ Hz.

- Similarly $I_{dc} = 0$.

Example (continued)

- The circuit contains a 120V, 60Hz lamp with in-phase voltage & current waveforms.



* Instantaneous Power: (it is a function of time!)

V = Vpeak

$$p(t) = (V\cos\omega_0 t)(I\cos\omega_0 t) =$$

$$\frac{1}{2}VI(1 + \cos 2\omega_0 t)$$

- Average power:

$$P_{ave} = \langle VI\frac{1+\cos 2\omega_0 t}{2}\rangle = \frac{VI}{2T_0}\int_{-T_0/2}^{T_0/2}(1+\cos 2\omega_0 t)dt = \frac{VI}{2}$$

Note:

$$P_{av} = \langle p(t)\rangle = \langle v(t)\cdot i(t)\rangle$$
$$P_{av} = V_{rms}\cdot I_{rms}$$

Example (continued)

Incandescent light bulbs flicker at twice the AC frequency, because the filament grows a bit hotter each time the current peaks.
So: 50 Hz AC => 100 Hz flicker;

- **RMS values:**

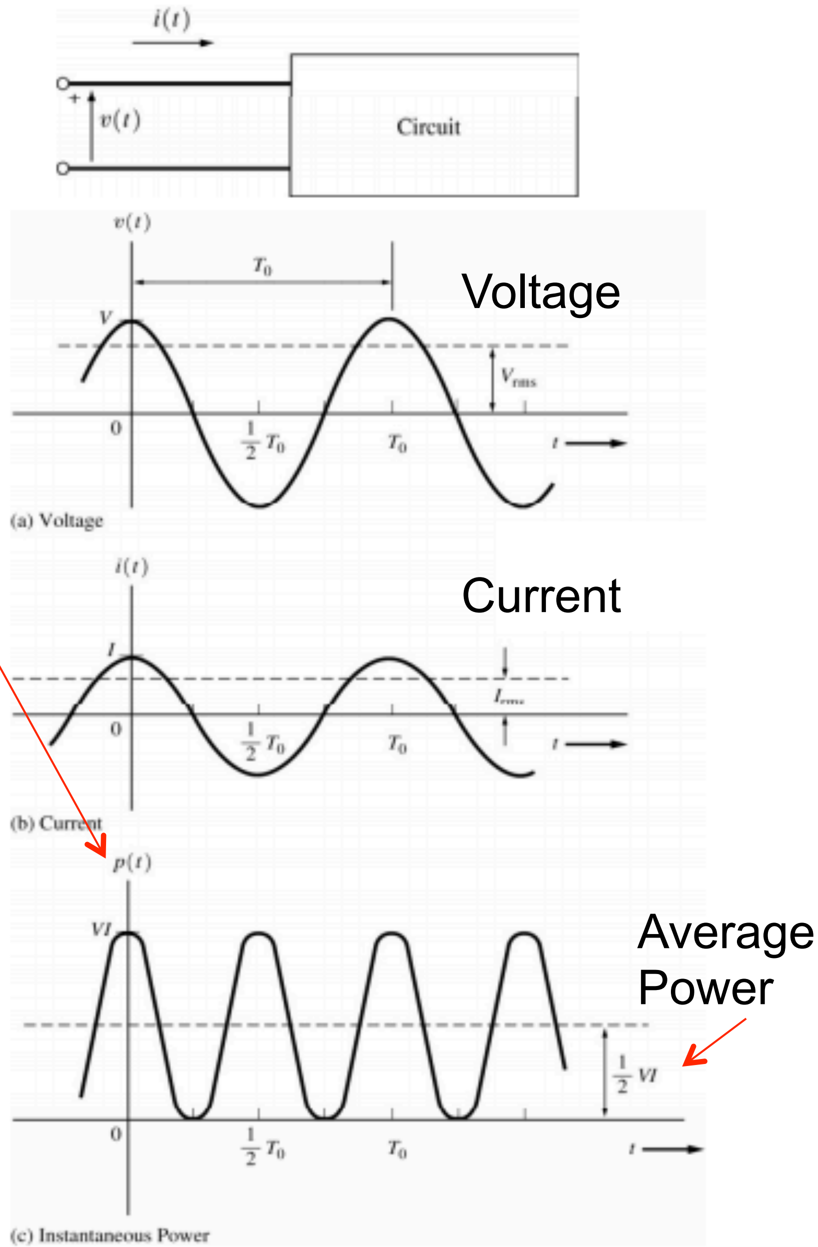
$V_{rms} = V/\sqrt{2}$, $I_{rms} = I/\sqrt{2}$, and $P_{ave} = \frac{1}{2}VI$

> Note that this is only true when V(t) is a sinusoidal. In this case V is the Peak amplitude of v(t)

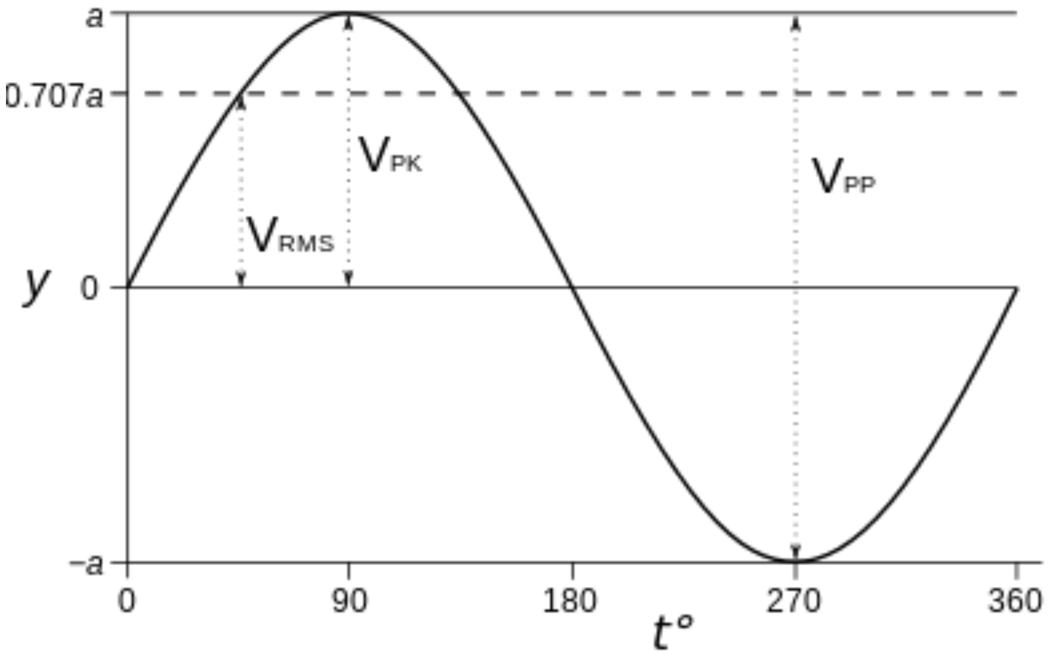
$$V_{rms} = \sqrt{\langle v^2(t) \rangle} = \sqrt{\frac{1}{T_0} \int_{-To/2}^{To/2} [V\cos(w_o t)]^2 \, dt}$$

$$V_{rms} = \frac{V}{\sqrt{2}}; I_{rms} = \frac{I}{\sqrt{2}};$$ V = Vpeak

$$P_{av} = V_{rms} \cdot I_{rms} = \frac{V \cdot I}{2}$$



RMS Values

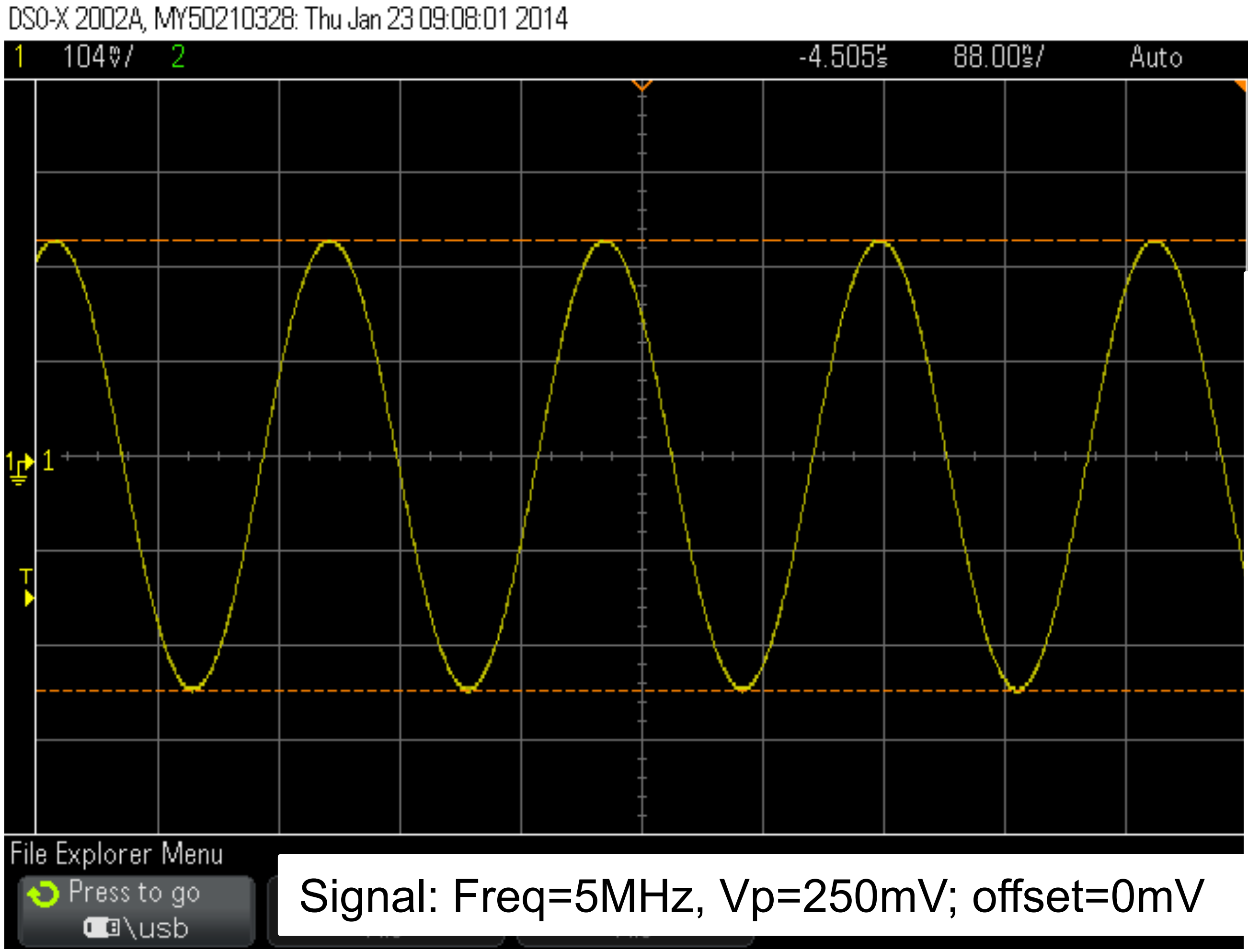


Waveform	Equation	RMS
DC, constant	$y = a$	a
Sine wave	$y = a\sin(2\pi f t)$	$\frac{a}{\sqrt{2}}$
Square wave	$y = \begin{cases} a & \{ft\} < 0.5 \\ -a & \{ft\} > 0.5 \end{cases}$	a

http://en.wikipedia.org/wiki/Root_mean_square

Example – Using the Scope

Note that RMS is Vpeak/SQRT(2)

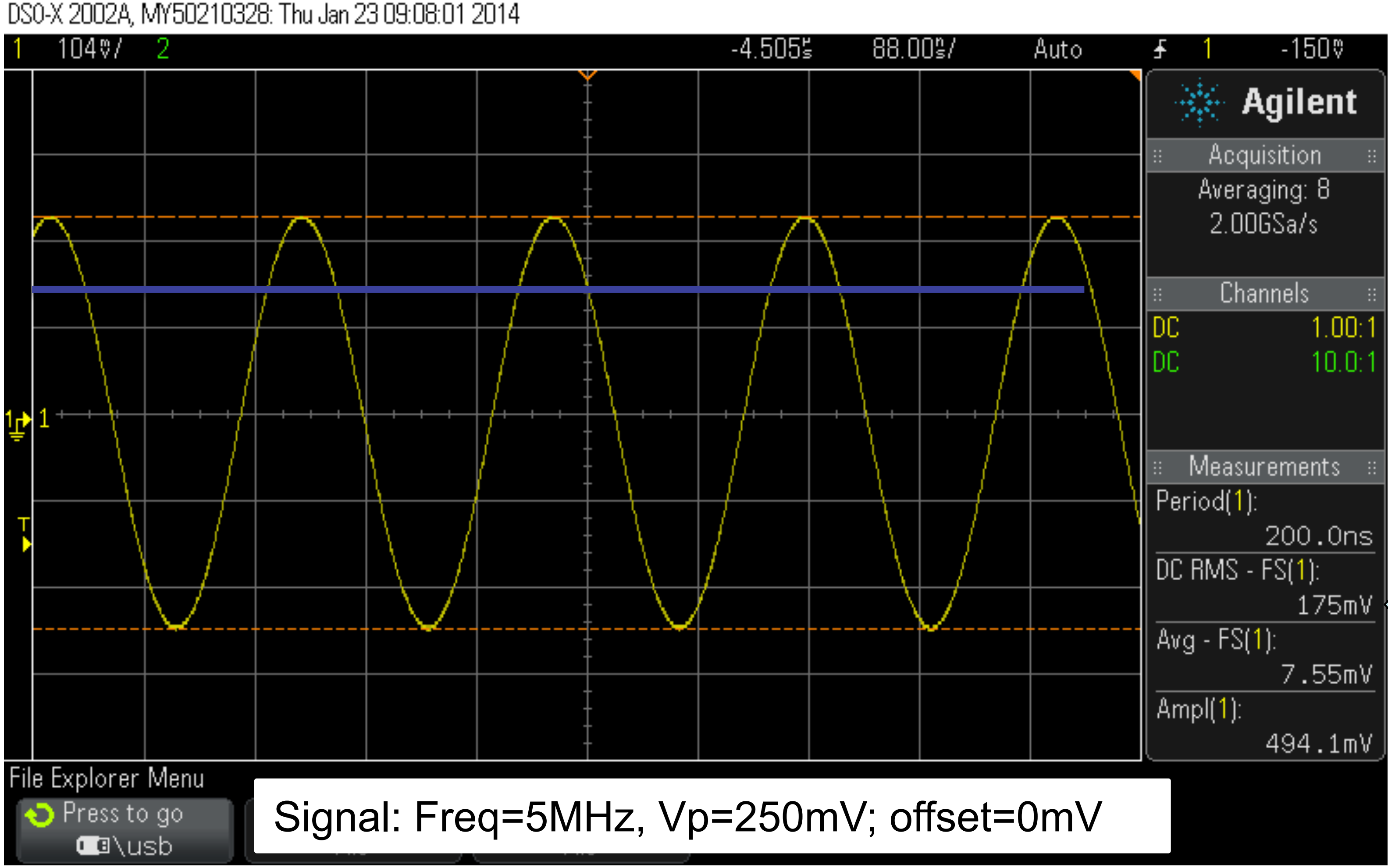


Signal: Freq=5MHz, Vp=250mV; offset=0mV

Find Period, RMS, Average (DC), & Vpp!

Example – Using the Scope

Note that RMS is Vpeak/SQRT(2)

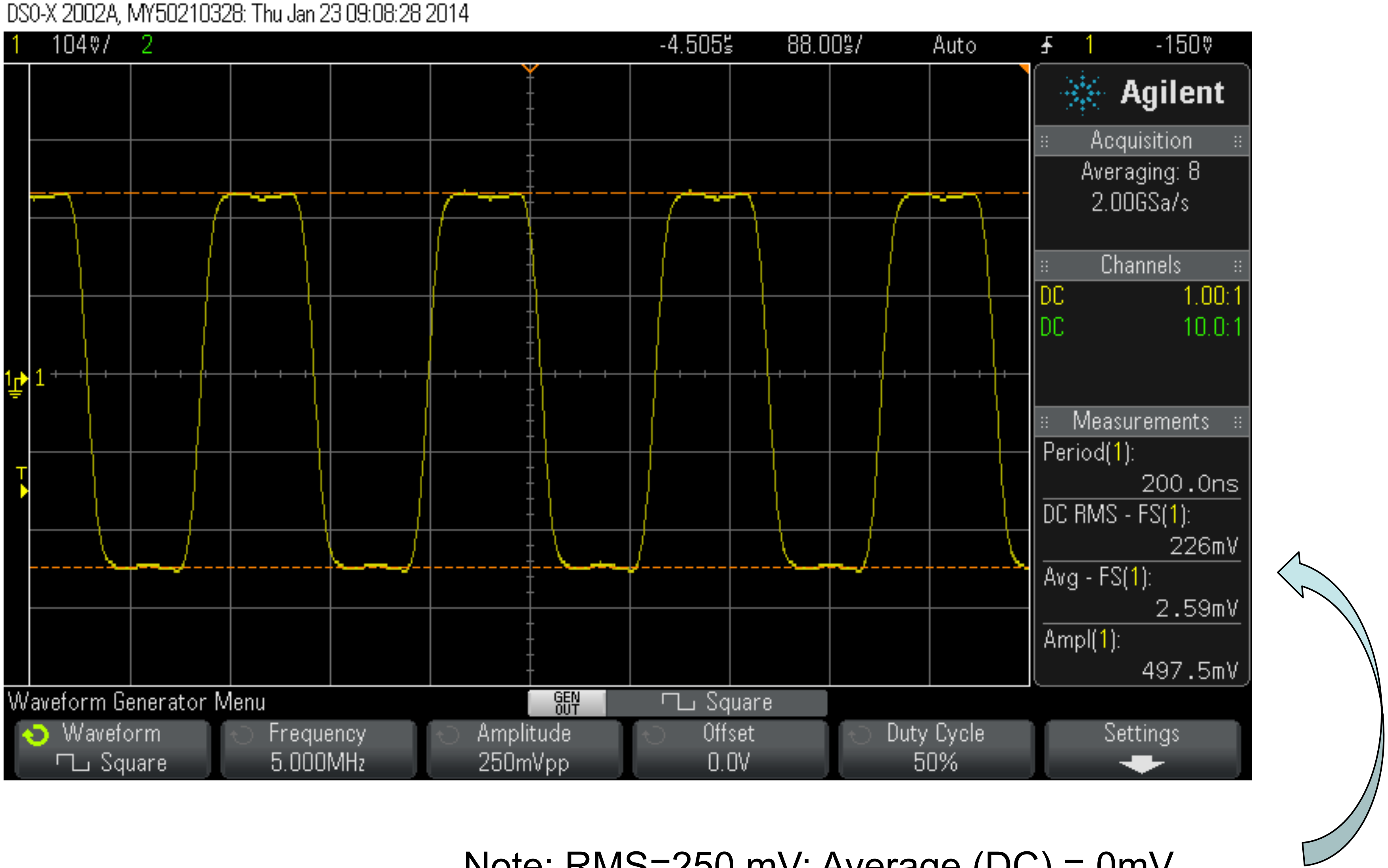


Signal: Freq=5MHz, Vp=250mV; offset=0mV

Note: RMS=250/sqrt(2) = **176mV**; Average (DC) = 0mV

Example - Scope

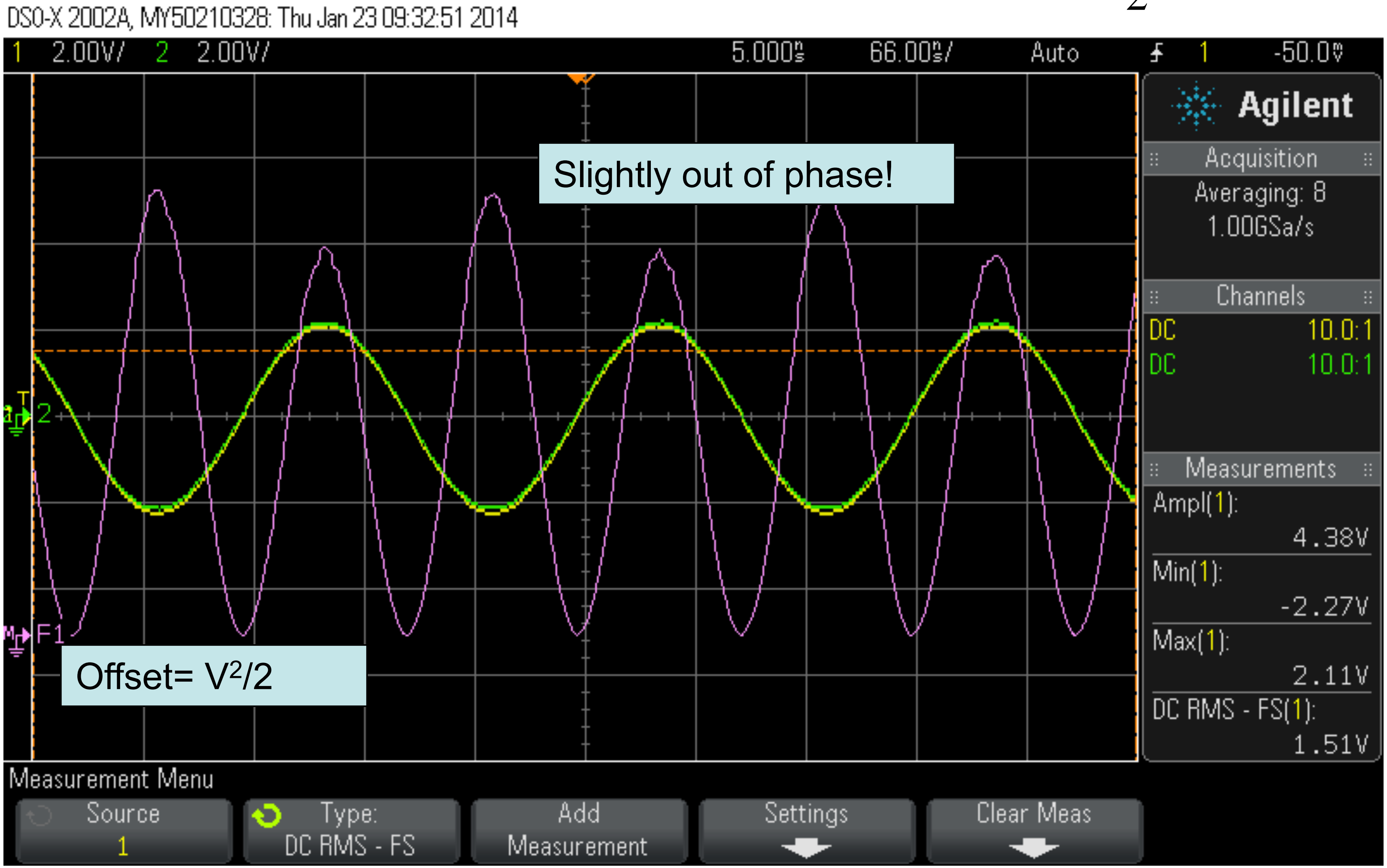
Note that RMS changes as the waveform
Changes; independent of the frequency



Note: RMS=250 mV; Average (DC) = 0mV

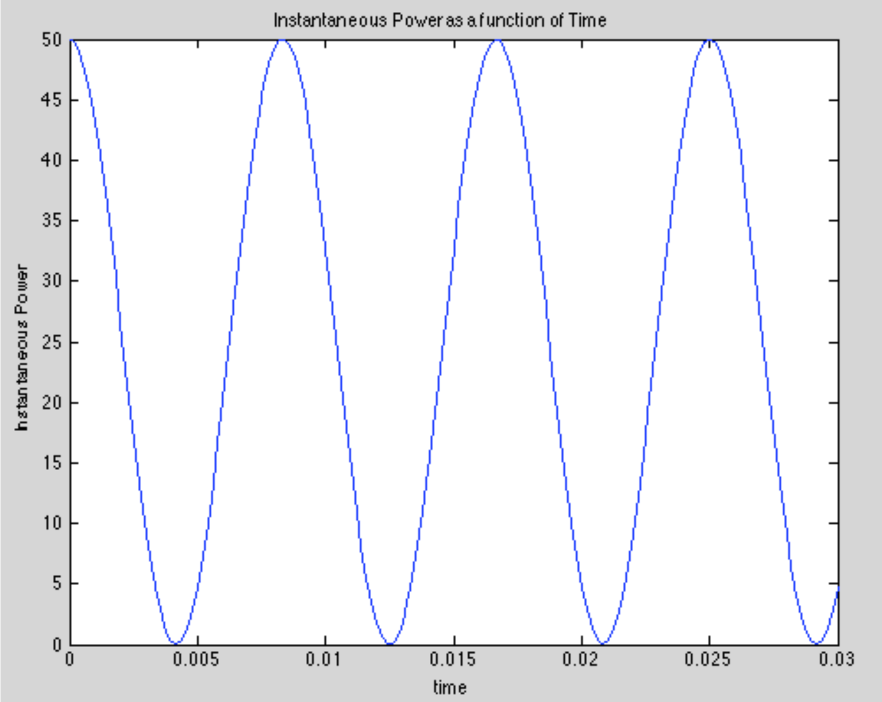
Example - Scope

Two signals being multiplied by each other! $V_{mul} = [V\cos(w_o t)]^2 = \frac{V^2}{2}(1+\cos(2w_o t))$



Example - Matlab

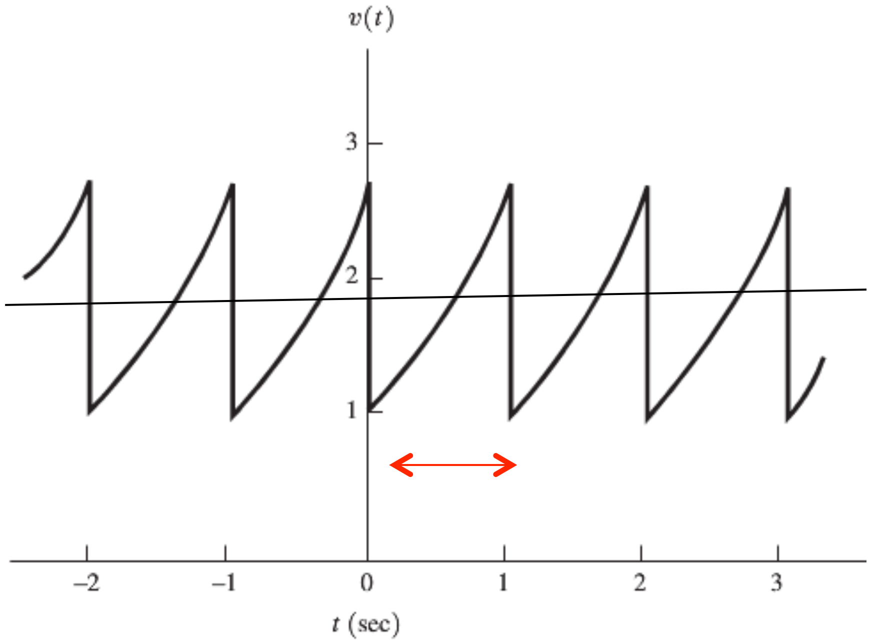
Assume v(t) and i(t) are in phase.
Plot the p(t).




```
clear;

fo = 60;
t = 0:0.0002:0.03;
wo = 2*pi*fo;

% Select theta to be the phase shift of current in degrees
theta = 0;

current = 5*cos(wo*t + theta*(pi/180));
volts = 10*cos(wo*t);


instpower = current.*volts;

plot(t,instpower);
xlabel('time');
ylabel('Instantaneous Power');
title('Instantaneous Power as a function of Time');
```

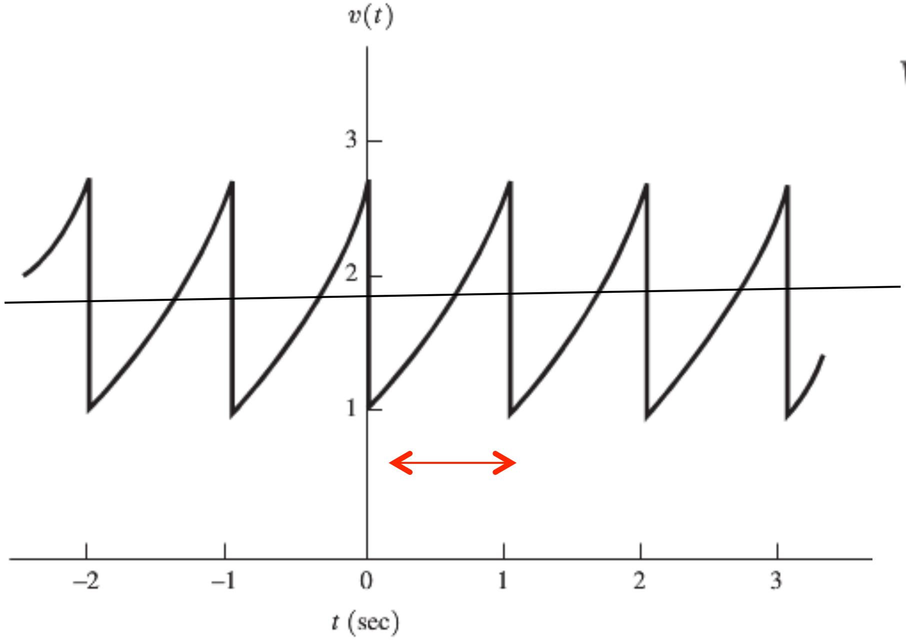
Example

- v(t)= e^t is a periodic voltage signal over time interval 0<t<1. Find DC & RMS values of the waveform



Example

- v(t)= e^t is a periodic voltage signal over time interval 0<t<1. Find DC & RMS values of the waveform



$$V_{dc} = \langle v(t) \rangle = \frac{1}{T_0}\int_0^{T_0} v(t)\,dt = \int_0^1 e^t dt = e^1 - e^0$$

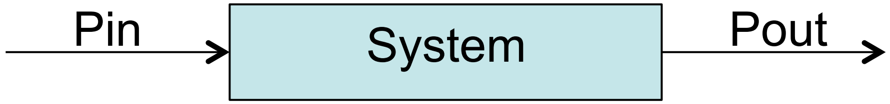
$$V_{dc} = e - 1 = 1.72\,\text{V}$$

$$V_{rms}^2 = \langle v^2(t) \rangle = \int_0^1 (e^t)^2 dt = \frac{1}{2}(e^2 - e^0) = 3.19$$

$$V_{rms} = \sqrt{3.19} = 1.79\,\text{V}$$

Decibel

Power and dB expression:



Decibel is logarithm of power ratio.

$$dB = 10\log_{10}\left(\frac{avePower_{out}}{avePower_{in}}\right) = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

For resistive load

$$dB = 20\log_{10}\left(\frac{V_{rms\ out}}{V_{rms\ in}}\right) + 10\log_{10}\left(\frac{R_{in}}{R_{load}}\right)$$

$$dB = 20\log_{10}\left(\frac{I_{rms\ out}}{I_{rms\ in}}\right) + 10\log_{10}\left(\frac{R_{load}}{R_{in}}\right)$$

For normalized powers, $R_{in} = R_{out}$, then

$$dB = 20\log_{10}\left(\frac{V_{rms\ out}}{V_{rms\ in}}\right) = 20\log_{10}\left(\frac{I_{rms\ out}}{I_{rms\ in}}\right)$$

Given dB, the power ratio is $\frac{P_{out}}{P_{in}} = 10^{dB/10}$

If power ratio is positive → GAIN
If power ratio is negative → ATTEN
If power ratio is zero → Unity GAIN

Remember:
$P_{av} = V^2rms/R$

Remember:
$P_{av} = I^2rms \times R$

Signal-to-Noise Ratio

The decibel signal-to-noise ratio is

$$(S/N)_{dB} = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) = 10\log_{10}\left(\frac{\langle s^2(t)\rangle}{\langle n^2(t)\rangle}\right)$$

Because the signal power is

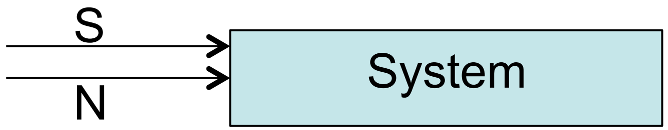
$$\langle s^2(t)\rangle / R = V^2_{rms\,signal} / R$$

and noise power is

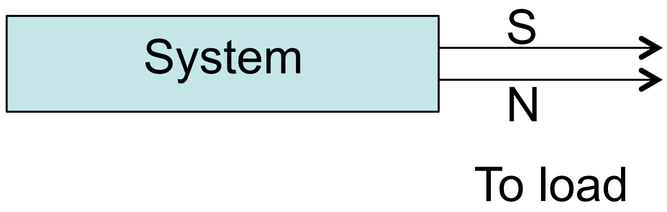
$$\langle n^2(t)\rangle / R = V^2_{rms\,noise} / R$$

This definition is equivalent to

$$(S/N)_{dB} = 20\log_{10}\left(\frac{V_{rms\,signal}}{V_{rms\,noise}}\right)$$



OR



dB vs dBm

dBm definition

dBm or *decibel-milliwatt* is an electrical power unit in decibels (dB), referenced to 1 milliwatt (mW).

The power in decibel-milliwatts ($P_{(dBm)}$) is equal to 10 times base 10 logarithm of the power in milliwatts ($P_{(mW)}$):

$$P_{(dBm)} = 10 \cdot \log_{10}(P_{(mW)} / 1mW)$$

The power in milliwatts ($P_{(mW)}$) is equal to 1mW times 10 raised by the power in decibel-milliwatts ($P_{(dBm)}$) divided by 10:

$$P_{(mW)} = 1mW \cdot 10^{(P_{(dBm)} / 10)}$$

1 milliwatt is equal to 0 dBm:

$$1mW = 0dBm$$

http://www.rapidtables.com/electric/dBW.htm

dBW vs dB

dBW definition

dBW or *decibel-watt* is a unit of power in decibel scale, referenced to 1 watt (W).

The power in decibel-watts ($P_{(dBW)}$) is equal to 10 times base 10 logarithm of the power in watts ($P_{(W)}$):

$$P_{(dBW)} = 10 \cdot \log_{10}(P_{(W)} / 1W)$$

The power in watts ($P_{(W)}$) is equal to 10 raised by the power in decibel-watts ($P_{(dBW)}$) divided by 10:

$$P_{(W)} = 1W \cdot 10^{(P_{(dBW)}/ 10)}$$

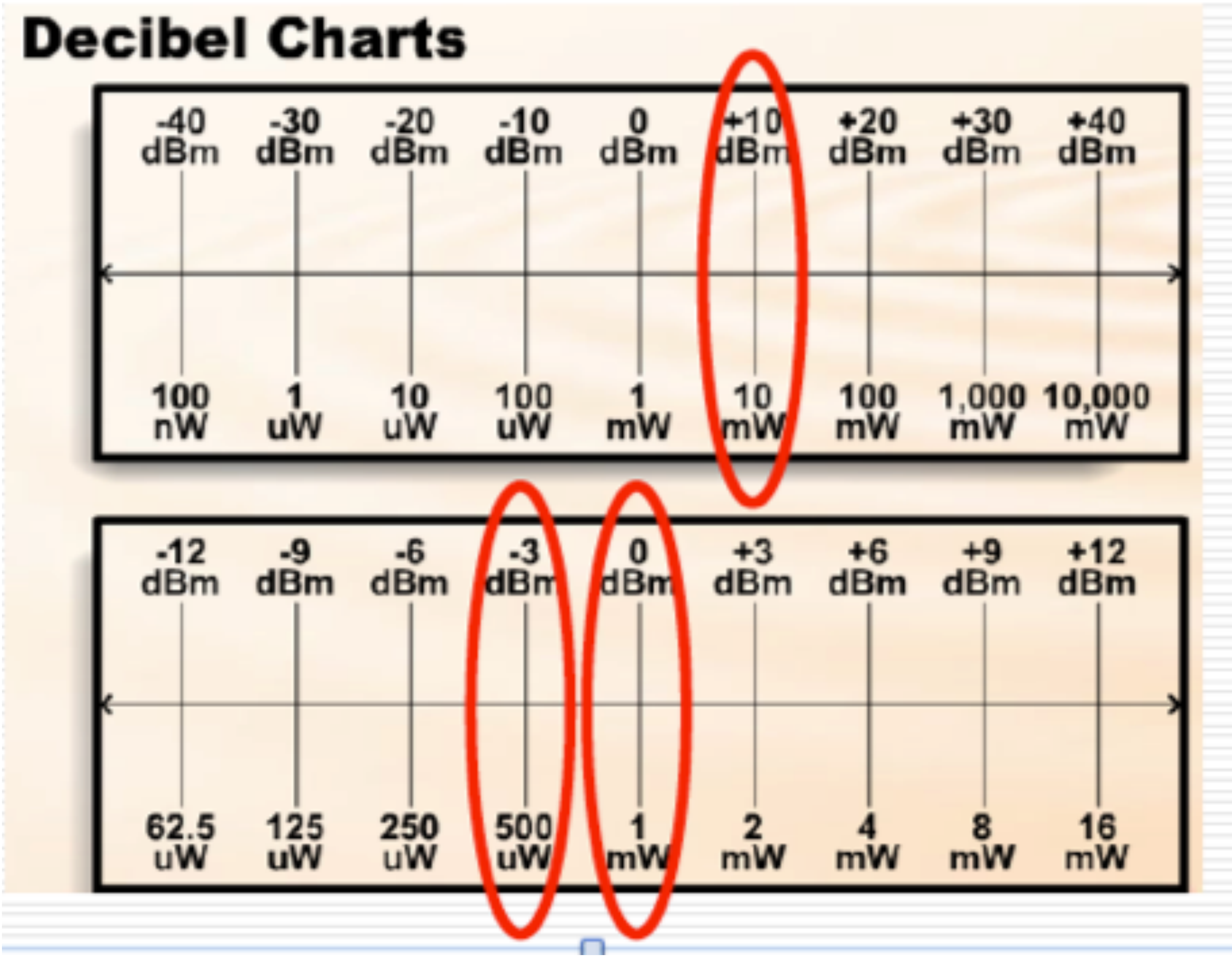
1 watt is equal to 0 dBW:

$$1W = 0dBW$$

1 milliwatt is equal to -30dBW:

$$1mW = 0.001W = -30dBW$$

dBm vs dBW


Decibel Charts
-40 dBm
-30 dBm
-20 dBm
-10 dBm
0 dBm
+10 dBm
+20 dBm
+30 dBm
+40 dBm
100 nW
1 uW
10 uW
100 uW
1 mW
10 mW
100 mW
1,000 mW
10,000 mW
-12 dBm
-9 dBm
-6 dBm
-3 dBm
0 dBm
+3 dBm
+6 dBm
+9 dBm
+12 dBm
62.5 uW
125 uW
250 uW
500 uW
1 mW
2 mW
4 mW
8 mW
16 mW

Example

The periodic voltage waveform appears across a *600Ω* resistive load. Find average power dissipated in the load & corresponding dBm value.

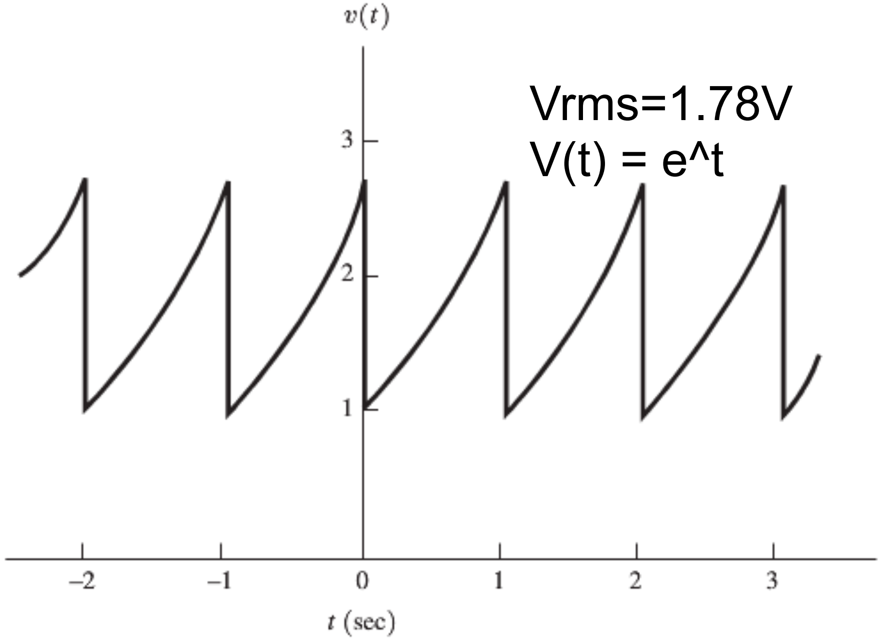
$$P = V_{rms}^2 / R = (1.79)^2 / 600 = 5.32\ mW \text{ and}$$

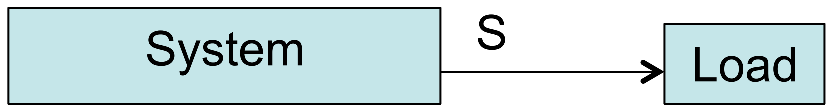
$$10\log\left(\frac{P}{10^{-3}}\right) = 10\log\left(\frac{5.32x10^{-3}}{10^{-3}}\right) = 7.26dBm$$

Note: The peak instantaneous power is

$$\max[p(t)] = \max[v(t)i(t)] = \max[v(t)^2 / R]$$

$$= (e)^2 / 600 = 12.32mW \quad \text{(at t=0)}$$





Phasor Complex Number

- Complex number *c*

$$c = x + jy = |c| e^{j\varphi}$$
$$|c| = \sqrt{x^2 + y^2}, \varphi = \tan^{-}(y/x)$$

where *x*, *y*, & φ are real numbers.

- *c is a phasor* if it is used to represent a sinusoidal waveform

i.e., $\omega(t) = |c| \cos[\omega_0 t + \angle c] = \mathrm{Re}\{c e^{j\omega_0 t}\}$

where the phasor $c = |c| e^{j\angle c}$ and Re{.} denotes the real part of complex quantity {.}.

- $c e^{j\omega_0 t}$ is a rotating phasor as distinguished from phasor *c*.

Examples:

$$v_1 = 10\cos(\omega_0 t + 35^o) = 10\angle 35^o$$

$$v_2 = 25\sin(2\pi 500 t + 45^o) =$$
$$25\cos(2\pi 500 t + 45 - 90) =$$
$$25\cos(2\pi 500 t - 45) = 25\angle -45^o$$

where $\omega_0 = 2\pi f_0, f_0 = 500\ Hz$

Remember: Complex numbers can be expressed using **Cartesian** or **Polar** Coordinate Systems

Note that when we use Phasor representation the frequency information is ignored!

More on dB….

- Read on your own

Power in Telecommunication Systems – Power change can have large dynamic range

- Remember:

$$10^x = y \xrightarrow{then} \log(10^x) = \log y \xrightarrow{Hence} x = \log y$$

- Example 1: if P2=2mW and P1 = 1mW →

  $10\log_{10}(P2/P1) = 3.01$ dB

- Example 2: if P2=1KW and P1=10W →20dB
- What if dB is given and you must find P2/P1?
  - P2/P1 = Antilog(dB/10) = $10^{dB/10}$.
- Example 3: if dB is +10 what is P2/P1?
  - P2/P1 = Antilog(+10/10) = $10^{+10/10}$ = 10

We tend to express power in dBW or dBm

Decibel values refer to relative magnitudes or changes in magnitude, not to an absolute level. It is convenient to be able to refer to an absolute level of power or voltage in decibels so that gains and losses with reference to an initial signal level may be calculated easily. The **dBW (decibel-Watt)** is used extensively in microwave applications. The value of 1 W is selected as a reference and defined to be 0 dBW. The absolute decibel level of power in dBW is defined as

$$\text{Power}_{\text{dBW}} = 10 \log \frac{\text{Power}_{\text{W}}}{1\ \text{W}}$$

EXAMPLE 3.9 A power of 1000 W is 30 dBW, and a power of 1 mW is −30 dBW.

Another common unit is the **dBm (decibel-milliWatt)**, which uses 1 mW as the reference. Thus 0 dBm = 1 mW. The formula is

$$\text{Power}_{\text{dBm}} = 10 \log \frac{\text{Power}_{\text{mW}}}{1\ \text{mW}}$$

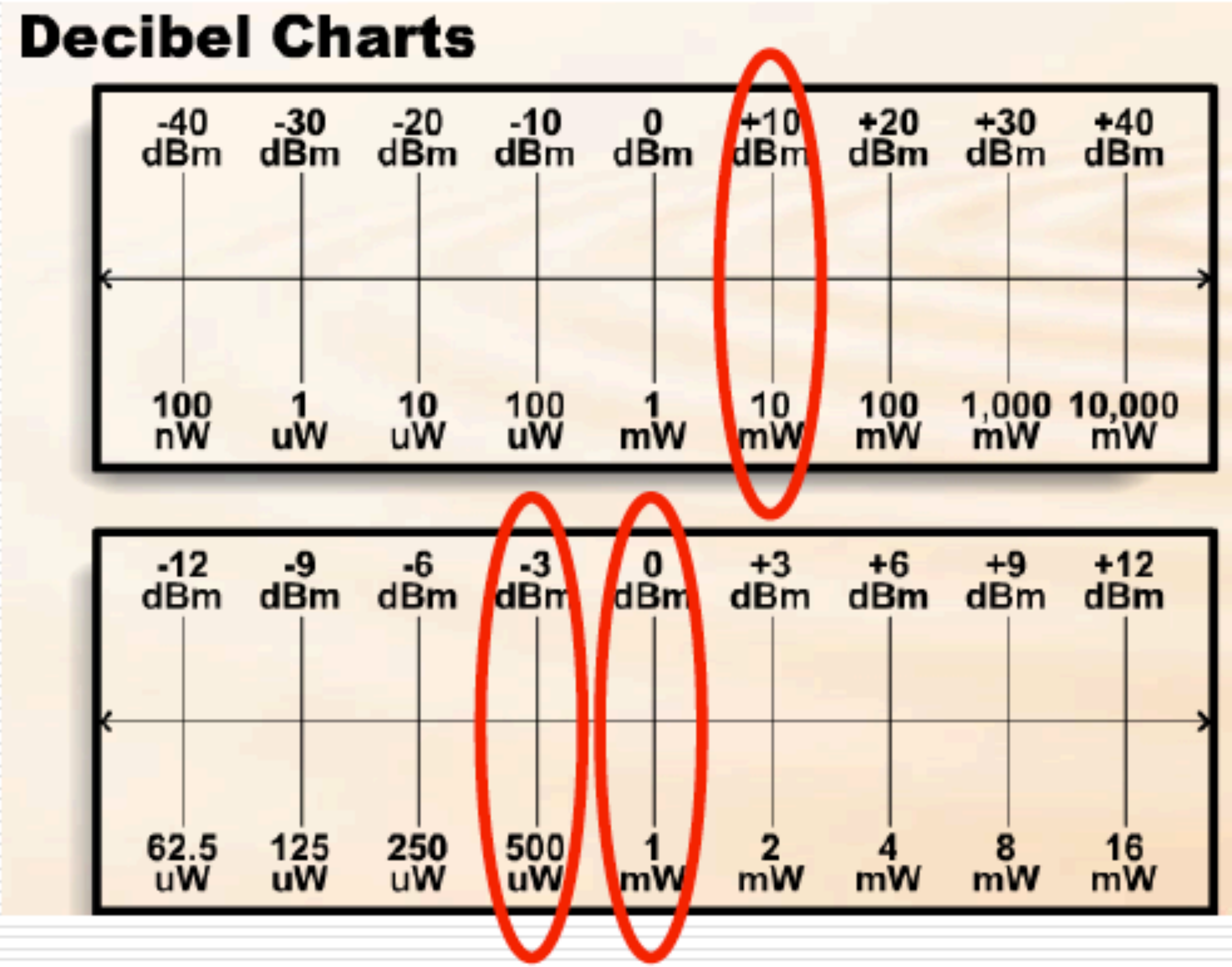
Note the following relationships:

$$+30\ \text{dBm} = 0\ \text{dBW}$$
$$0\ \text{dBm} = -30\ \text{dBW}$$

A unit in common use in cable television and broadband LAN applications is the **dBmV (decibel-millivolt)**. This is an absolute unit with 0 dBmV equivalent to 1 mV. Thus

$$\text{Voltage}_{\text{dBmV}} = 20 \log \frac{\text{Voltage}_{\text{mV}}}{1\ \text{mV}}$$

dBm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1,000 mW
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-12 dBm
-9 dBm
-6 dBm
-3 dBm
0 dBm
+3 dBm
+6 dBm
+9 dBm
+12 dBm
62.5 uW
125 uW
250 uW
500 uW
1 mW
2 mW
4 mW
8 mW
16 mW

You Should Know…

- Time Average
- Dc Value
- Power
- Instantaneous Power
- Average Power
- RMS
- Normalized Power
- Energy And Power Waveforms
- Decibel
- dBm And dB And dBW
- Signal-to-noise Ratio
- Phasor Representation

DEMO

References

- Leon W. Couch II, Digital and Analog Communication Systems, 8th edition, Pearson / Prentice, Chapter 1
- Electronic Communications System: Fundamentals Through Advanced, Fifth Edition by Wayne Tomasi – Chapter 2
  (https://www.goodreads.com/book/show/209442.Electronic_Communications_System)