6. MAXWELL’S EQUATIONS IN TIME-VARYING FIELDS

Applets

- http://www.oerrecommender.org/visits/119103
- http://webphysics.davidson.edu/physlet_resources/bu_semester2/
- Make a video http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=687.0
- Very Cool Applets http://micro.magnet.fsu.edu/electromag/java/faraday2/

So far….

- Static Electromagnetic ….
  - No change in time (static)
- We now look at cases where currents and charges vary in time → H & E fields change accordingly
  - Examples: light, x-rays, infrared waves, gamma rays, radio waves, etc
- We refer to these waves as **time-varying** electromagnetic waves
  - A set of new equations are required!

Maxwell's Equations

Maxwell's equations.

Same for Dynamic & Static

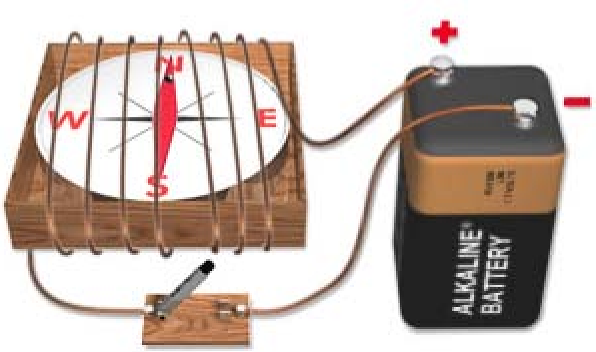
Reference	Differential Form	Integral Form	
Gauss's law	$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$	(6.1)
Faraday's law	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ (Static :Zero)	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$ (Static :Zero)	(6.2)*
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$	(6.3)
Ampère's law	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ (Static :J)	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{s}$ (Static :I)	(6.4)

*For a stationary surface S.

In this chapter, we will examine Faraday's and Ampère's laws

A little History

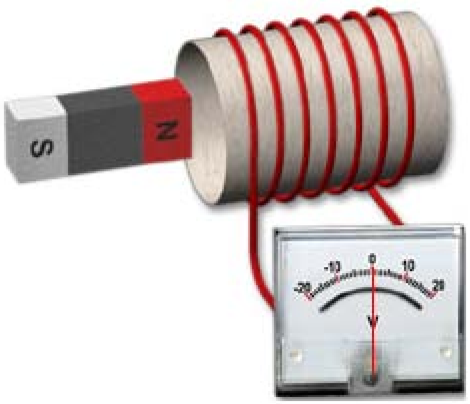
- **Oersted** demonstrated the relation between electricity and magnetism
  - Current impacts (excerpts force on) a compass needle
    - Fm is due to the magnetic field
    - When current in Z then needle moves to Phi direction
- http://micro.magnet.fsu.edu/electromag/java/compass/index.html



Induced magnetic field can influence the direction of the compass needle. When we connect the circuit, the conducting wire wrapped around the compass is energized creating a magnetic field that counteracts the effects of the Earth's magnetic field and changes the direction of the compass needle.

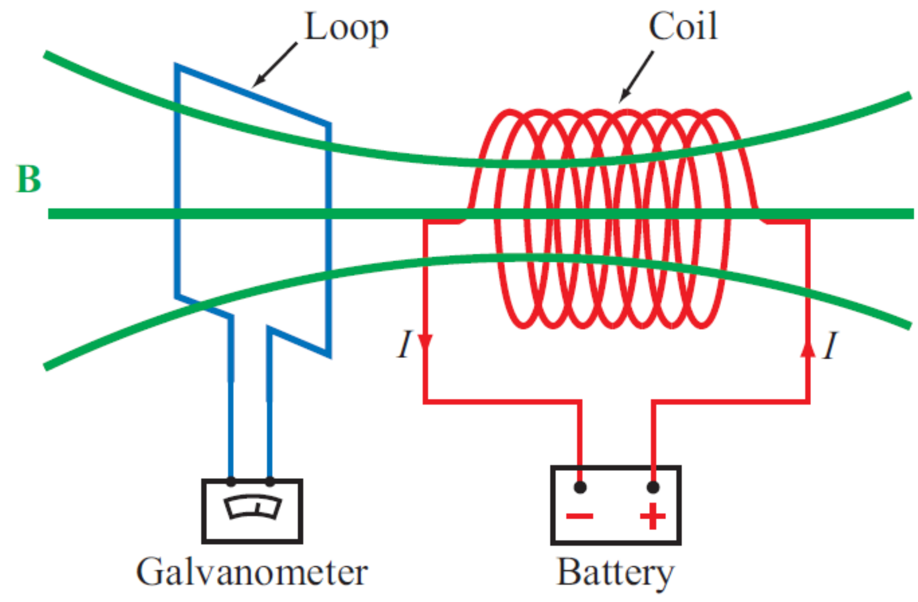
A Little History

- Faraday (in London) hypothesized that magnetic field should induce current!
  - Henry in Albany independently trying to prove this!
- They showed that magnetic fields can produce electric current
  - The key to this induction process is CHANGE
  - http://micro.magnet.fsu.edu/electro 2/index.html
  - Another applet: http://phet.colorado.edu/sims law/faradays-law_en.html



galvanometer needle moves

Faraday's Law



Electromotive force (voltage) induced by time-varying magnetic flux:

$$V_{\text{emf}} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s} \qquad \text{(V)}$$

Magnetic Flux (Wb)

We can generate the current through the loop By moving the loop or changing direction of current

Magnetic fields can produce an electric current in a closed loop,

When the meter detects current → voltage has been induced → electromotive force has been created → this process is called emf induction

Three types of EMF

1. A time-varying magnetic field linking a stationary loop; the induced emf is then called the ***transformer emf***, $V_{\text{emf}}^{\text{tr}}$.

Stationary loop; B changing

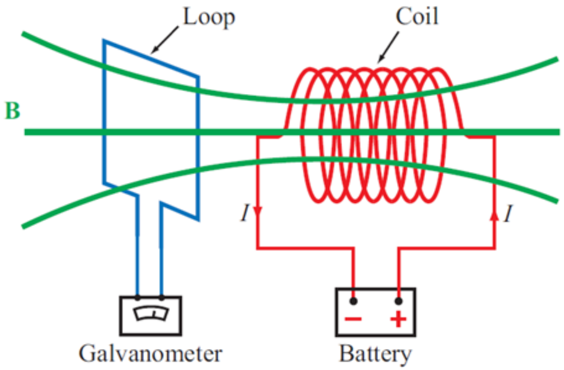
2. A moving loop with a time-varying surface area (relative to the normal component of **B**) in a static field **B**; the induced emf is then called the ***motional emf***, $V_{\text{emf}}^{\text{m}}$.

Moving loop; B fixed

3. A moving loop in a time-varying field **B**.

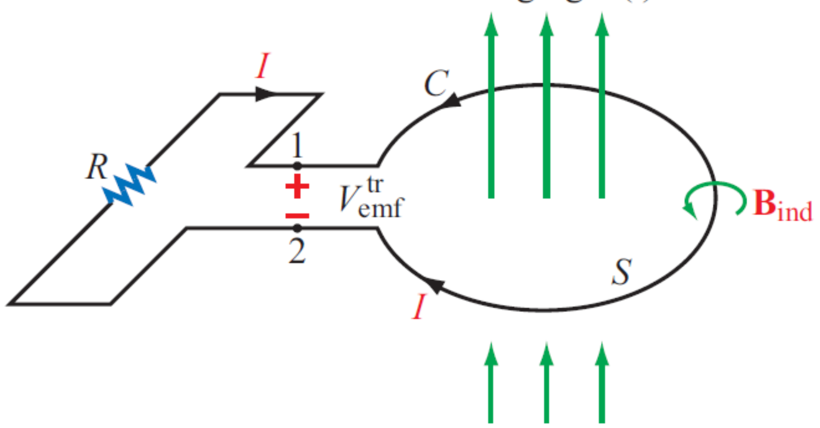
The total emf is given by

$$V_{\text{emf}} = V_{\text{emf}}^{\text{tr}} + V_{\text{emf}}^{\text{m}},$$



Stationary Loop in Time-Varying B

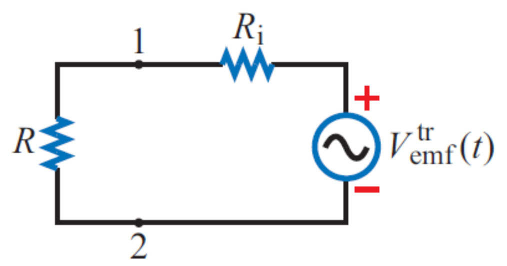
$$V_{\text{emf}}^{\text{tr}} = -N \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} \quad \text{(transformer emf)},$$



(a) Loop in a changing **B** field

$$I = \frac{V_{\text{emf}}^{\text{tr}}}{R + R_{\text{i}}} . \qquad (6.9)$$

For good conductors, R_{i} usually is very small, and it may be ignored in comparison with practical values of R.



(b) Equivalent circuit

Figure 6-2: (a) Stationary circular loop in a changing magnetic field **B**(t), and (b) its equivalent circuit.

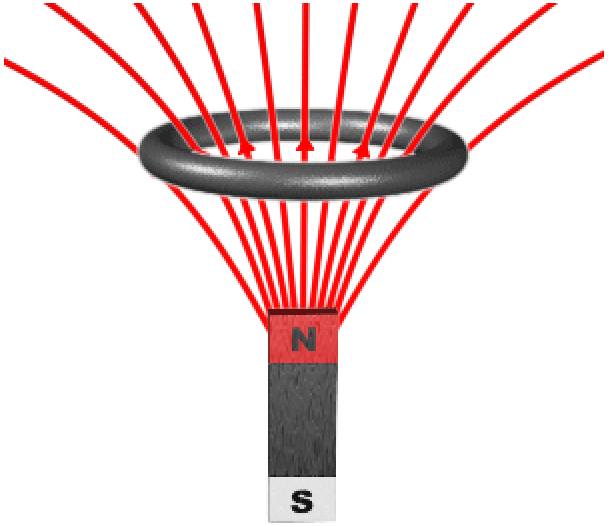
Stationary Loop in Time-Varying **B**

- Assuming S is stationary and B is varying: Transformer EMF

$$V_{\text{emf}}^{\text{tr}} = -N \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} \quad \text{(transformer emf)},$$

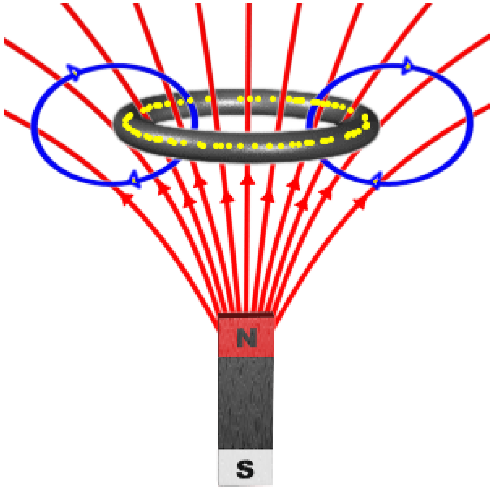
No change of surface

- Two types of B fields are generated
  - Changing B(t)
  - Induced B (Bind)
- Applet: [http://micro.magnet.fsu.edu/ele aw/index.html](http://micro.magnet.fsu.edu/eleaw/index.html)



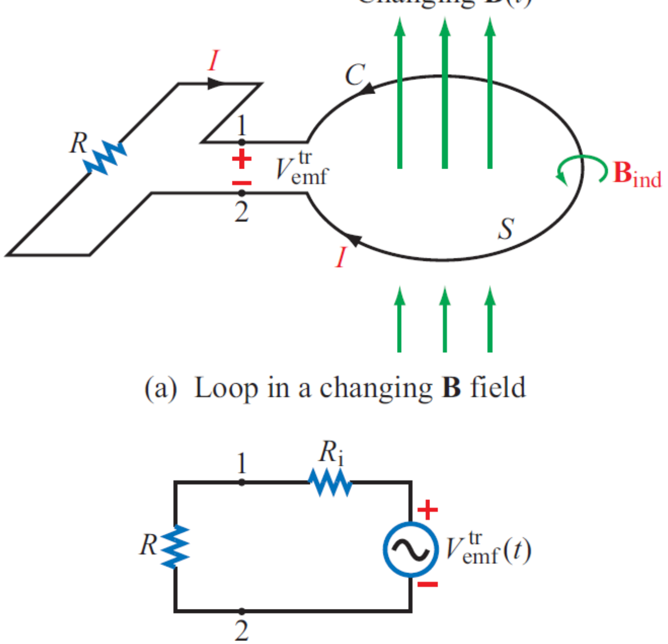
Lenz's Law

- Increasing B(t) → change of magnetic flux →I generates;
  - I(t) → Bind
  - Bind with be opposite of B(t) change
  - Direction of Bind → Direction of I(t)



Lenz's Law

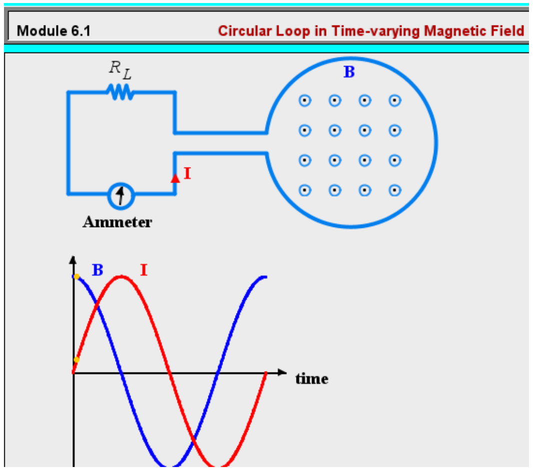
- Increasing B(t) → change of magnetic flux →I generates;
  - I(t) → Bind
  - Bind with be opposite of B(t) change
  - Direction of Bind → Direction of I(t)
  - If I(t) clockwise → I moving from + to –
    - V2> V1→ emf is negative



(a) Loop in a changing **B** field

(b) Equivalent circuit

Figure 6-2: (a) Stationary circular loop in a changing magnetic field **B**(t), and (b) its equivalent circuit.



Faraday's Law

magnetic field induces an E field whose CURL is equal to the negative of the time derivative of B

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$$

Faraday's Law

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{s} = V_{\text{emf}}^{\text{tr}} + V_{\text{emf}}^{\text{m}}$$

Transformer

$$V_{\text{emf}}^{\text{tr}} = -N\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} \qquad (N \text{ loops})$$

Motional

$$V_{\text{emf}}^{\text{m}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

Note A0

Example

An inductor is formed by winding N turns of a thin conducting wire into a circular loop of radius a. The inductor loop is in the x–y plane with its center at the origin, and connected to a resistor R, as shown in Fig. 6-3. In the presence of a magnetic field $\mathbf{B} = B_0(\hat{\mathbf{y}}2 + \hat{\mathbf{z}}3)\sin\omega t$, where ω is the angular frequency, find

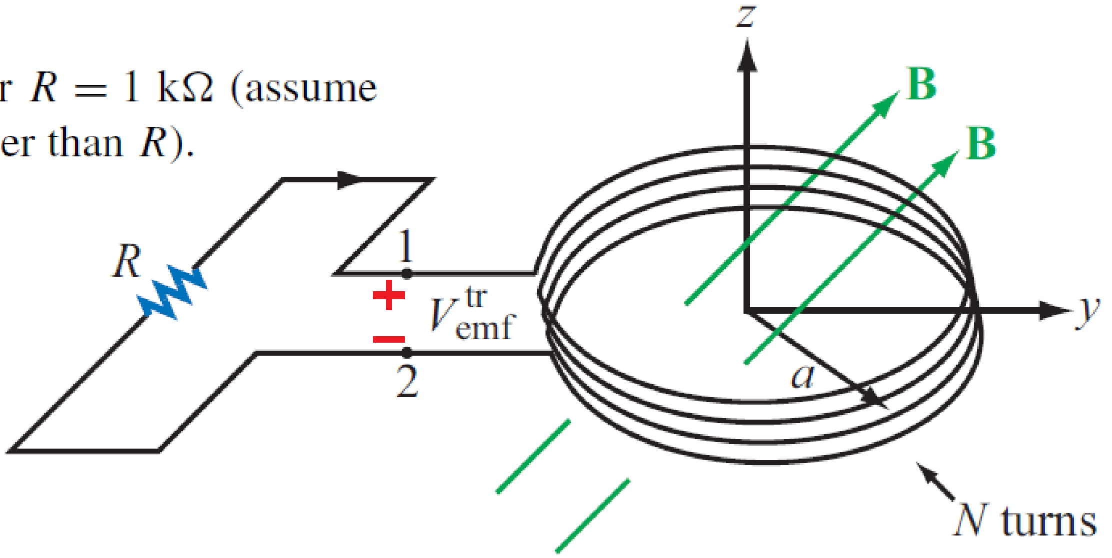
(a) the magnetic flux linking a single turn of the inductor,

(b) the transformer emf, given that $N = 10$, $B_0 = 0.2$ T, $a = 10$ cm, and $\omega = 10^3$ rad/s,

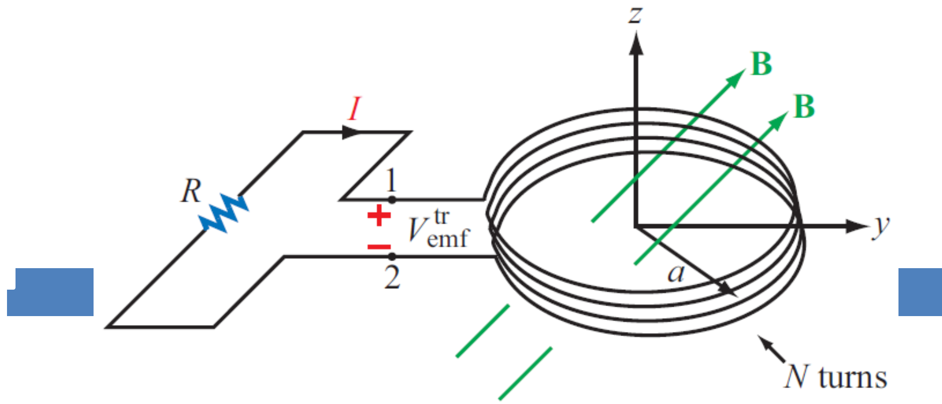
(c) the polarity of V_{emf}^{tr} at $t = 0$, and

(d) the induced current in the circuit for $R = 1$ kΩ (assume the wire resistance to be much smaller than R).

Note the direction of B



Note B0



Finding the Magnetic Flux:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$$

Find ds!

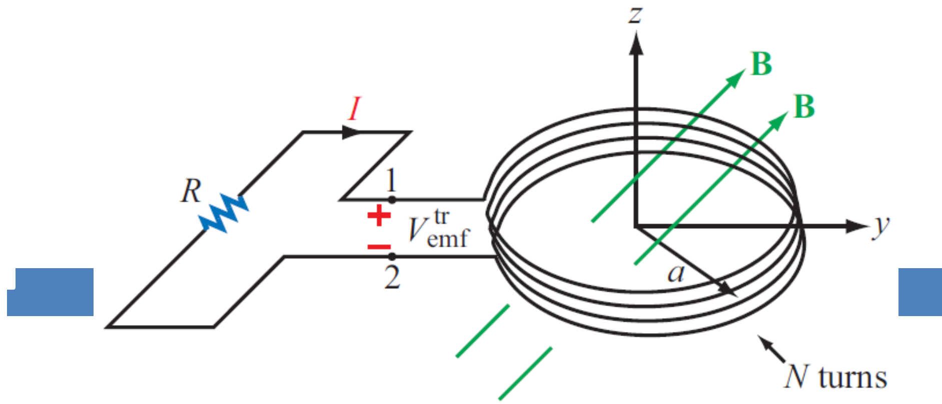
Find Polarity of Vtr at t=0

Find Vtr:

What is V1 – V2 based on the Given polarity?

Find Vrt!

Note B0



Finding the Magnetic Flux:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$$
$$= \int_S [B_0(\hat{\mathbf{y}}\,2 + \hat{\mathbf{z}}\,3)\sin\omega t] \cdot \hat{\mathbf{z}}\, ds$$
$$= 3\pi a^2 B_0 \sin\omega t.$$

Find Vtr:

$$V_{emf}^{tr} = -N\frac{d\Phi}{dt}$$
$$= -\frac{d}{dt}(3\pi N a^2 B_0 \sin\omega t)$$
$$= -3\pi N\omega a^2 B_0 \cos\omega t.$$

For $N = 10$, $a = 0.1$ m, $\omega = 10^3$ rad/s, and $B_0 = 0.2$ T,

$$V_{emf}^{tr} = -188.5 \cos 10^3 t \qquad \text{(V)}.$$

Find Polarity of Vtr at t=0

$$V_{emf}^{tr} = V_1 - V_2$$
$$= -188.5 \qquad \text{(V)}.$$

Ideal Transformer

$$V_1 = -N_1 \frac{d\Phi}{dt}.$$

A similar relation holds true on the secondary side:

Make sure you can prove these!

$$V_2 = -N_2 \frac{d\Phi}{dt}.$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

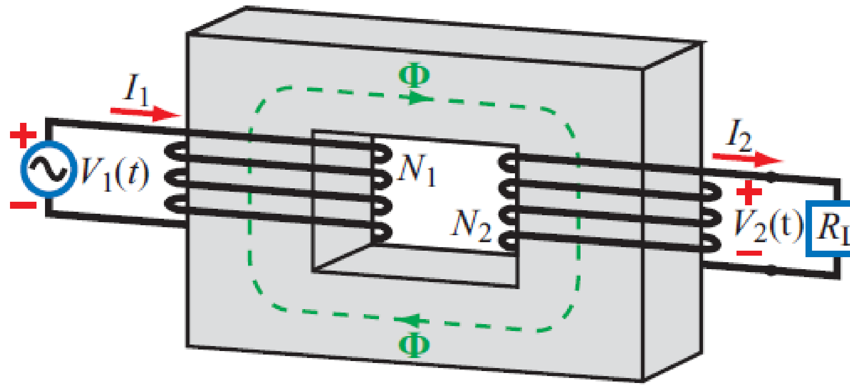
$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$R_{in} = \frac{V_1}{I_1}$$ Due to the primary coil

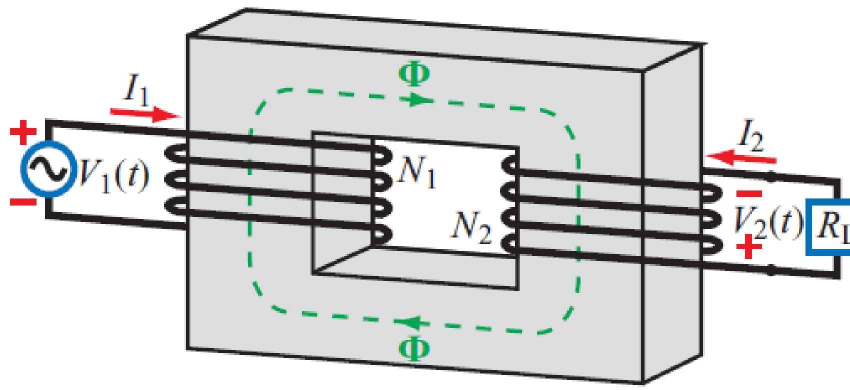
$$R_{in} = \frac{V_2}{I_2}\left(\frac{N_1}{N_2}\right)^2 = \left(\frac{N_1}{N_2}\right)^2 R_L. \qquad (6.20)$$

When the load is an impedance Z_L and V_1 is a sinusoidal source, the phasor-domain equivalent of Eq. (6.20) is

$$Z_{in} = \left(\frac{N_1}{N_2}\right)^2 Z_L. \qquad (6.21)$$



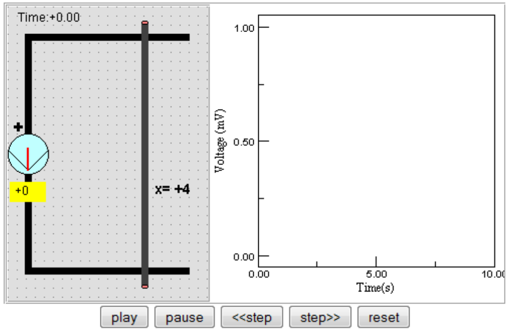
(a)



(b)

Primary and sec. coils
Separated by the magnetic core
(permittivity is infinity)
Magnetic flux is confined in the core

Motional EMF



- In the existence of constant (static) magnetic field the wire is moving
  - Fm is generated in charges
  - Thus, Em = Fm/q = UXB
  - Em is motional emf
- http://webphysics.davidson.edu/physlet_resources/bu_semester2/ (motional EMF)

Motional EMF

Magnetic force on charge *q* moving with velocity **u** in a magnetic field **B**:

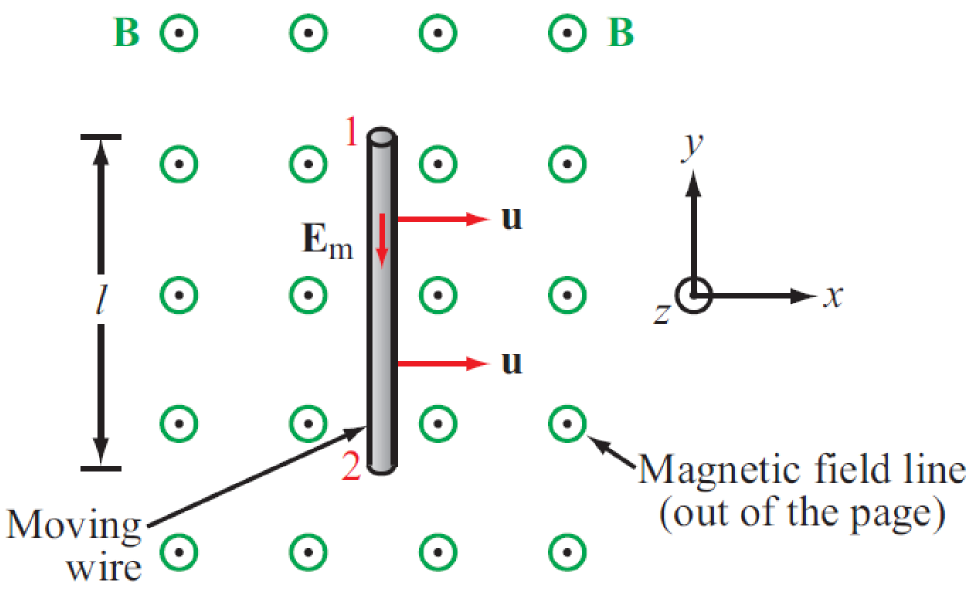
$$\mathbf{F}_m = q(\mathbf{u} \times \mathbf{B}).$$

This magnetic force is equivalent to the electrical force that would be exerted on the particle by the electric field Em given by

$$\mathbf{E}_m = \frac{\mathbf{F}_m}{q} = \mathbf{u} \times \mathbf{B}.$$

This, in turn, induces a voltage difference between ends 1 and 2, with end 2 being at the higher potential. The induced voltage is

$$V_{emf}^{m} = V_{12} = \int_{2}^{1} \mathbf{E}_m \cdot d\mathbf{l} = \int_{2}^{1} (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}.$$



Note that U → X direction
B→ Z direction
dl → Y direction
UxB → -Y direction

$$V_{emf}^{m} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} \qquad \text{(motional emf).}$$

Induced emf

Example : Sliding Bar

$$V_{\text{emf}}^{\text{m}} = V_{12} = V_{43} = \int_3^4 (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

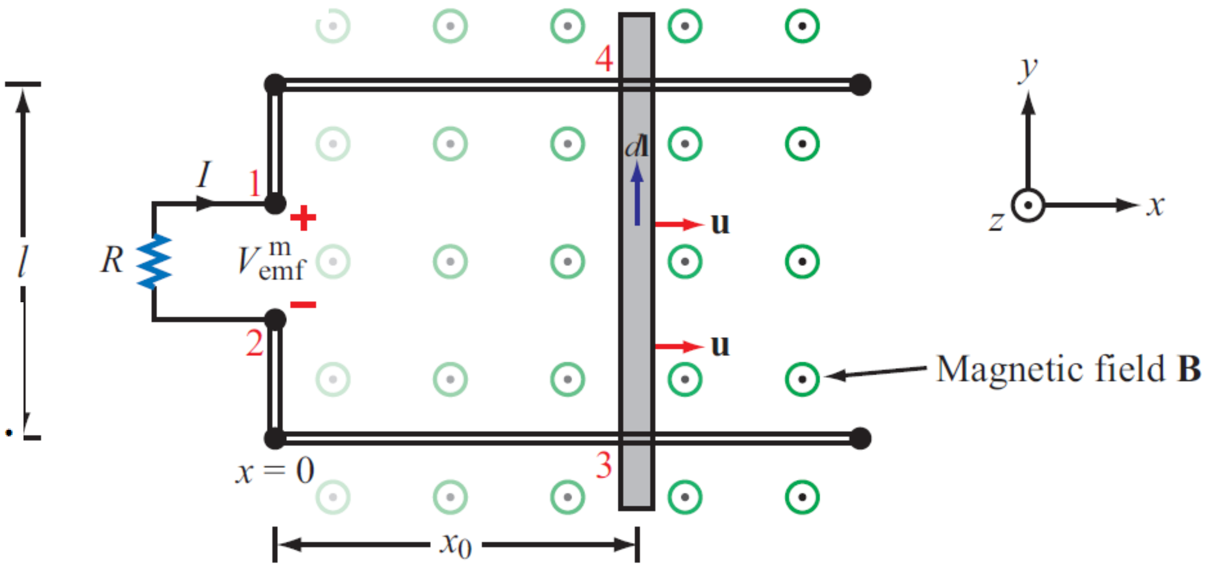
$$= \int_3^4 (\hat{\mathbf{x}}u \times \hat{\mathbf{z}}B_0x_0) \cdot \hat{\mathbf{y}}\, dl = -uB_0x_0l.$$

Note that B increases with x

$$\mathbf{B} = \hat{\mathbf{z}}B_0x$$

The length of the loop is related to u *by* $x_0 = ut$*. Hence*

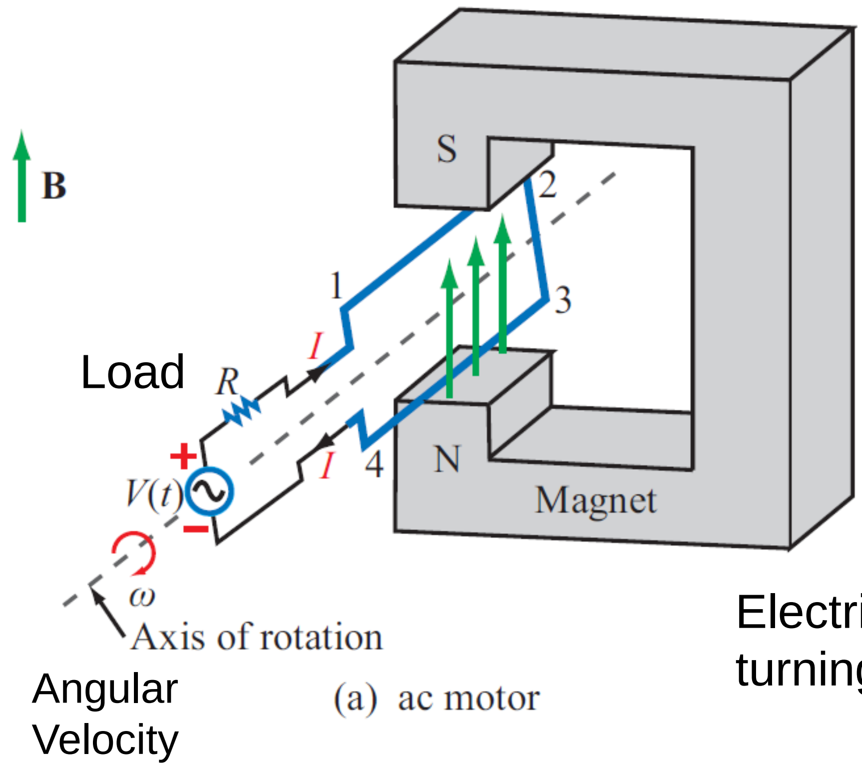
$$V_{\text{emf}}^{\text{m}} = -B_0u^2lt \qquad \text{(V)}.$$



Note A1

EM Generator

EM Motor/ Generator Reciprocity



(a) ac motor

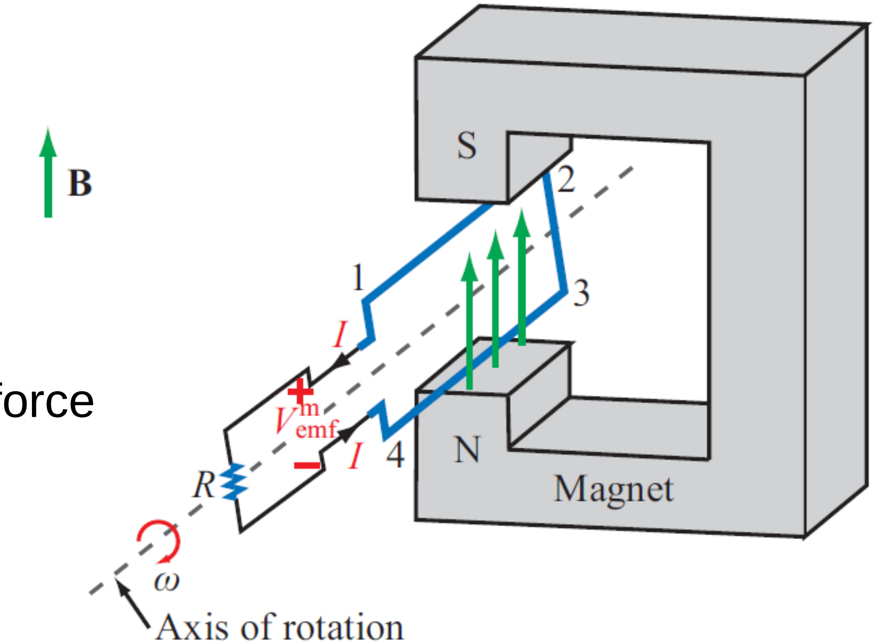
Current passing through the loop

Electrical energy being converted to mechanical turning the loop

Motor: Electrical to mechanical energy conversion

EM Motor/ Generator Reciprocity

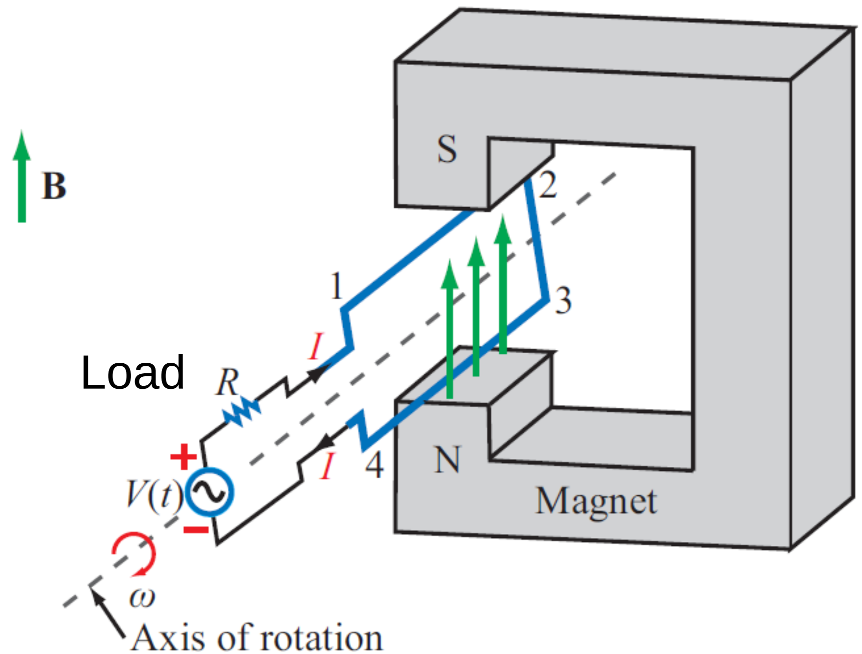
The loop is turning due to external force
B = Bo in Z direction



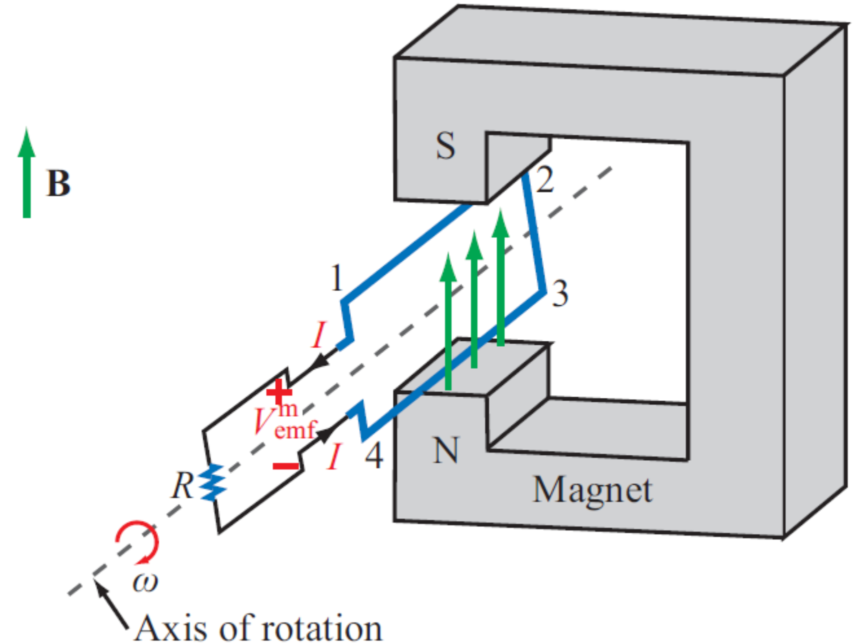
(b) ac generator

Generator: Mechanical to electrical energy conversion

EM Motor/ Generator Reciprocity



(a) ac motor



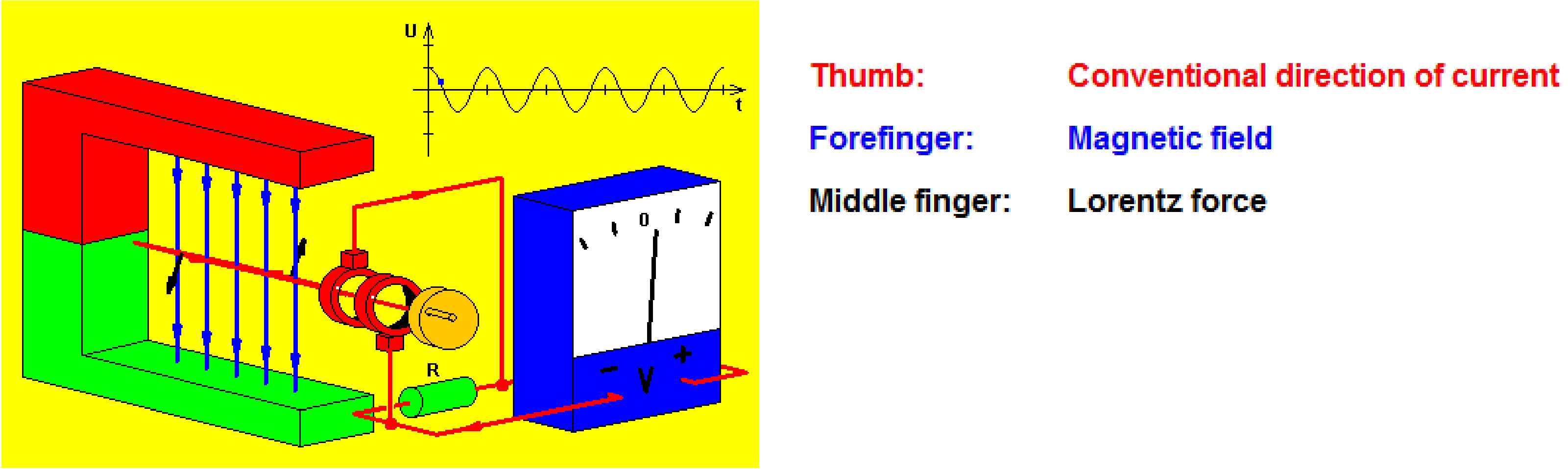
(b) ac generator

Motor: Electrical to mechanical energy conversion

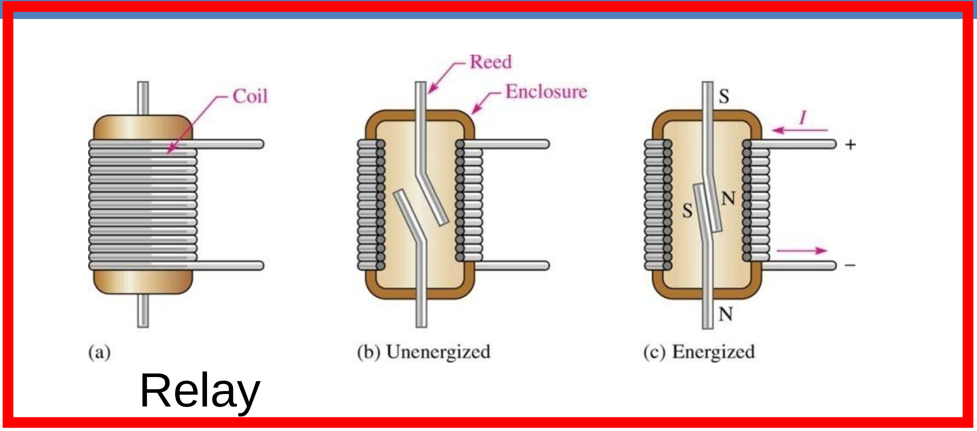
Generator: Mechanical to electrical energy conversion

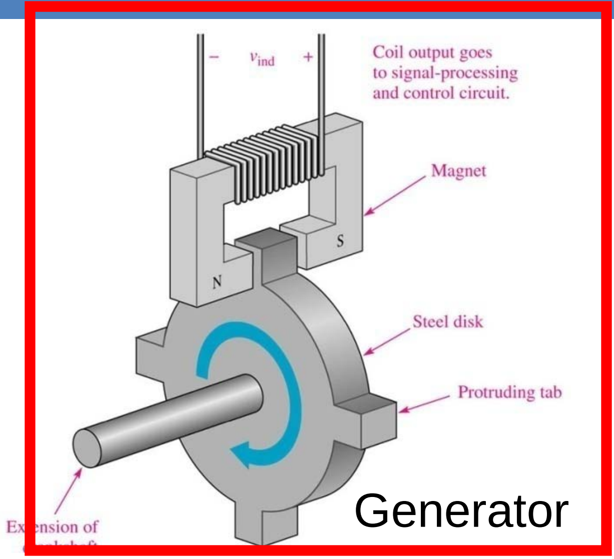
Applet

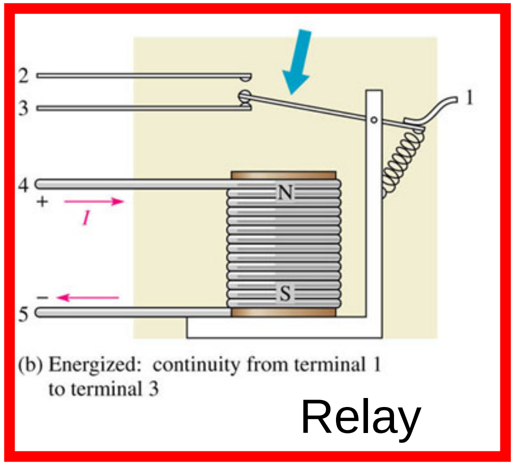
- http://www.walter-fendt.de/ph14e/electricmotor.htm
- http://www.walter-fendt.de/ph14e/generator_e.htm
- Good tutorial: http://micro.magnet.fsu.edu/electromag/electricity/generators/index.html

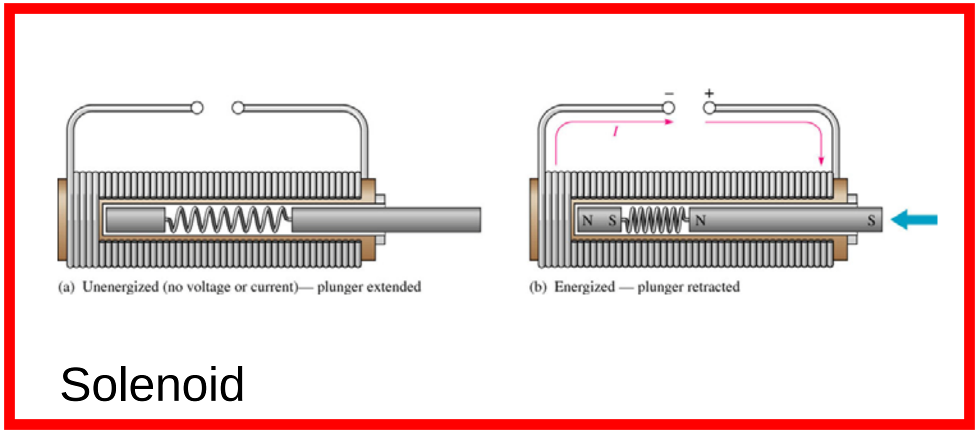


Other Applications



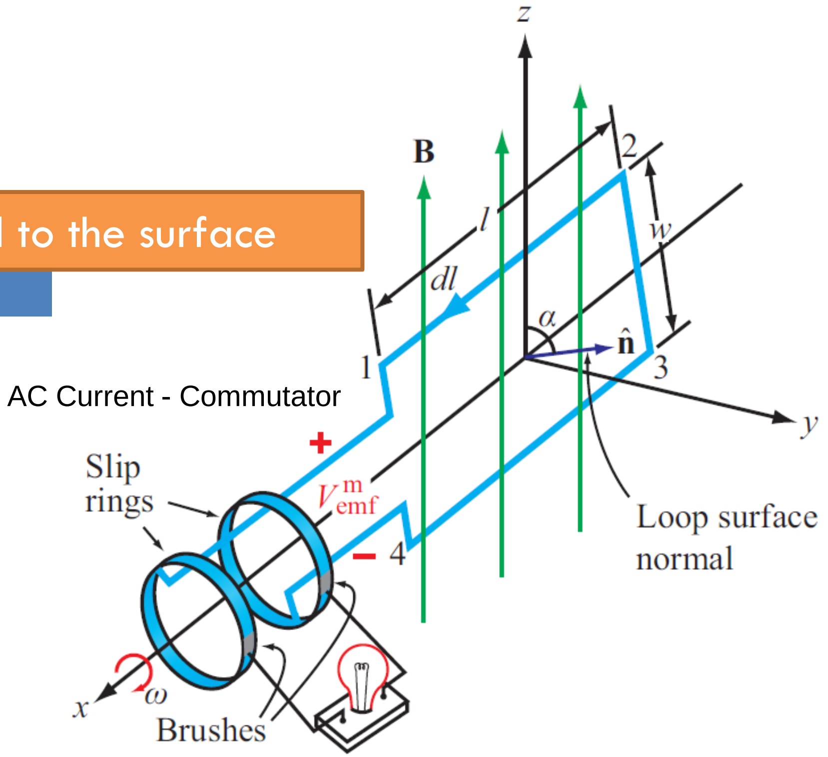






EM Generator EMF

n is normal to the surface



As the loop rotates with an angular velocity *ω about its own* axis, segment 1–2 moves with velocity **u** given by

$$\mathbf{u} = \hat{\mathbf{n}}\omega \frac{w}{2}$$

Also: $\hat{\mathbf{n}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}} \sin\alpha$.

Segment 3-4 moves with velocity **–u**. Hence:

$$V_{emf}^{m} = V_{14} = \int_{2}^{1} (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} + \int_{4}^{3} (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$= \int_{-l/2}^{l/2} \left[\left(\hat{\mathbf{n}}\omega \frac{w}{2}\right) \times \hat{\mathbf{z}} B_0\right] \cdot \hat{\mathbf{x}}\, dx$$

$$+ \int_{l/2}^{-l/2} \left[\left(-\hat{\mathbf{n}}\omega \frac{w}{2}\right) \times \hat{\mathbf{z}} B_0\right] \cdot \hat{\mathbf{x}}\, dx.$$

Note A2

$$V_{emf}^{m} = wl\omega B_0 \sin\alpha = A\omega B_0 \sin\alpha,$$

$$\alpha = \omega t + C_0,$$

$$V_{emf}^{m} = A\omega B_0 \sin(\omega t + C_0) \quad \text{(V)}.$$

A = W . l

EM Generator EMF Alternative Approach

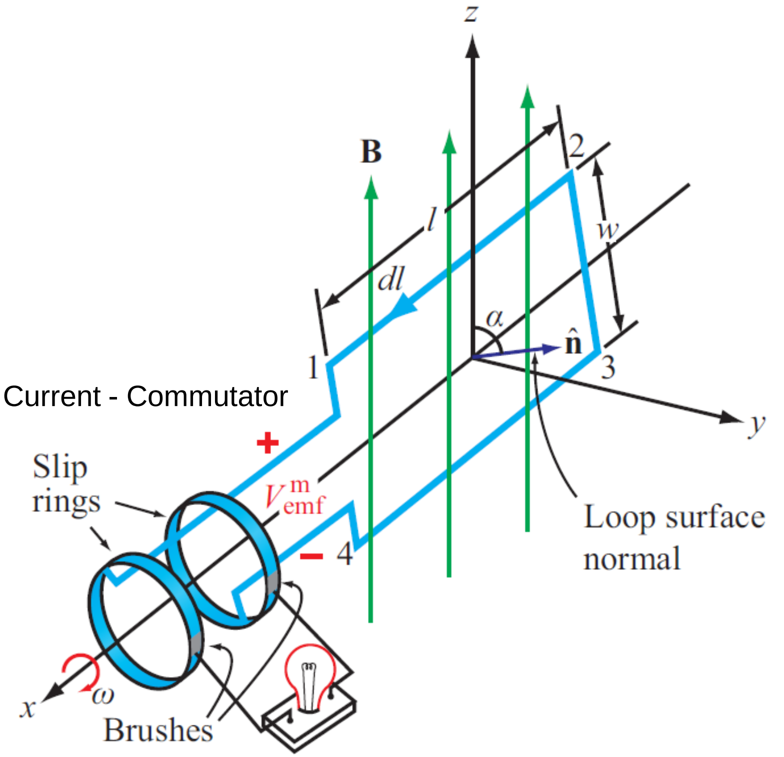
Using Magnetic Flux and Faraday's Law

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$$

$$V_{\text{emf}} = -\frac{d\Phi}{dt}$$

$$V_{\text{emf}}^{\text{m}} = A\omega B_0 \sin(\omega t + C_0) \qquad \text{(V)}.$$

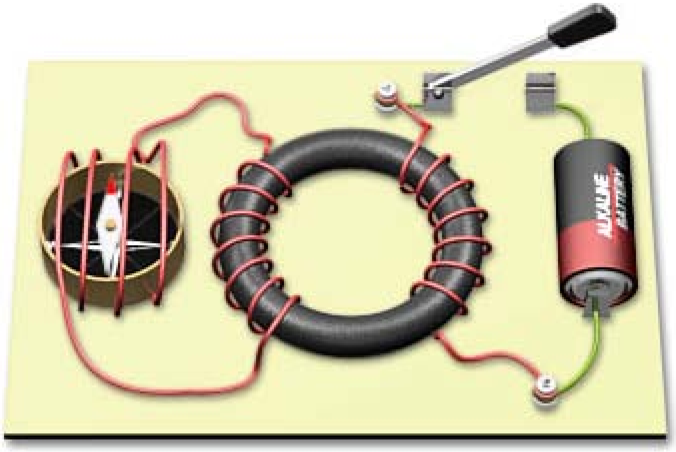
AC Current - Commutator



Note A3

Faraday’s Law

- The compass in the second coil deflects momentarily and returns immediately to its original position
- The deflection of the compass is an indication that an electromotive force was induced causing current to flow momentarily in the second coil.
- The closing and opening of the switch cause the magnetic field in the ring to change to expand and collapse respectively.
- Faraday discovered that changes in a magnetic field could induce an electromotive force and current in a nearby circuit.
- The generation of an electromotive force and current by a changing magnetic field is called electromagnetic induction.



http://micro.magnet.fsu.edu/electromag/java/faraday/

Displacement Current

Ampère's law in differential form is given by

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = Jc + Jd$$

For arbitrary open surface

$$\int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{s} = \int_S \mathbf{J} \cdot d\mathbf{s} + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}.$$

This term is conduction current I_C

This term must represent a current

Conduction current :
(transporting charges)
Displacement current:
(does not transport)

Current Density

Conduction	$\mathbf{J}_c = \sigma \mathbf{E}$
Displacement	$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$

Application of Stokes's theorem gives:

Total Current = $$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_c + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s} \quad \text{(Ampère's law)}$$

Note: Convection Current & Conduction Current are difference

Displacement Current

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_c + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$$ = Total Current = Ic + Id

Define the displacement current as:

$$I_d = \int_S \mathbf{J}_d \cdot d\mathbf{s} = \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s},$$

The displacement current does not involve real charges;
it is an equivalent current that depends on $\partial \mathbf{D}/\partial t$

$J_c = \sigma E$ $\quad \mathbf{J}_d = \partial \mathbf{D}/\partial t$

Note:
If dE/dt = 0 → Id = 0

Maxwell's contributions: defining the concept of displacement current & unifying time-varying electricity and magnetism

Remember: Conduction Current

Conduction current density:

$$\mathbf{J} = \sigma \mathbf{E} \qquad (\text{A/m}^2) \quad (\text{Ohm's law}),$$

Materials: Conductors & Dielectrics

Conductors: Loose electrons → Conduction current can be created due to E field

Dielectrics: electrons are tightly bound to the atom → no current when E is applied

Perfect Dielectric:

Conductivity = 0 → $J_c = 0$

Perfect Conductor:

Conductivity = INF → $E = J_c/\sigma = 0$

$$\text{Perfect dielectric:} \quad \mathbf{J} = 0,$$

$$\text{Perfect conductor:} \quad \mathbf{E} = 0.$$

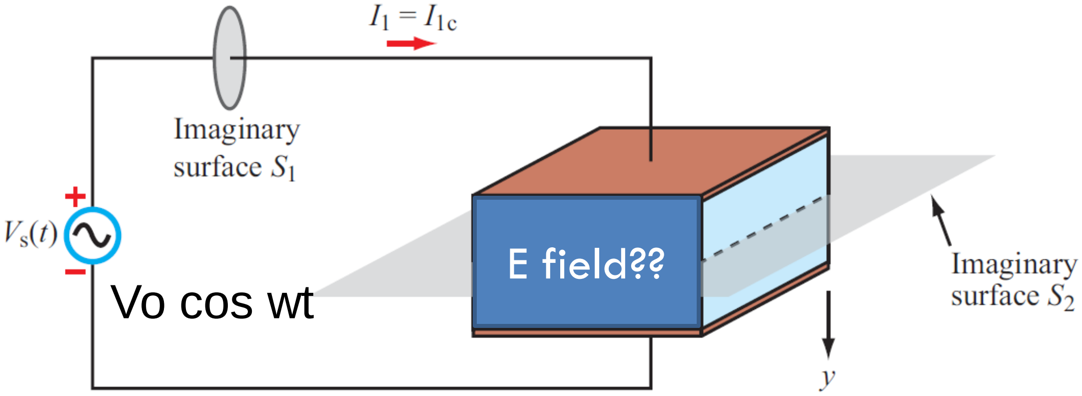
From Chapter 4

Conductivity depends on impurity and temperature!

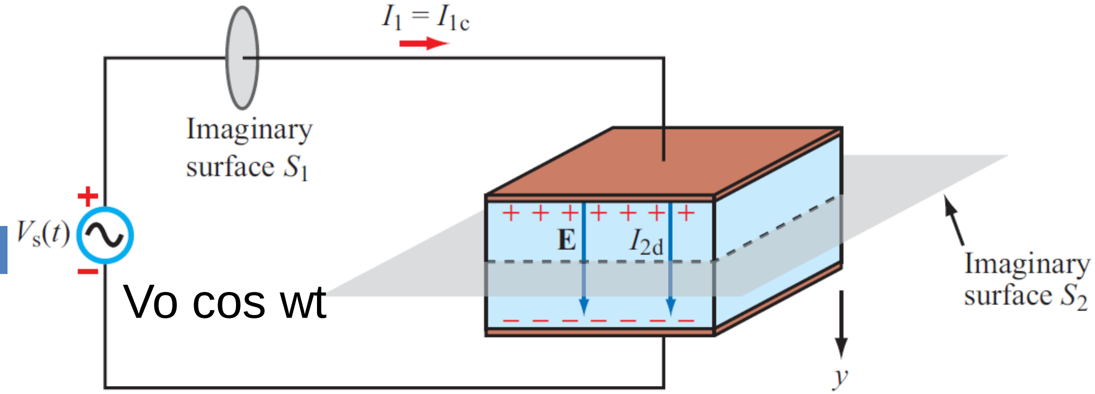
For metals: T inversely proportional to Conductivity!

Note A3b

Capacitor Circuit



Capacitor Circuit



Given: Wires are perfect conductors and capacitor insulator material is perfect dielectric.

For Surface S_1:

$$I_1 = I_{1c} + I_{1d}$$

$$I_{1c} = C\frac{dV_C}{dt} = C\frac{d}{dt}(V_0\cos\omega t) = -CV_0\omega\sin\omega t$$

$$I_{1d} = 0$$ (**D** = 0 in perfect conductor)

Remember: $V_{21} = V_1 - V_2 = -\int_{x_2}^{x_1} \mathbf{E}\cdot d\mathbf{l}$

$V_{12} = \int_2^1 \mathbf{E}_m \cdot d\mathbf{l}$ = Ey. d → Ey = V/d

For Surface S_2:

$$I_2 = I_{2c} + I_{2d}$$

$I_{2c} = 0$ (perfect dielectric) $J_c = \sigma E$

$$\mathbf{E} = \hat{\mathbf{y}}\frac{V_c}{d} = \hat{\mathbf{y}}\frac{V_0}{d}\cos\omega t$$

$$I_{2d} = \int_S \frac{\partial \mathbf{D}}{\partial t}\cdot d\mathbf{s}$$ Note: $\mathbf{J}_d = \partial\mathbf{D}/\partial t$

$$= \int_A \left[\frac{\partial}{\partial t}\left(\hat{\mathbf{y}}\frac{\varepsilon V_0}{d}\cos\omega t\right)\right]\cdot(\hat{\mathbf{y}}\,ds)$$

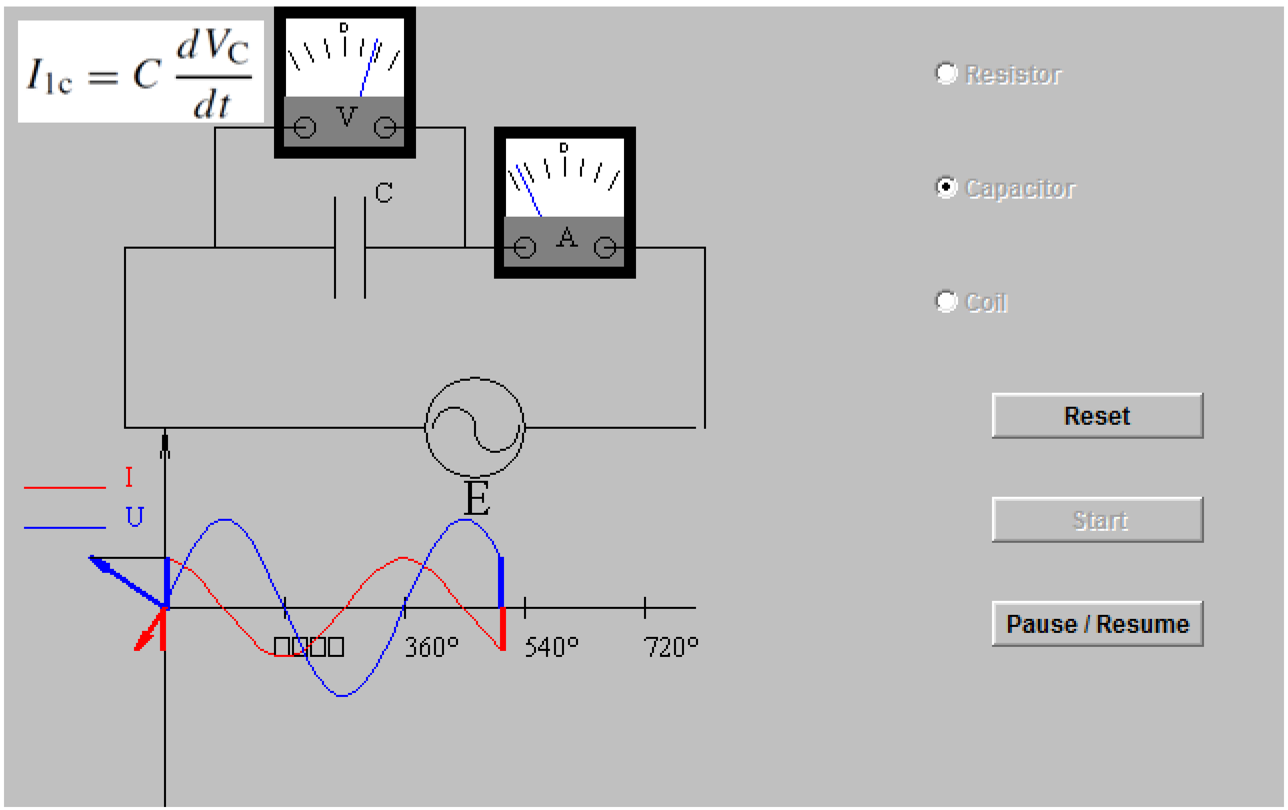
$$= -\frac{\varepsilon A}{d}V_0\omega\sin\omega t = -CV_0\omega\sin\omega t$$

C

Conclusion: $I_1 = I_2$

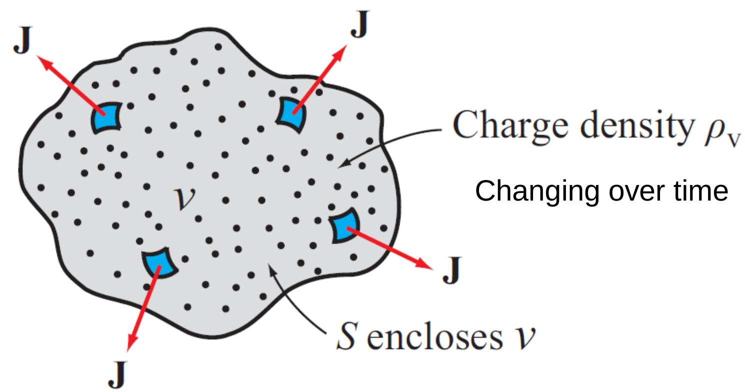
Applet

http://www.circuit-magic.com/capacitor.htm

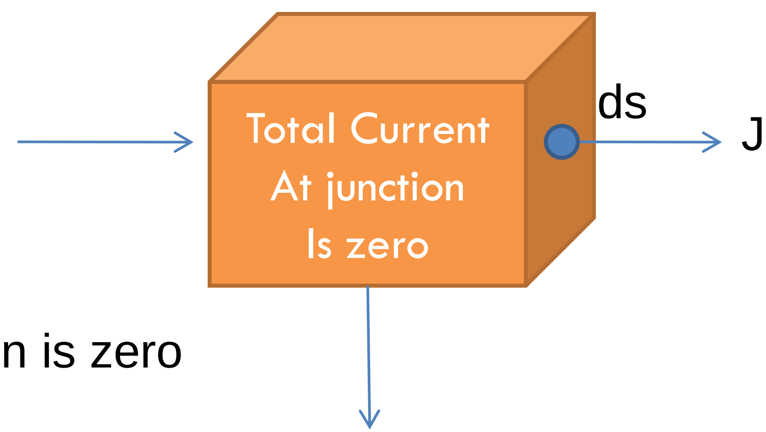


Charge Current Continuity Equation

Current I out of a volume is equal to rate of decrease of charge Q contained in that volume:



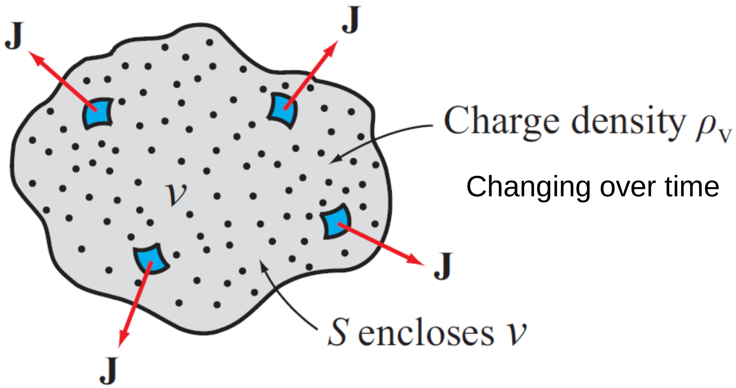
$$I = -\frac{dQ}{dt}$$



Kirchhoff's current law:
Algebraic sum of currents following out of a junction is zero

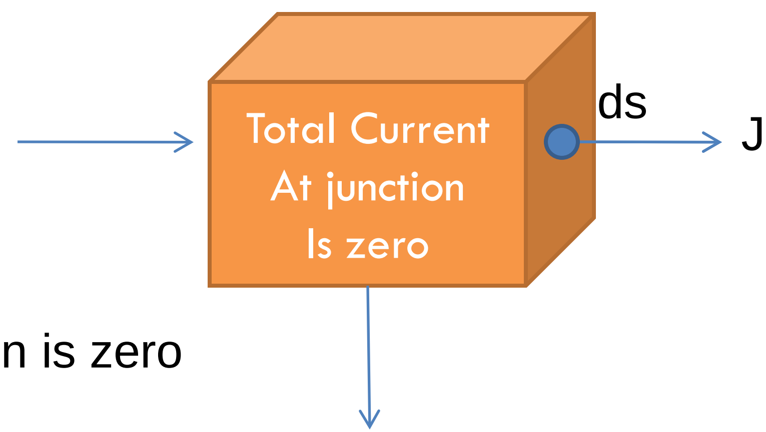
Charge Current Continuity Equation

Current I out of a volume is equal to rate of decrease of charge Q contained in that volume:



$$I = -\frac{dQ}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} \rho_v \, d\mathcal{V}$$

$$\oint_S \mathbf{J} \cdot d\mathbf{s} = -\frac{d}{dt}\int_{\mathcal{V}} \rho_v \, d\mathcal{V}$$



Kirchhoff's current law:
Algebraic sum of currents following out of a junction is zero

Maxwell's Equations – General Set

POINT FORM	INTEGRAL FORM
$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$ (Ampère's law)
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot d\mathbf{l} = \int_S \left(-\frac{\partial \mathbf{B}}{\partial t} \right) \cdot d\mathbf{S}$ (Faraday's law; S fixed)
$\nabla \cdot \mathbf{D} = \rho$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho \, dv$ (Gauss's law)
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$ (nonexistence of monopole)

Maxwell's Equations – Free Space Set

- We assume there are **no charges** in free space and thus, $J_c = \sigma E = 0$

POINT FORM	INTEGRAL FORM
$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot d\mathbf{l} = \int_S \left(-\frac{\partial \mathbf{B}}{\partial t} \right) \cdot d\mathbf{S}$
$\nabla \cdot \mathbf{D} = 0$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = 0$
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$

Time-varying E and H cannot exist independently!
If dE/dt non-zero→ dD/dt is non-zero →Curl of H is non-zero → H is non-zero

If H is a function of time → E must exist!

Maxwell Equations – Electrostatics and Magnetostatics

Governing equations

- **Differential form**

$$\nabla \cdot \mathbf{D} = \rho_v \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = 0 \qquad \nabla \times \mathbf{H} = \mathbf{J}$$

- **Integral form**

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q \qquad \oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0 \qquad \oint_C \mathbf{H} \cdot d\mathbf{l} = I$$

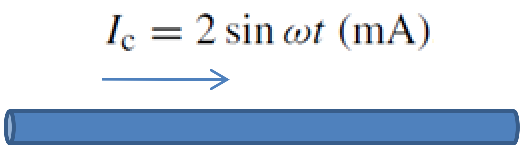
Boundary Conditions

General Form // Time-varying

Field Components	General Form	Medium 1 Dielectric	Medium 2 Dielectric	Medium 1 Dielectric	Medium 2 Conductor
Tangential E	$\hat{\mathbf{n}}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$	$E_{1t} = E_{2t}$		$E_{1t} = E_{2t} = 0$	
Normal D	$\hat{\mathbf{n}}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$	$D_{1n} - D_{2n} = \rho_s$		$D_{1n} = \rho_s$	$D_{2n} = 0$
Tangential H	$\hat{\mathbf{n}}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$	$H_{1t} = H_{2t}$		$H_{1t} = J_s$	$H_{2t} = 0$
Normal B	$\hat{\mathbf{n}}_2 \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$	$B_{1n} = B_{2n}$		$B_{1n} = B_{2n} = 0$	

Example: Displacement Current

The conduction current flowing through a wire with conductivity $\sigma = 2 \times 10^7$ S/m and relative permittivity $\varepsilon_r = 1$ is given by $I_c = 2 \sin \omega t$ (mA). If $\omega = 10^9$ rad/s,



Does E exist? Why?

Using Ohm's law; this is not a perfect conductor → $J_c = \sigma E$

Does Id exist?

If E exists → D exists (assuming it is time-varying, which is because Ic is time-varying!) → Id exists

Are Ic and Id related?

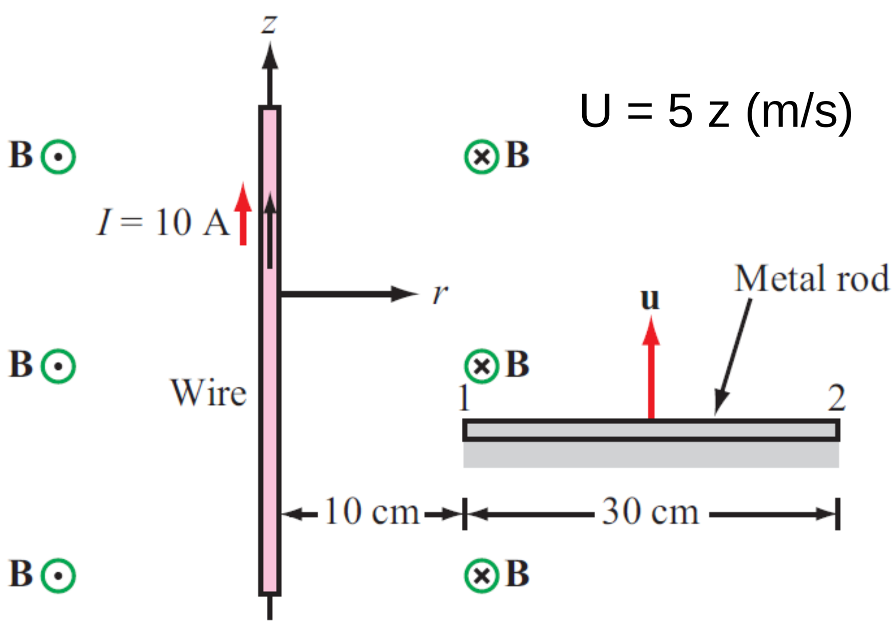
E is related to Jc ; Jd is defined as change of Electric flux density (εE) in time → They MUST be related!

Find the displacement current.

Note A4

Example

Find V12



Note A5