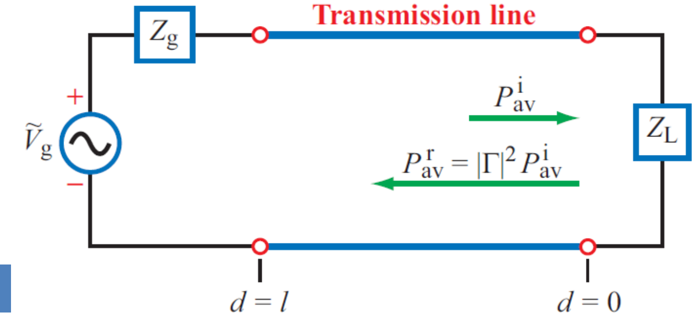
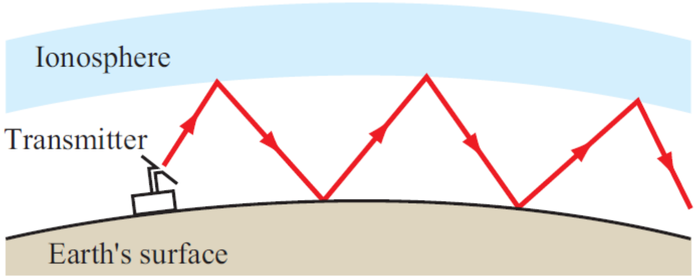
7. PLANE WAVE PROPAGATION

Review



- We have learned about wave propagation
  - Guided propagation
    - Skywave [f=3-3- Mhz]
    - Cable
- Transmission Line
  - Reflected wave
    - Constructive Parameters
    - Standing waves
  - Wave equation
    - Time representation
    - Phasor representation
      - Propagation constant

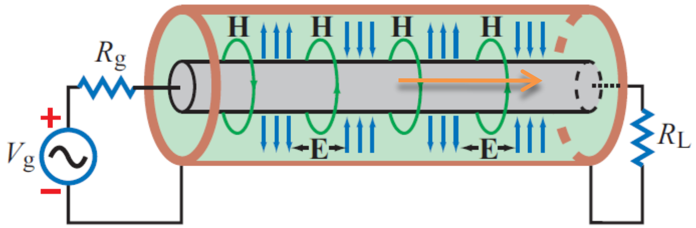


We have been assuming TEM waves:

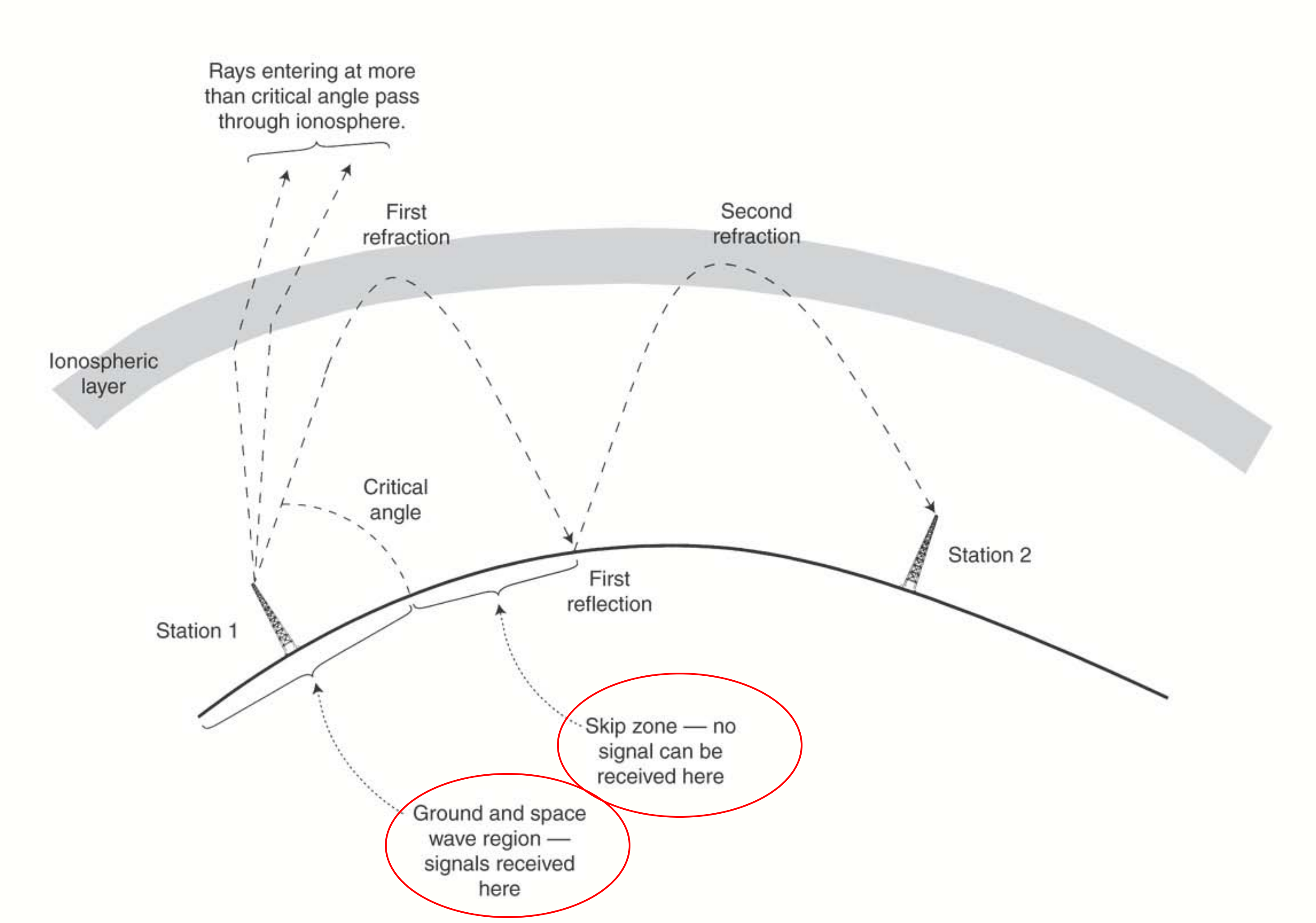
Direction of propagation is in Z

E is in r direction

H is radial

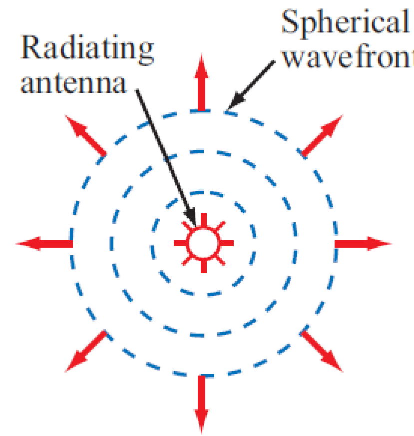


Sky Wave (Skip) Propagation

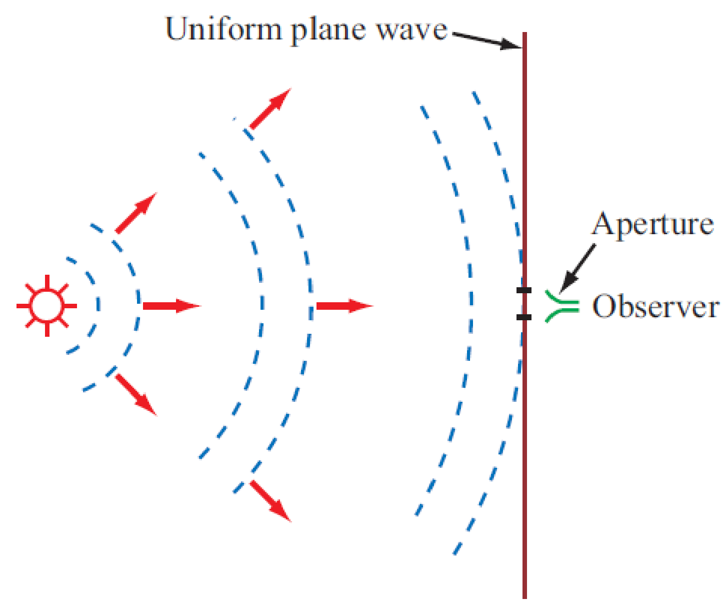


EM Waves can be unguided:

1. EM Source radiates → Spherical wave
2. Spherical wave → Planer wave (far-field effect)
3. Planer waves are **uniform**
4. We consider **TEM waves**



(a) Spherical wave



(b) Plane-wave approximation

Unbounded EM Waves

Planer Waves

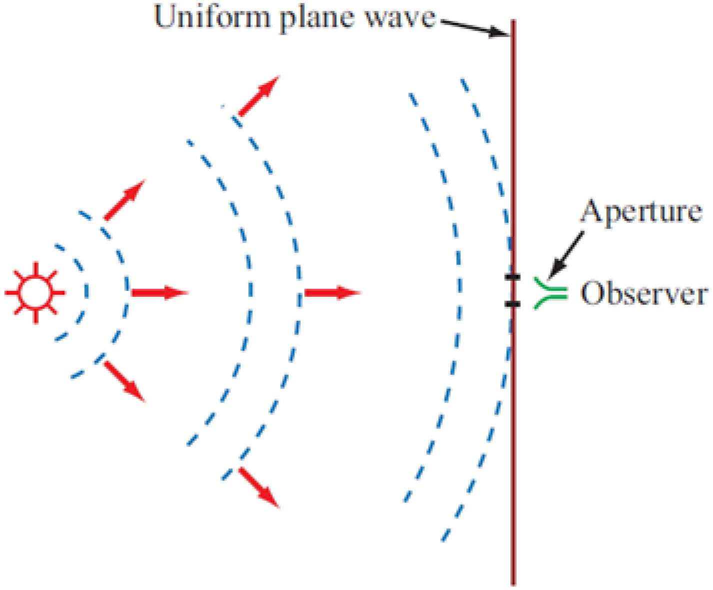
Unbounded EM Waves:

1. Waves are traveling in dielectric (perfect dielectric → lossless media)
2. We use **wave equations** instead of transmission line equations
3. We refer to **intrinsic impedance** rather than characteristic impedance, Zo
4. Propagation constant = loss + Phase constant
5. **k = wave number** (same as phase constant in transmission line)

$$u_p = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \text{(m/s)},$$

$$\lambda = \frac{2\pi}{k} = \frac{u_p}{f} \qquad \text{(m)}.$$

We start by considering phasor form!



(b) Plane-wave approximation

Review of Maxwell's Equations

POINT FORM
$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$

THIS IS WHAT WE HAVE LEARNED SO FAR……

We now express these in phasor form. HOW?

Review of Maxwell's Equations – General Form

POINT FORM

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \widetilde{\mathbf{E}} = \tilde{\rho}_{\mathrm{v}}/\varepsilon,$$

$$\nabla \times \widetilde{\mathbf{E}} = -j\omega\mu\widetilde{\mathbf{H}},$$

$$\nabla \cdot \widetilde{\mathbf{H}} = 0,$$

$$\nabla \times \widetilde{\mathbf{H}} = \widetilde{\mathbf{J}} + j\omega\varepsilon\widetilde{\mathbf{E}}.$$

All the fields are in phasor form

Time derivatives are expressed differently:d/dt → jw

Maxwell's Equations – Free Space Set

- We assume there are **no charges** in free space and thus, $J_c = \sigma E = 0$

POINT FORM	INTEGRAL FORM
$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot d\mathbf{l} = \int_S \left(-\frac{\partial \mathbf{B}}{\partial t} \right) \cdot d\mathbf{S}$
$\nabla \cdot \mathbf{D} = 0$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = 0$
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$

Time-varying E and H cannot exist independently!
If dE/dt non-zero→ dD/dt is non-zero →Curl of H is non-zero → H is non-zero

If H is a function of time → E must exist!

Maxwell's Equations – Free Space Set

- We assume there are **no charges** in free space and thus, $J_c = \sigma E = 0$

POINT FORM	Phasor Form
$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$	$\nabla \cdot \tilde{\mathbf{E}} = 0,$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \tilde{\mathbf{E}} = -j\omega\mu\tilde{\mathbf{H}},$
$\nabla \cdot \mathbf{D} = 0$	$\nabla \cdot \tilde{\mathbf{H}} = 0,$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \times \tilde{\mathbf{H}} = j\omega\varepsilon\,\tilde{\mathbf{E}}.$

We will use these to derive the wave equation for EM waves.

Wave Equations - Assume no volume charges

Complex permittivity

$$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega},$$

$$\nabla \cdot \widetilde{\mathbf{E}} = 0,$$

$$\nabla \times \widetilde{\mathbf{E}} = -j\omega\mu\widetilde{\mathbf{H}},$$

$$\nabla \cdot \widetilde{\mathbf{H}} = 0,$$

$$\nabla \times \widetilde{\mathbf{H}} = j\omega\varepsilon_c\widetilde{\mathbf{E}}.$$

$$\nabla \times (\nabla \times \widetilde{\mathbf{E}}) = -j\omega\mu(\nabla \times \widetilde{\mathbf{H}}).$$

For electrostatic Del of E is zero

$$\nabla \times (\nabla \times \widetilde{\mathbf{E}}) = -j\omega\mu(j\omega\varepsilon_c\widetilde{\mathbf{E}}) = \omega^2\mu\varepsilon_c\widetilde{\mathbf{E}}.$$

Special Property:

$$\nabla \times (\nabla \times \widetilde{\mathbf{E}}) = \nabla(\nabla \cdot \widetilde{\mathbf{E}}) - \nabla^2\widetilde{\mathbf{E}},$$

Laplacian of E

$$\nabla^2\widetilde{\mathbf{E}} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\widetilde{\mathbf{E}}.$$

$$\nabla^2\widetilde{\mathbf{E}} + \omega^2\mu\varepsilon_c\widetilde{\mathbf{E}} = 0,$$

Homogeneous Wave Equations for E and H

$$\nabla^2 \tilde{\mathbf{E}} + \omega^2 \mu \varepsilon_c \tilde{\mathbf{E}} = 0$$

$$\nabla^2 \tilde{\mathbf{E}} - \gamma^2 \tilde{\mathbf{E}} = 0.$$

Propagation Constant:

$$\gamma^2 = -\omega^2 \mu \varepsilon_c,$$

Similarly:

$$\nabla^2 \tilde{\mathbf{H}} - \gamma^2 \tilde{\mathbf{H}} = 0.$$

Complex permittivity

$$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega},$$

Homogeneous Wave Equations for E and H (Lossless case)

$$\nabla^2\widetilde{\mathbf{E}} + \omega^2\mu\varepsilon_c\widetilde{\mathbf{E}} = 0$$

$$\nabla^2\widetilde{\mathbf{E}} - \gamma^2\widetilde{\mathbf{E}} = 0.$$

Propagation Constant:

$$\gamma^2 = -\omega^2\mu\varepsilon_c,$$

Complex permittivity

$$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega},$$

Note: if lossless conductivity =0

$$\gamma^2 = -\omega^2\mu\varepsilon.$$

$$k = \omega\sqrt{\mu\varepsilon}\,.$$

$$\nabla^2\widetilde{\mathbf{E}} + k^2\widetilde{\mathbf{E}} = 0.$$

Our assumption was having a uniform plane

*A **uniform plane wave** is characterized by electric and magnetic fields that have uniform properties at all points across an infinite plane.*

- There is no **change** of field
  - For example, in x-y plane: $dE/dx = dE/dy = 0$

Uniform Plane Wave (x-y plane)

Consider vector field E:

$$\widetilde{\mathbf{E}} = \hat{\mathbf{x}}\widetilde{E}_x + \hat{\mathbf{y}}\widetilde{E}_y + \hat{\mathbf{z}}\widetilde{E}_z,$$

$$\nabla^2\widetilde{\mathbf{E}} + k^2\widetilde{\mathbf{E}} = 0.$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(\hat{\mathbf{x}}\widetilde{E}_x + \hat{\mathbf{y}}\widetilde{E}_y + \hat{\mathbf{z}}\widetilde{E}_z) + k^2(\hat{\mathbf{x}}\widetilde{E}_x + \hat{\mathbf{y}}\widetilde{E}_y + \hat{\mathbf{z}}\widetilde{E}_z) = 0.$$

Must satisfy→
There is no change in X and Y (uniform)
Same thing for Ey and Ez:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right)\widetilde{E}_x = 0,$$

Only non-zero vector component
Same thing for Ey and Ez:

$$\boxed{\frac{d^2\widetilde{E}_x}{dz^2} + k^2\widetilde{E}_x = 0.}$$

Uniform Plane Wave (x-y plane) - Solution

$$\frac{d^2\widetilde{E}_x}{dz^2} + k^2\widetilde{E}_x = 0.$$

Note A

Application of $\nabla \times \widetilde{\mathbf{E}} = -j\omega\mu\widetilde{\mathbf{H}}$ yields:

$$\widetilde{H}_y(z) = \frac{k}{\omega\mu}E_{x0}^{+}e^{-jkz} = H_{y0}^{+}e^{-jkz}$$

General Form of the Solution:

$$\widetilde{E}_x(z) = \widetilde{E}_x^{+}(z) + \widetilde{E}_x^{-}(z) = E_{x0}^{+}e^{-jkz} + E_{x0}^{-}e^{jkz}$$

Propagating in +Z

Propagating in -Z

For a wave travelling along +z only:

$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\widetilde{E}_x^{+}(z) = \hat{\mathbf{x}}E_{x0}^{+}e^{-jkz}$$

Summary: This is a plane wave with

$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\widetilde{E}_x^{+}(z) = \hat{\mathbf{x}}E_{x0}^{+}e^{-jkz},$$

$$\widetilde{\mathbf{H}}(z) = \hat{\mathbf{y}}\frac{\widetilde{E}_x^{+}(z)}{\eta} = \hat{\mathbf{y}}\frac{E_{x0}^{+}}{\eta}e^{-jkz}.$$

Power and Impedance

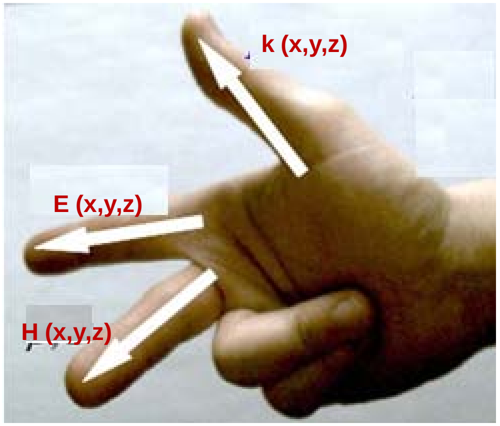
$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\widetilde{E}_x^+(z) = \hat{\mathbf{x}}E_{x0}^+ e^{-jkz},$$

$$\widetilde{\mathbf{H}}(z) = \hat{\mathbf{y}}\frac{\widetilde{E}_x^+(z)}{\eta} = \hat{\mathbf{y}}\frac{E_{x0}^+}{\eta}e^{-jkz}.$$

Represents Voltage In a transmission lines

Represents Current in a Transmission Lines

TEM traveling wave!



Phasor Form
Remember: *HEK*

Intrinsic Impedance of a lossless medium (analogy to Zo)

$$\eta = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} \qquad (\Omega)$$

Time Domain Representation

TEM Traveling Wave Solution

$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\widetilde{E}_x^+(z) = \hat{\mathbf{x}}E_{x0}^+ e^{-jkz},$$

$$\widetilde{\mathbf{H}}(z) = \hat{\mathbf{y}}\frac{\widetilde{E}_x^+(z)}{\eta} = \hat{\mathbf{y}}\frac{E_{x0}^+}{\eta}e^{-jkz}.$$

$$E_{x0}^+ = |E_{x0}^+|e^{j\phi^+}$$

$$\mathbf{E}(z,t) = \mathfrak{Re}\left[\widetilde{\mathbf{E}}(z)\, e^{j\omega t}\right]$$

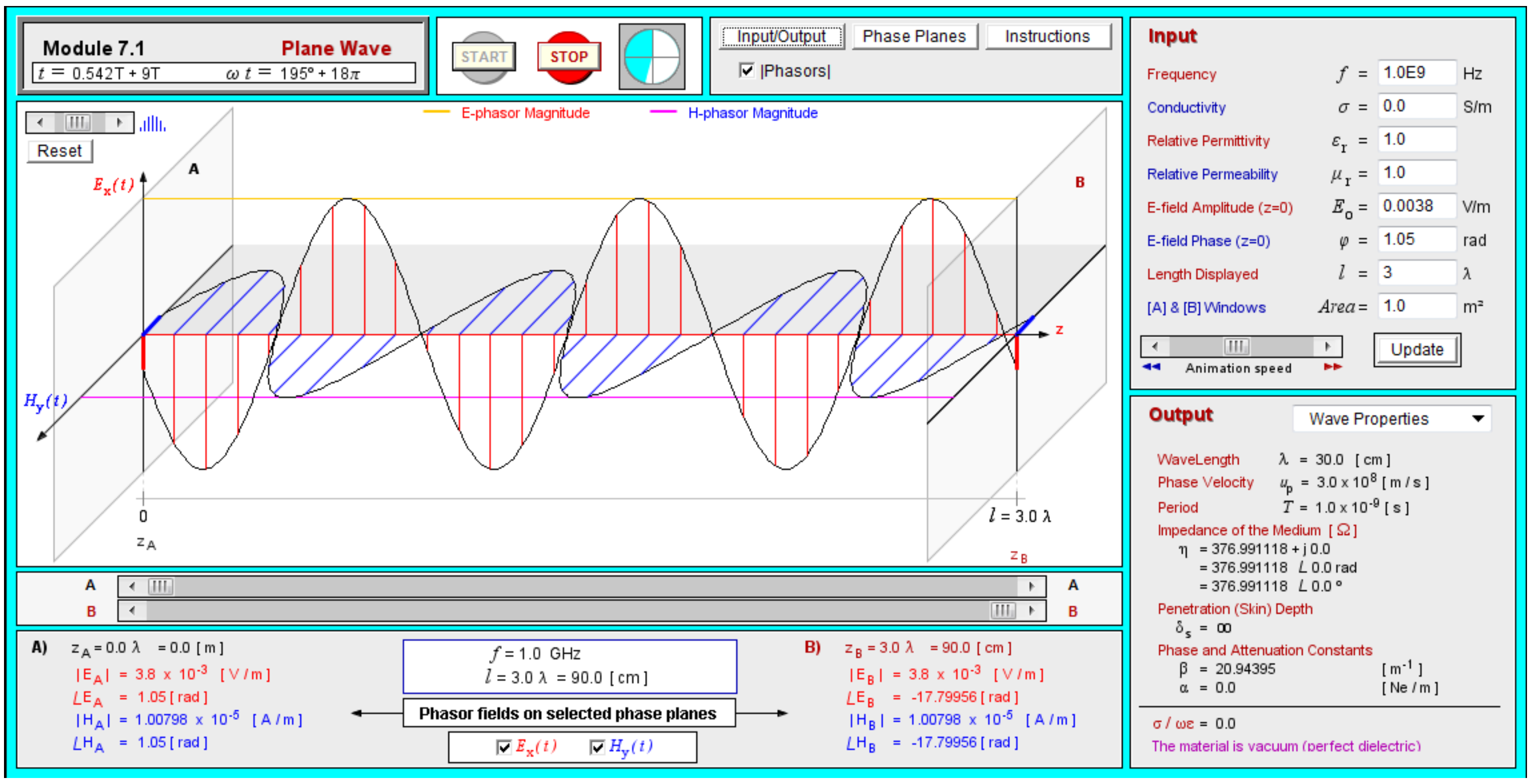
Time-Domain Solution

$$\mathbf{E}(z,t) = \mathfrak{Re}\left[\widetilde{\mathbf{E}}(z)\, e^{j\omega t}\right] = \hat{\mathbf{x}}|E_{x0}^+|\cos(\omega t - kz + \phi^+) \quad \text{(V/m)},$$

and

$$\mathbf{H}(z,t) = \mathfrak{Re}\left[\widetilde{\mathbf{H}}(z)\, e^{j\omega t}\right] = \hat{\mathbf{y}}\frac{|E_{x0}^+|}{\eta}\cos(\omega t - kz + \phi^+) \quad \text{(A/m)}.$$

Check the Simulator

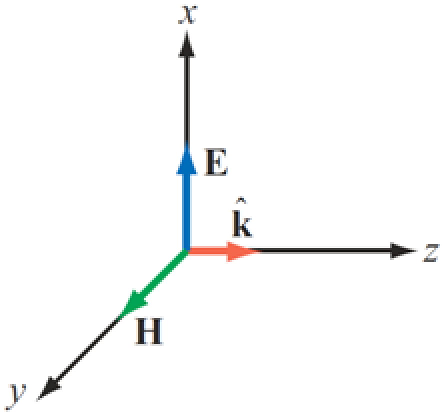


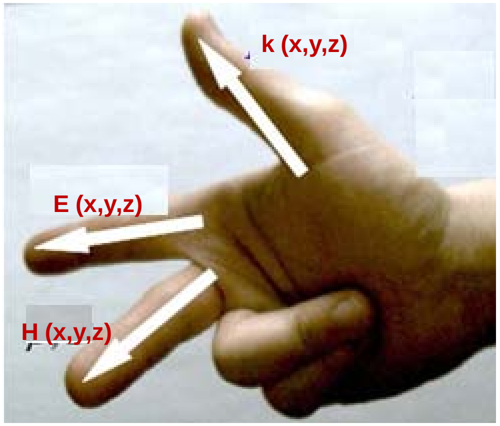
Directional Relation Between **E** and **H**

For Any TEM Wave

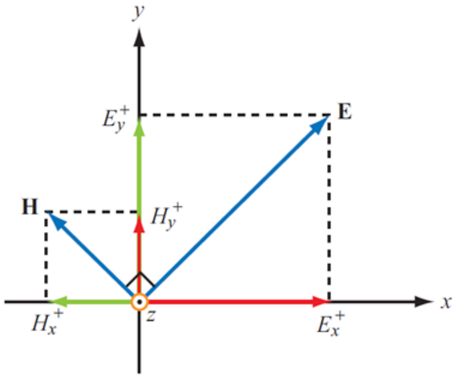
$$\widetilde{\mathbf{H}} = \frac{1}{\eta}\,\hat{\mathbf{k}} \times \widetilde{\mathbf{E}},$$

$$\widetilde{\mathbf{E}} = -\eta\,\hat{\mathbf{k}} \times \widetilde{\mathbf{H}}.$$





Phasor Form



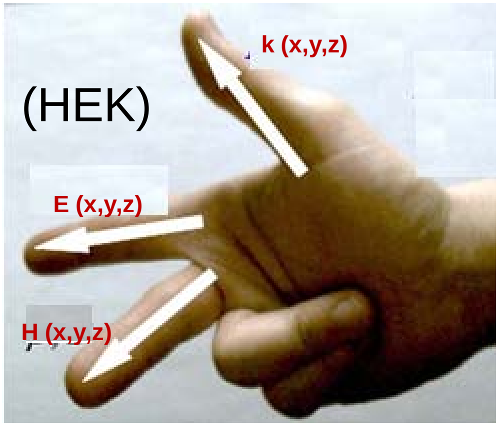
Note:
E and H may have x & y components
However, they travel in Z direction and
They are perpendicular to each other!

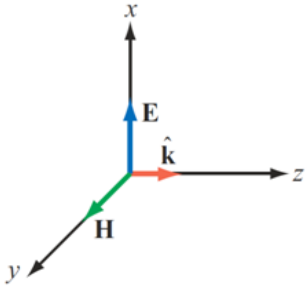
Example

The electric field of a 1-MHz plane wave traveling in the $+z$-direction in air points along the x-direction. If this field reaches a peak value of 1.2π (mV/m) at $t = 0$ and $z = 50$ m, obtain expressions for $\mathbf{E}(z, t)$ and $\mathbf{H}(z, t)$, and then plot them as a function of z at $t = 0$.

What is k? (it is a function of what? Which direction is it pointing at?)
What is E?
What is H?

K(z) in +Z direction
E(z,t) in +X direction
H(z,t) in +Y direction





Note B

Example

The electric field of a 1-MHz plane wave traveling in the $+z$-direction in air points along the x-direction. If this field reaches a peak value of 1.2π (mV/m) at $t = 0$ and $z = 50$ m, obtain expressions for $\mathbf{E}(z, t)$ and $\mathbf{H}(z, t)$, and then plot them as a function of z at $t = 0$.

Find l, k, E(z,t), H(z,t)

$$\lambda = \frac{c}{f} = \frac{3 \times 10^{8}}{1 \times 10^{6}} = 300 \text{ m},$$

$$k = (2\pi/300) \text{ (rad/m)}.$$

$$\mathbf{E}(z, t) = \hat{\mathbf{x}}|E_{x0}^{+}| \cos(\omega t - kz + \phi^{+})$$

$$= \hat{\mathbf{x}}\, 1.2\pi \cos\left(2\pi \times 10^{6} t - \frac{2\pi z}{300} + \phi^{+}\right) \text{ (mV/m)}.$$

$$-\frac{2\pi \times 50}{300} + \phi^{+} = 0 \qquad \text{or} \qquad \phi^{+} = \frac{\pi}{3}$$

Notes B

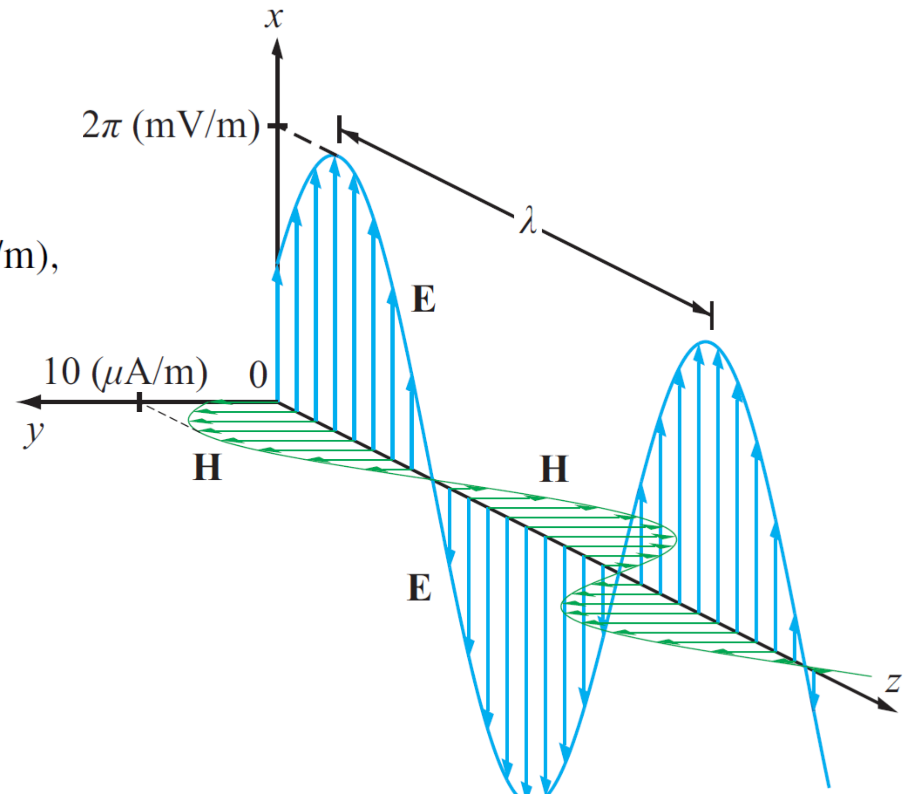
Example cont.

Hence,

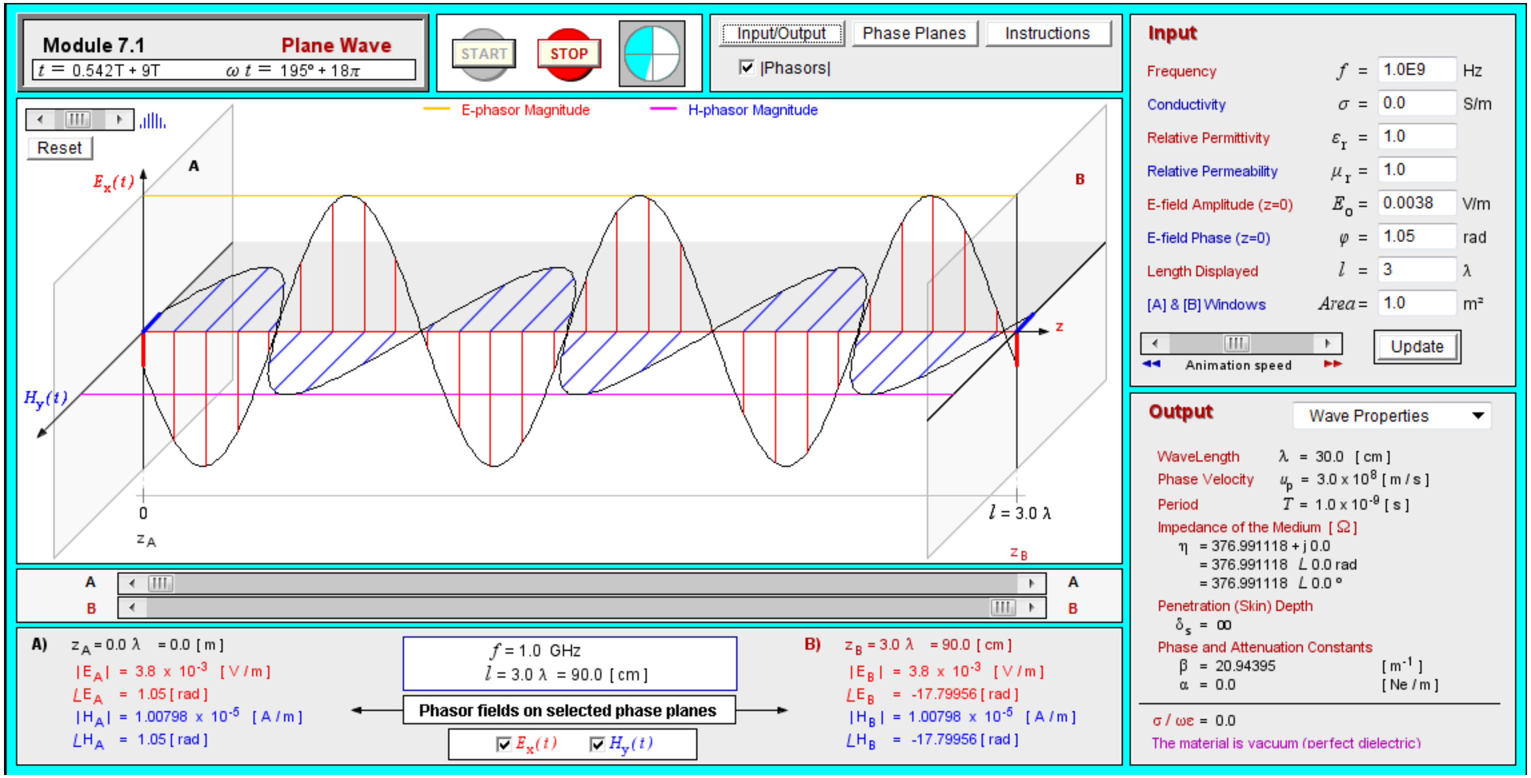
$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\, 1.2\pi \cos\left(2\pi \times 10^6 t - \frac{2\pi z}{300} + \frac{\pi}{3}\right) \quad \text{(mV/m)},$$

and from Eq. (7.34b) we have

$$\mathbf{H}(z,t) = \hat{\mathbf{y}}\, \frac{E(z,t)}{\eta_0}$$
$$= \hat{\mathbf{y}}\, 10 \cos\left(2\pi \times 10^6 t - \frac{2\pi z}{300} + \frac{\pi}{3}\right) \quad (\mu\text{A/m}),$$



Check out the Simulator

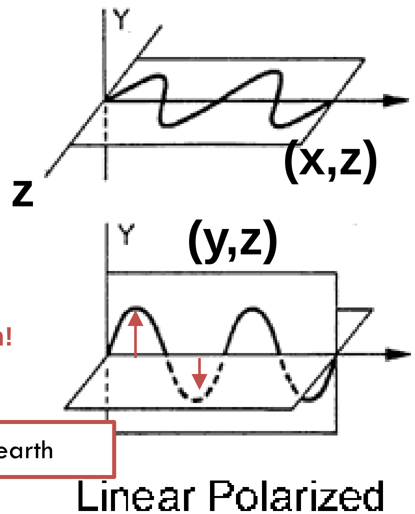


Polarization - General

- Polarization is the orientation of electric field component of an electromagnetic wave relative to the Earth's surface.
- Polarization is important to get the maximum performance from the antennas
- There are different types of polarization (depending on existence and changes of different electric fields)
  - Linear
    - Horizontal (E field changing in parallel with respect to earth's surface)
    - Vertical (E field going up/down with respect to earth's surface)
    - Dual polarized
  - Circular (Ex and Ey)
    - Similar to satellite communications
    - TX and RX antennas must agree on direction of rotation
  - Elliptical
- Linear polarization is used in WiFi communications

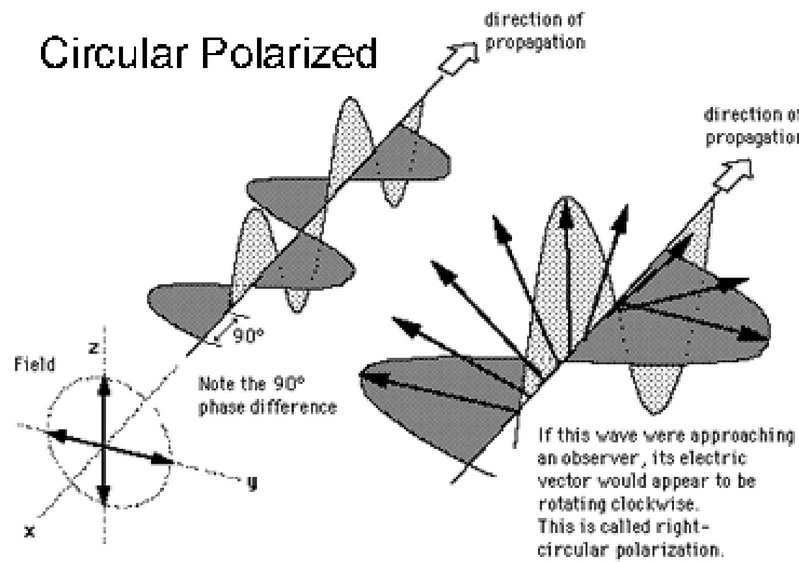
Polarization can change as the signal travels away from the source!
-Due to the magnetic field of Earth (results in Faraday rotation)
-Due to reflection

Polarization - General



E-Field is Going up/down respect to Earth! (Vertical Polarization)

Propagating parallel to earth



E-Field is Rotating (or Corkscrewed) as they are traveling

- **Polarization is important to get the maximum performance from the antennas**
  - The polarization of the antennas at both ends of the path must use the same polarization
  - This is particularly important when the transmitted power is limited

Wave Polarization

*The **polarization** of a uniform plane wave describes the locus traced by the tip of the **E** vector (in the plane orthogonal to the direction of propagation) at a given point in space as a function of time.*

See Notes

Wave Polarization

Plane wave propagating along +z :

$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\widetilde{E}_x(z) + \hat{\mathbf{y}}\widetilde{E}_y(z),$$

If: $E_{x0} = a_x,$

$E_{y0} = a_y e^{j\delta},$

Delta = Angle difference

then

$$\widetilde{\mathbf{E}}(z) = (\hat{\mathbf{x}}a_x + \hat{\mathbf{y}}a_y e^{j\delta})e^{-jkz},$$

$$\widetilde{E}_x(z) = E_{x0}e^{-jkz},$$

$$\widetilde{E}_y(z) = E_{y0}e^{-jkz},$$

$$\begin{aligned}\mathbf{E}(z,t) &= \mathfrak{Re}\left[\widetilde{\mathbf{E}}(z)\, e^{j\omega t}\right] \\ &= \hat{\mathbf{x}}a_x \cos(\omega t - kz) \\ &\quad + \hat{\mathbf{y}}a_y \cos(\omega t - kz + \delta).\end{aligned}$$

$$\frac{d^2\widetilde{E}_x}{dz^2} + k^2\widetilde{E}_x = 0.$$

Polarization State

Polarization state describes the trace of **E** as a function of time at a fixed *z*

$$\widetilde{\mathbf{E}}(z) = (\hat{\mathbf{x}} a_x + \hat{\mathbf{y}} a_y e^{j\delta}) e^{-jkz},$$

$$\begin{aligned} \mathbf{E}(z,t) &= \mathfrak{Re}\left[\widetilde{\mathbf{E}}(z)\, e^{j\omega t}\right] \\ &= \hat{\mathbf{x}} a_x \cos(\omega t - kz) \\ &\quad + \hat{\mathbf{y}} a_y \cos(\omega t - kz + \delta). \end{aligned}$$

Time domain representation

Magnitude of **E**

$$\begin{aligned} |\mathbf{E}(z,t)| &= [E_x^2(z,t) + E_y^2(z,t)]^{1/2} \\ &= [a_x^2 \cos^2(\omega t - kz) \\ &\quad + a_y^2 \cos^2(\omega t - kz + \delta)]^{1/2} \end{aligned}$$

Inclination Angle

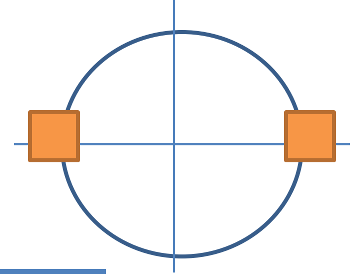
$$\psi(z,t) = \tan^{-1}\left(\frac{E_y(z,t)}{E_x(z,t)}\right)$$

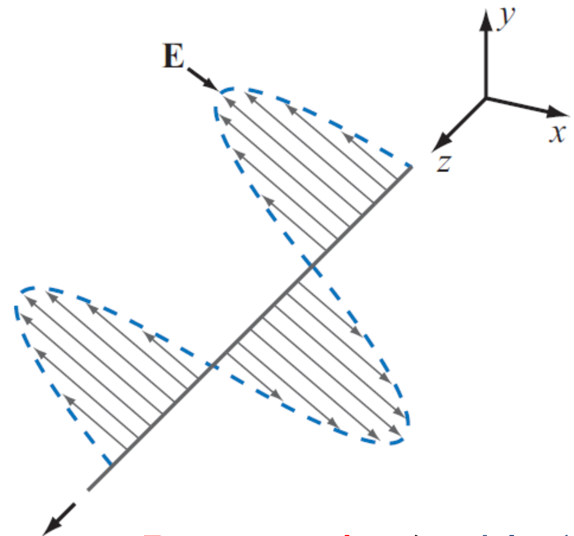
psi

Linear Polarization:

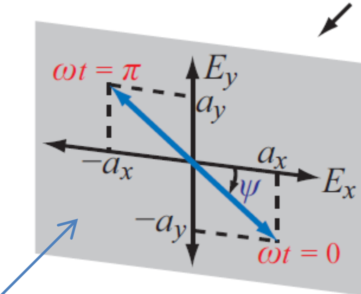
$\delta = 0$ or $\delta = \pi$

In-phase | Out-of-phase





$$\mathbf{E}(z,t) = \mathfrak{Re}\left[\widetilde{\mathbf{E}}(z)\, e^{j\omega t}\right]$$
$$= \hat{\mathbf{x}} a_x \cos(\omega t - kz)$$
$$+ \hat{\mathbf{y}} a_y \cos(\omega t - kz + \delta).$$
$$= (\hat{\mathbf{x}} a_x + \hat{\mathbf{y}} a_y) \cos(\omega t - kz) \quad \text{(in-phase)},$$
$$= (\hat{\mathbf{x}} a_x - \hat{\mathbf{y}} a_y) \cos(\omega t - kz) \quad \text{(out-of-phase)}$$



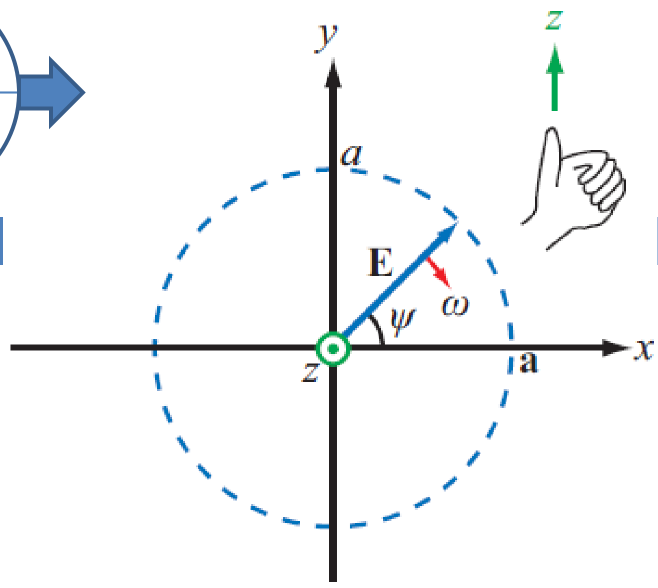
E traces a line(in blue) as the wave traverses a fixed plane

If $a_y = 0$, then $\psi = 0°$ or $180°$, and the wave is x-polarized; conversely, if $a_x = 0$, then $\psi = 90°$ or $-90°$, and the wave is y-polarized.

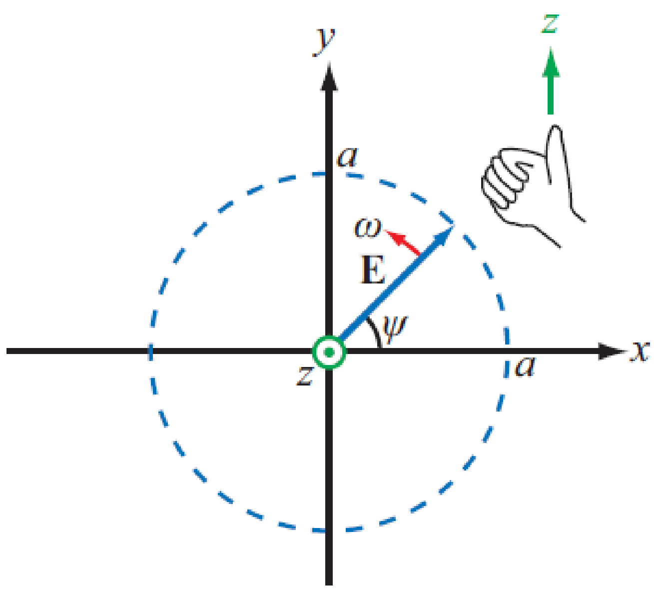
Circular Polarization

LHP: $a_x = a_y = a$ and $\delta = \pi/2$

RHP: $a_x = a_y = a$ and $\delta = -\pi/2$

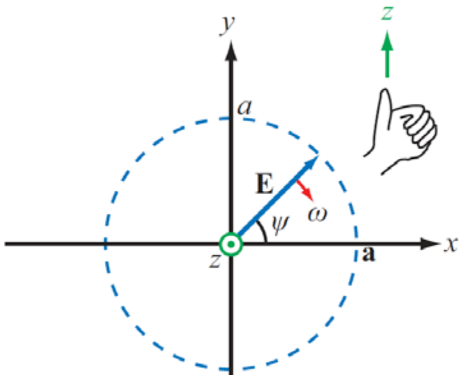


(a) LHC polarization



(b) RHC polarization

LH Circular Polarization



$$\mathbf{E}(z,t) = \Re\mathfrak{e}\left[\widetilde{\mathbf{E}}(z)\, e^{j\omega t}\right]$$
$$= \hat{\mathbf{x}} a \cos(\omega t - kz) + \hat{\mathbf{y}} a \cos(\omega t - kz + \pi/2)$$
$$= \hat{\mathbf{x}} a \cos(\omega t - kz) - \hat{\mathbf{y}} a \sin(\omega t - kz).$$

Magnitude

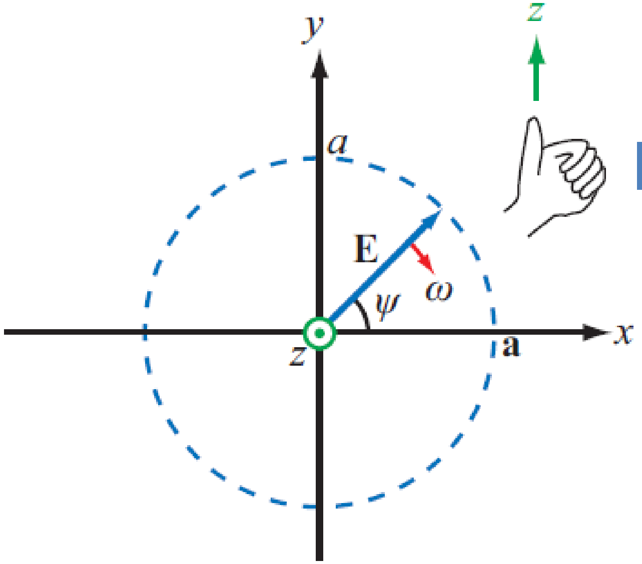
$$|\mathbf{E}(z,t)| = \left[E_x^2(z,t) + E_y^2(z,t)\right]^{1/2}$$
$$= [a^2 \cos^2(\omega t - kz) + a^2 \sin^2(\omega t - kz)]^{1/2}$$
$$= a,$$

Inclination Angle

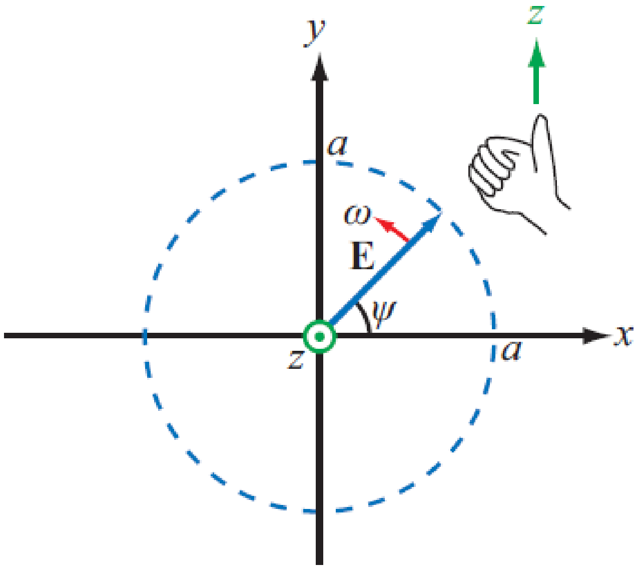
$$\psi(z,t) = \tan^{-1}\left[\frac{E_y(z,t)}{E_x(z,t)}\right]$$
$$= \tan^{-1}\left[\frac{-a \sin(\omega t - kz)}{a \cos(\omega t - kz)}\right]$$
$$= -(\omega t - kz).$$

RH Circular Polarization:

$a_x = a_y = a$ and $\delta = -\pi/2$



(a) LHC polarization

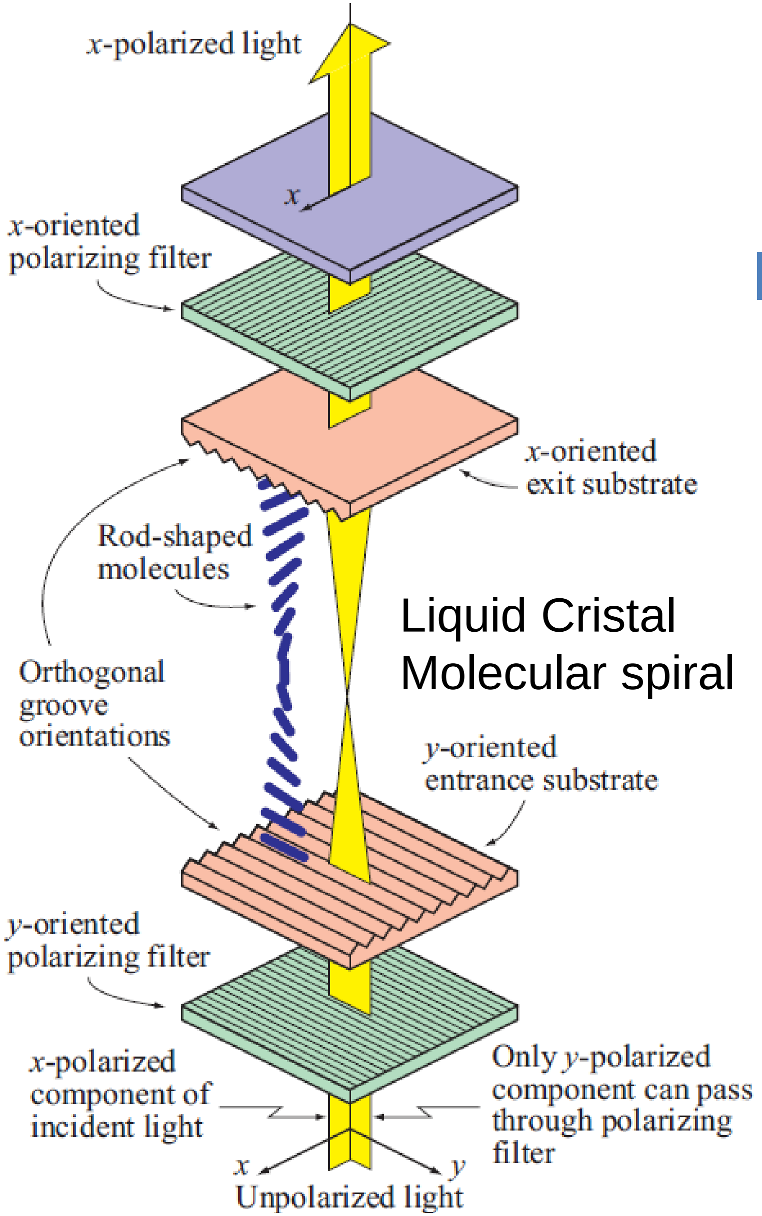


(b) RHC polarization

Example

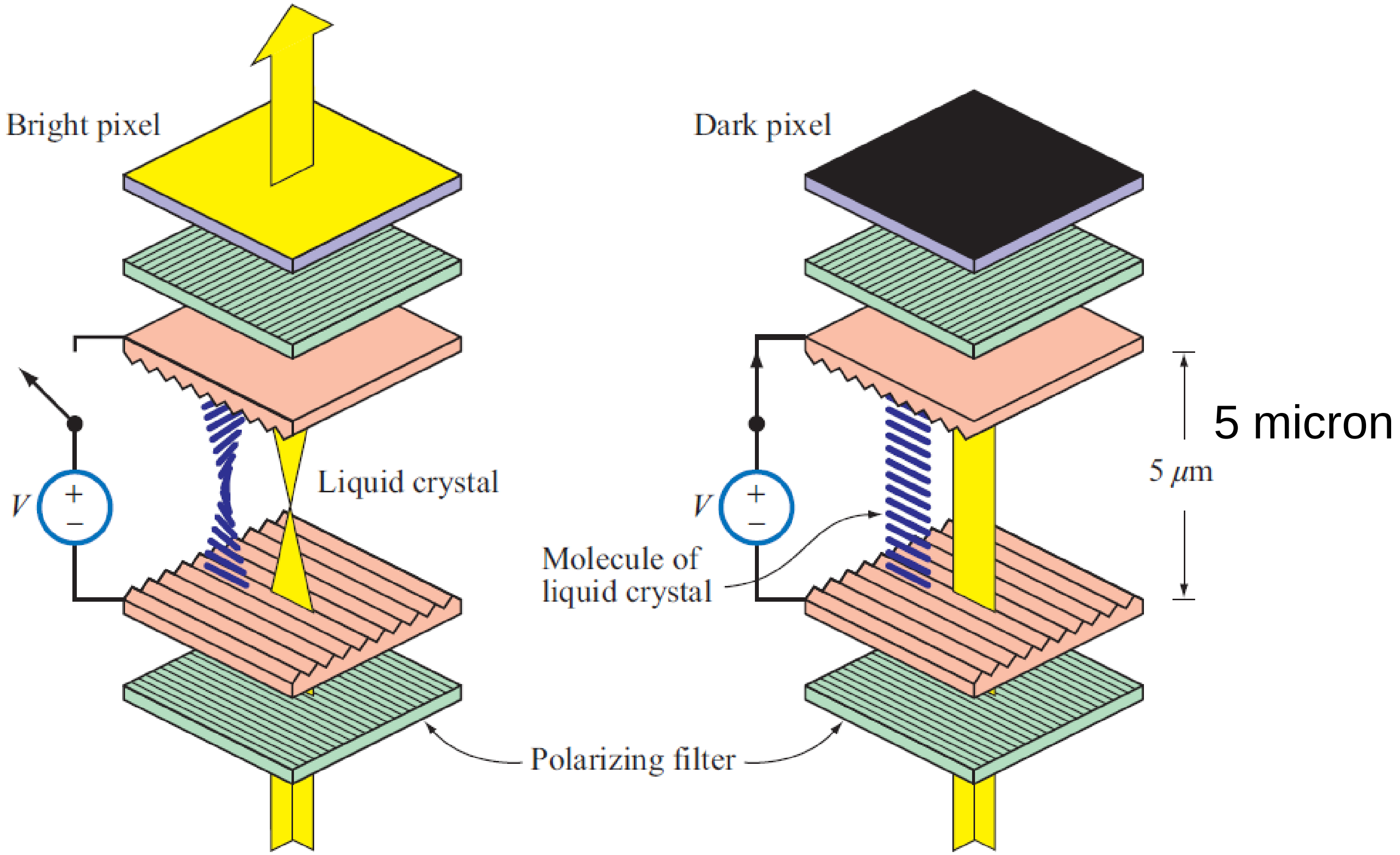
See notes

LCD

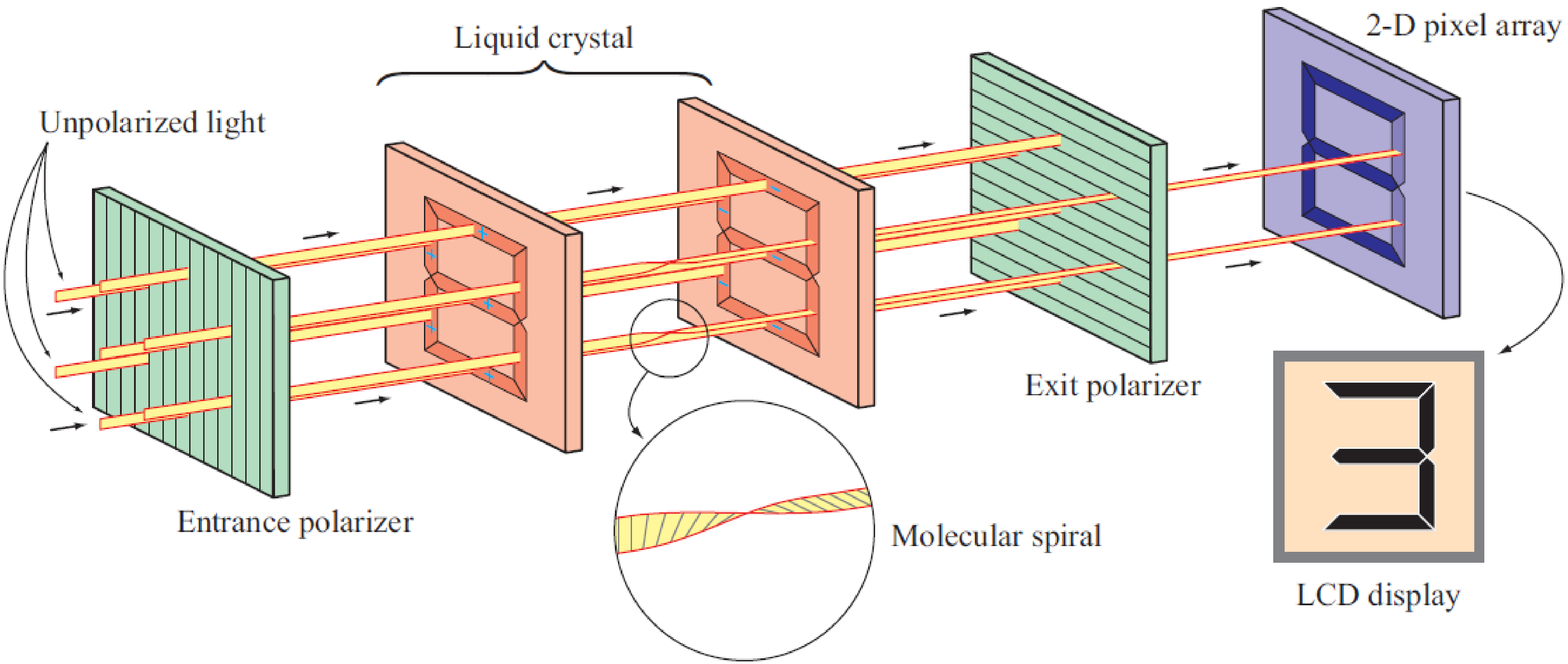


x-polarized light
x
x-oriented polarizing filter
x-oriented exit substrate
Rod-shaped molecules
Liquid Cristal Molecular spiral
Orthogonal groove orientations
y-oriented entrance substrate
y-oriented polarizing filter
x-polarized component of incident light
Only y-polarized component can pass through polarizing filter
x
y
Unpolarized light

Operation of a Single Pixel



LCD 2-D Array



Liquid crystal
2-D pixel array
Unpolarized light
Exit polarizer
Entrance polarizer
Molecular spiral
LCD display