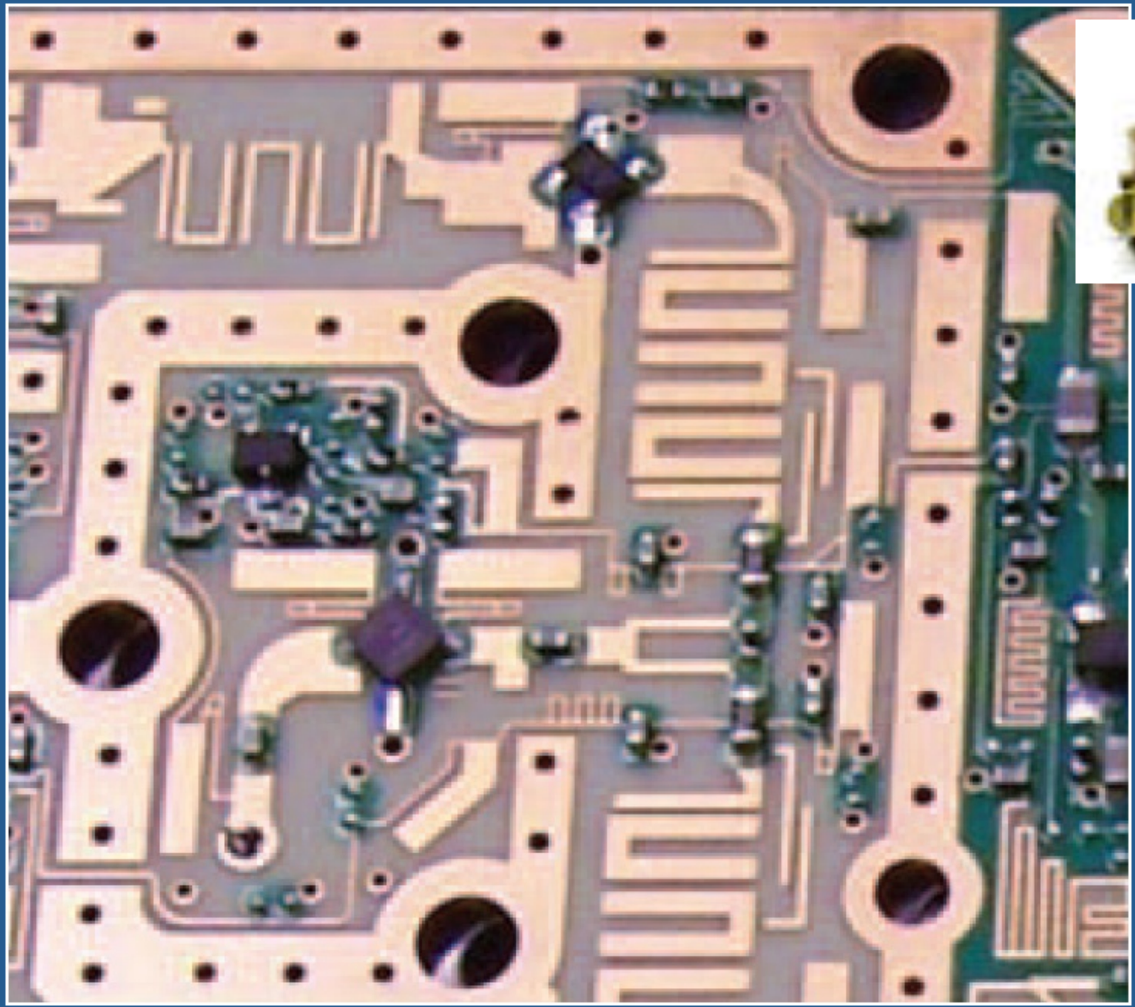


2. TRANSMISSION LINES

Transmission Lines

A transmission line connects a generator to a load

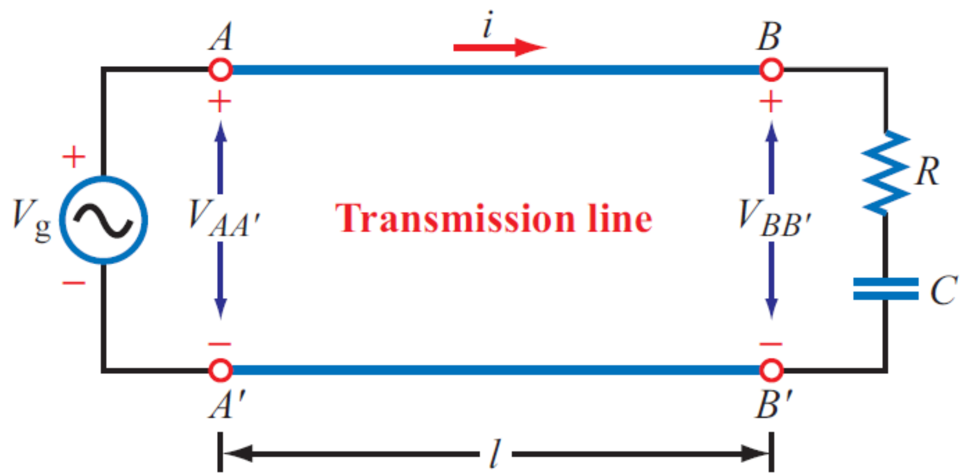
Transmission lines include:

- Two parallel wires
- Coaxial cable
- Microstrip line
- Optical fiber
- Waveguide
- etc.

Transmission Line Effects

$$V_{AA'} = V_g(t) = V_0 \cos \omega t \qquad \text{(V)}$$

$$V_{BB'}(t) = V_{AA'}(t - l/c) \qquad \text{Delayed by } l/c$$
$$= V_0 \cos[\omega(t - l/c)]$$
$$= V_0 \cos(\omega t - \phi_0),$$



At $t = 0$, and for $f = 1$ kHz , if:

(1) $l = 5$ cm:

$$V_{BB'} = V_0 \cos(2\pi f l/c) = 0.999999999998\, V_0$$

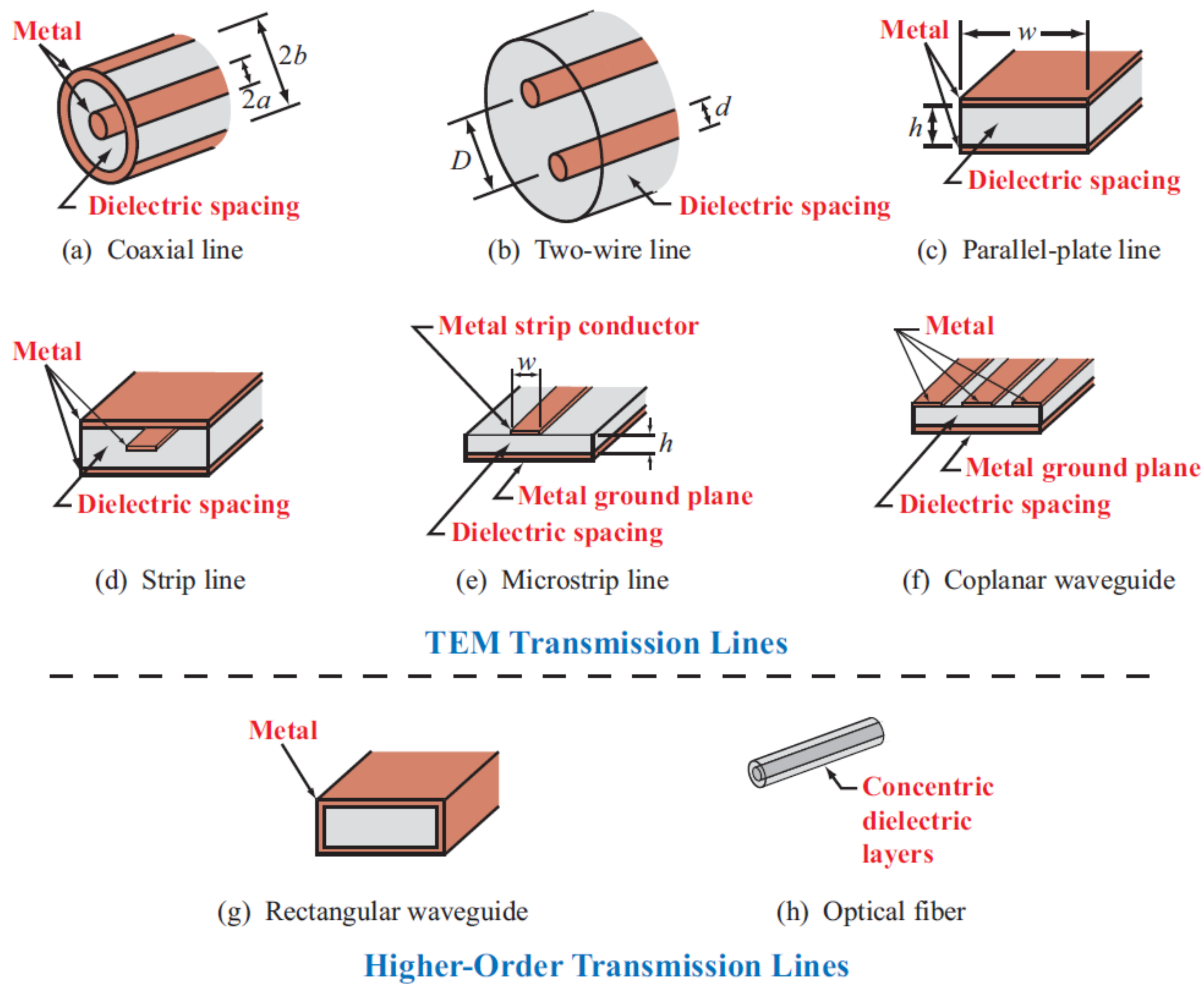
(2) But if $l = 20$ km:

$$V_{BB'} = 0.91 V_0$$

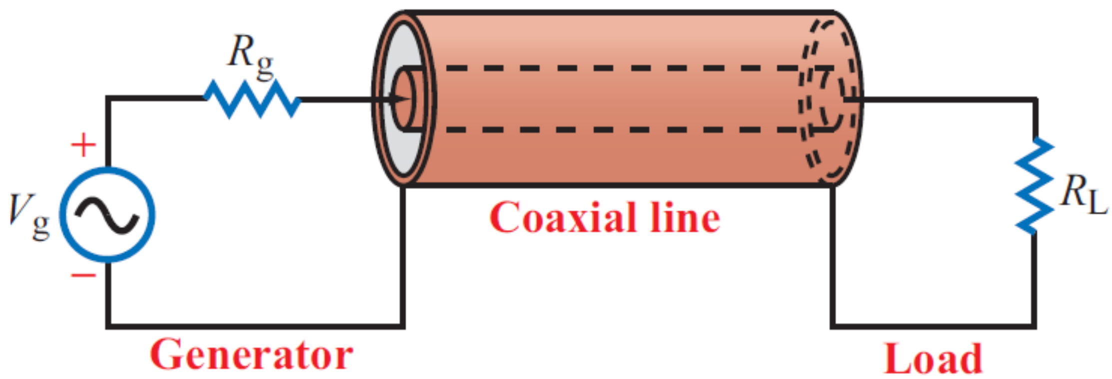
Dispersion and Attenuation

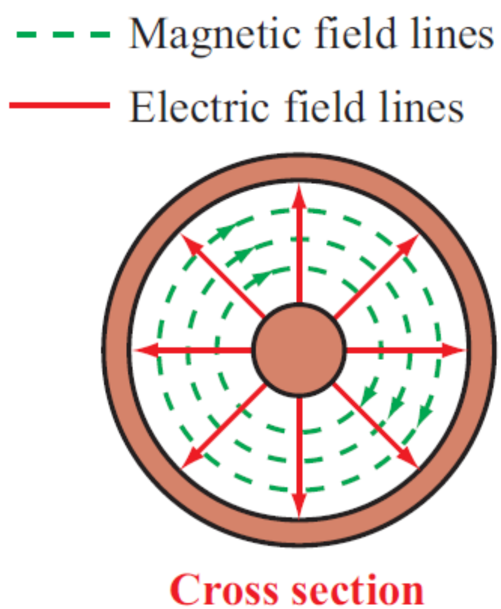
Types of Transmission Modes

TEM (Transverse Electromagnetic): Electric and magnetic fields are orthogonal to one another, and both are orthogonal to direction of propagation



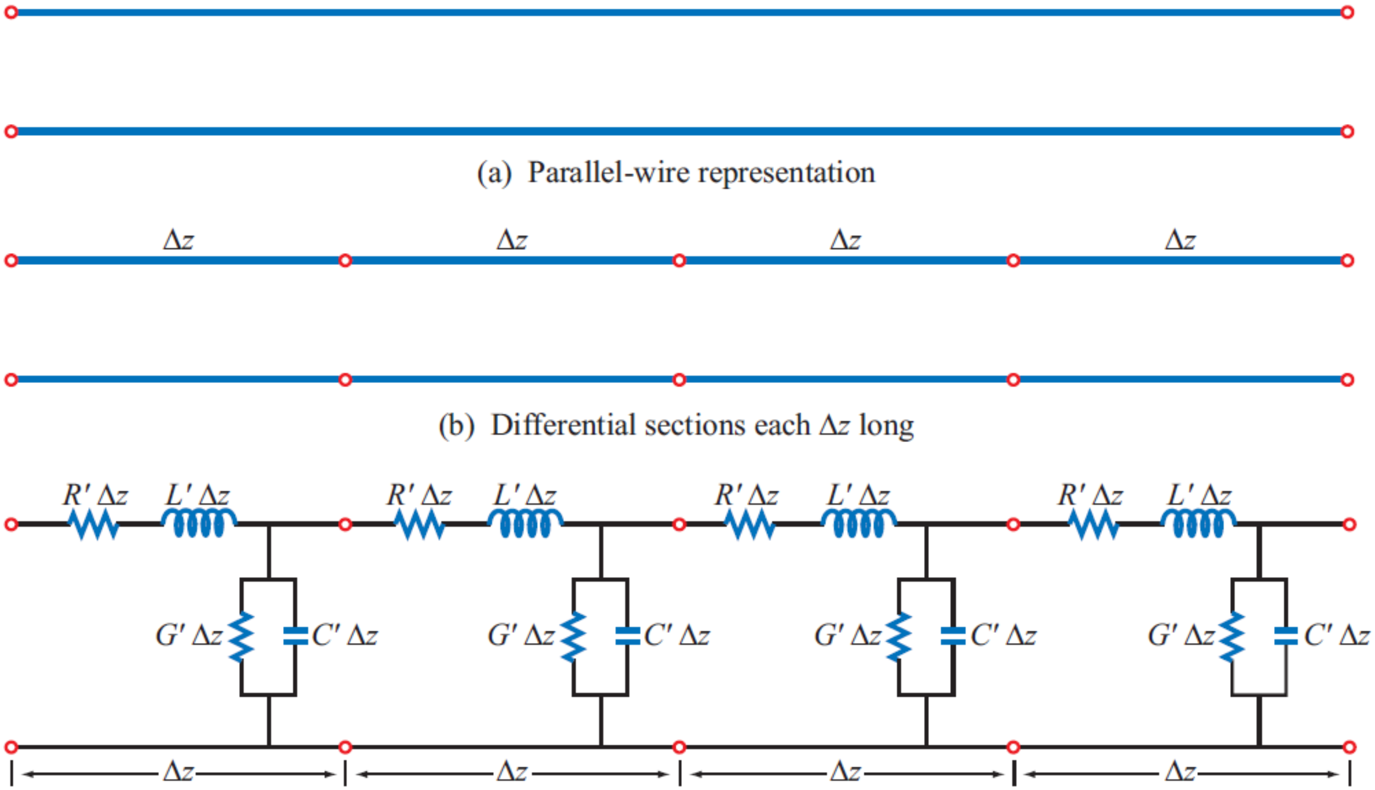
Example of TEM Mode





Electric Field E is radial
Magnetic Field H is azimuthal
Propagation is into the page

Transmission Line Model



(a) Parallel-wire representation

(b) Differential sections each Δz long

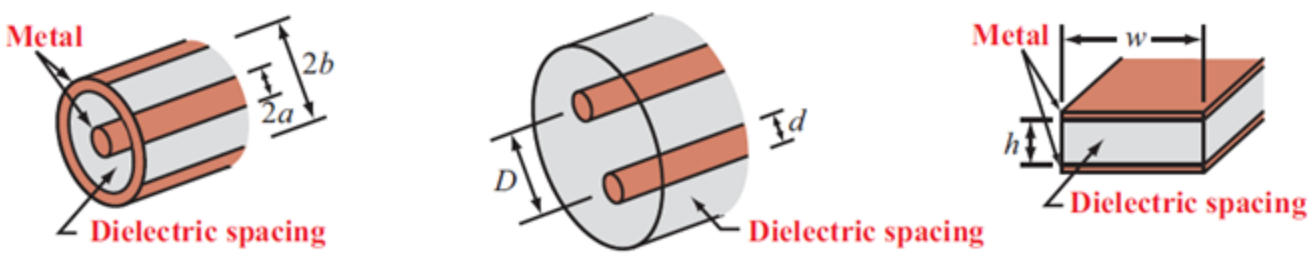
(c) Each section is represented by an equivalent circuit

- R': The combined ***resistance*** of both conductors per unit length, in Ω/m,
- L': The combined ***inductance*** of both conductors per unit length, in H/m,
- G': The ***conductance*** of the insulation medium between the two conductors per unit length, in S/m, and
- C': The ***capacitance*** of the two conductors per unit length, in F/m.

Table 2-1: Transmission-line parameters R', L', G', and C' for three types of lines.

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\varepsilon w}{h}$	F/m

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) μ, ε, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c/\sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right] \simeq \ln(2D/d)$.



(a) Coaxial line (b) Two-wire line (c) Parallel-plate line

Applied Electromagnetics 6e

Textbook CD*

GETTING STARTED

- » Welcome
- » Using this CD
- » Terms
- » Feedback

STUDENT RESOURCES

- » Exercise Solutions
- » CD Modules
- » Solved Problems
- » Technology Briefs
- » Frequency Allocation Chart

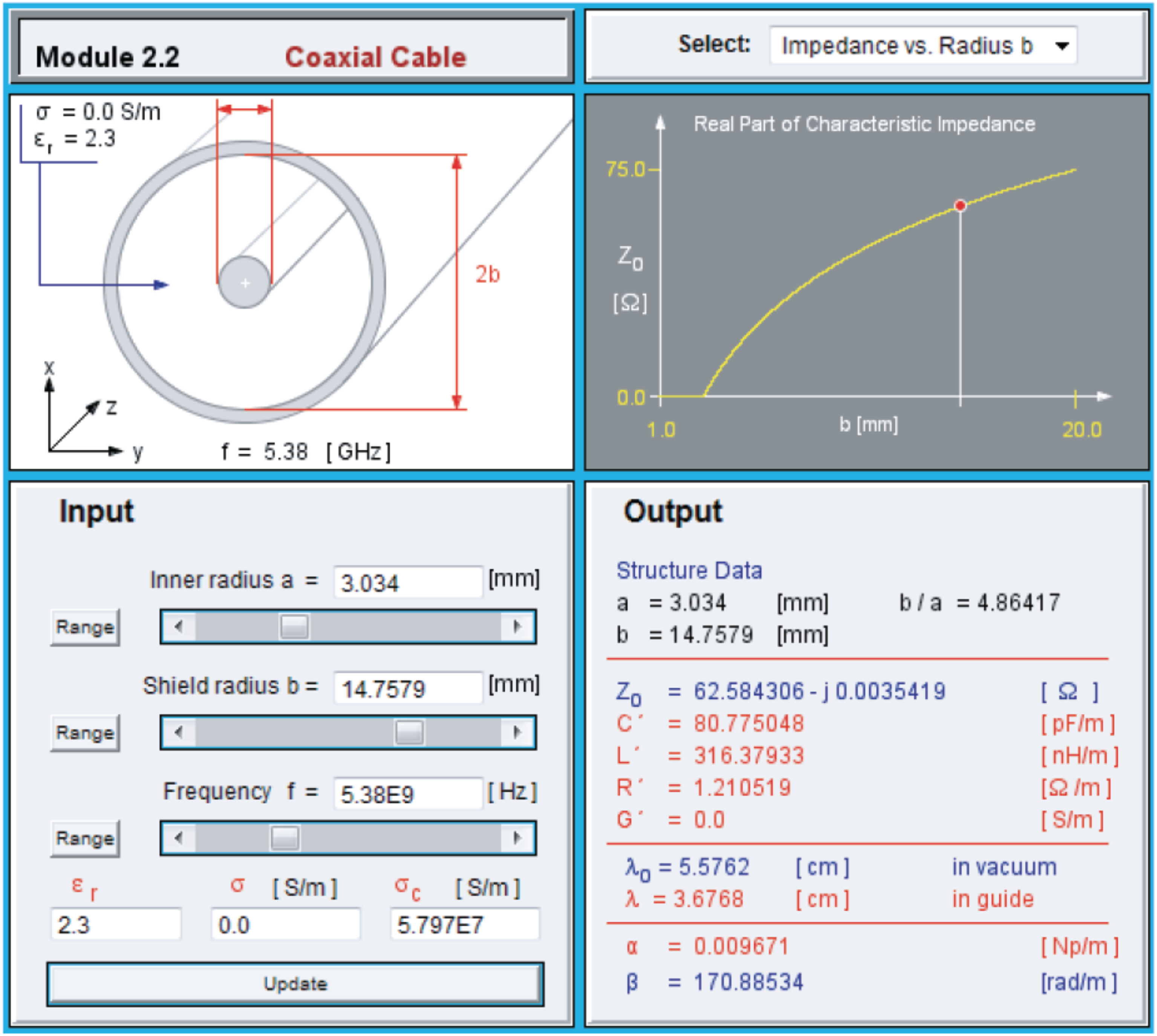
WELCOME

Welcome to the CD-ROM companion of the sixth edition of *Applied Electromagnetics*, developed to serve the student as an interactive self-study supplement to the text.

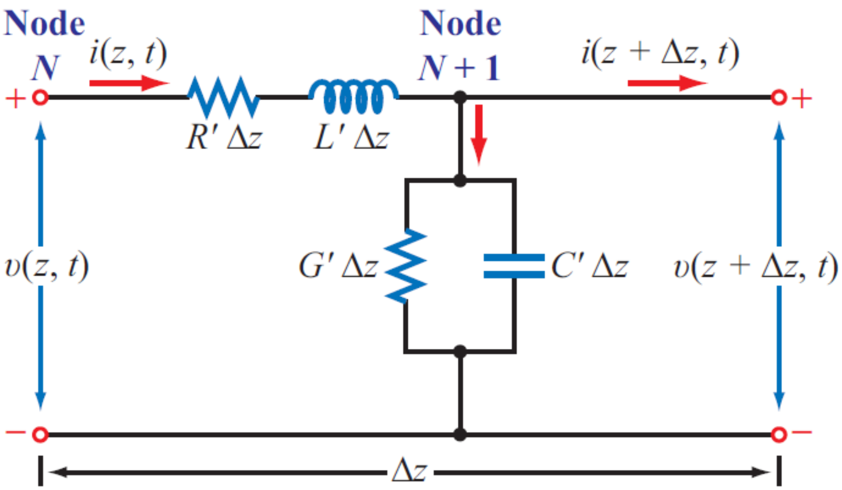
The navigation is highly flexible; the user may go though the material in the order outlined in the table of contents or may proceed directly to any exercise, module, technology brief or solved problem of interest.

We hope you find this CD-ROM helpful and we welcome your feedback and suggestions.

Fawwaz Ulaby
Eric Michielssen
Umberto Ravaioli


Module 2.2
Coaxial Cable
Select:
Impedance vs. Radius b
σ = 0.0 S/m
ε_r = 2.3
2b
x
z
y
f = 5.38 [GHz]
Real Part of Characteristic Impedance
75.0
Z0
[Ω]
0.0
1.0
b [mm]
20.0
Input
Inner radius a = 3.034 [mm]
Range
Shield radius b = 14.7579 [mm]
Range
Frequency f = 5.38E9 [Hz]
Range
ε r
σ [S/m]
σc [S/m]
2.3
0.0
5.797E7
Update
Output
Structure Data
a = 3.034 [mm]
b / a = 4.86417
b = 14.7579 [mm]
Z0 = 62.584306 - j 0.0035419 [Ω]
C´ = 80.775048 [pF/m]
L´ = 316.37933 [nH/m]
R´ = 1.210519 [Ω /m]
G´ = 0.0 [S/m]
λ0 = 5.5762 [cm] in vacuum
λ = 3.6768 [cm] in guide
α = 0.009671 [Np/m]
β = 170.88534 [rad/m]

Transmission-Line Equations



$$Ae^{j\theta} = A\cos(\theta) + Aj\sin(\theta)$$

$$\cos(\theta) = A\,\mathrm{Re}[Ae^{j\theta}]$$

$$\sin(\theta) = A\,\mathrm{Im}[Ae^{j\theta}]$$

$$E(z) = |E(z)|\,e^{j\theta_z}$$

$$|e^{j\theta}| = 1$$

$$C = A + jB \rightarrow \theta = \tan\frac{B}{A}; |C| = \sqrt{A^2 + B^2}$$

Remember:

Kirchhoff Voltage Law:

Vin-Vout – VR’ – VL’ =0

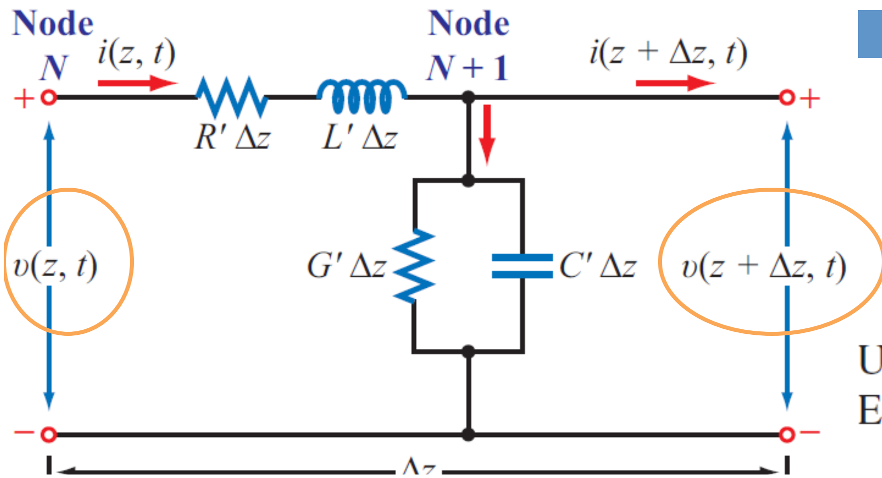
Kirchhoff Current Law:

Iin – Iout – Ic’ – IG’ =0

Note:

VL=L . di/dt
Ic=C . dv/dt

Transmission-Line Equations



$$v(z,t) - R'\Delta z\, i(z,t) - L'\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z, t) = 0.$$

Upon dividing all terms by Δz and rearranging them, we obtain

$$-\left[\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z}\right] = R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t}. \quad (2.13)$$

In the limit as $\Delta z \to 0$, Eq. (2.13) becomes a differential equation:

$$-\frac{\partial v(z,t)}{\partial z} = R'\, i(z,t) + L' \frac{\partial i(z,t)}{\partial t}. \quad (2.14)$$

$$i(z,t) - G'\Delta z\, v(z+\Delta z,t) - C'\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0. \quad (2.15)$$

Upon dividing all terms by Δz and taking the limit $\Delta z \to 0$, Eq. (2.15) becomes a second-order differential equation:

$$-\frac{\partial i(z,t)}{\partial z} = G'\, v(z,t) + C' \frac{\partial v(z,t)}{\partial t}. \quad (2.16)$$

ac signals: use phasors

$$v(z,t) = \mathfrak{Re}[\widetilde{V}(z)\, e^{j\omega t}],$$

$$i(z,t) = \mathfrak{Re}[\widetilde{I}(z)\, e^{j\omega t}],$$

$$-\frac{d\widetilde{V}(z)}{dz} = (R' + j\omega L')\, \widetilde{I}(z),$$

$$-\frac{d\widetilde{I}(z)}{dz} = (G' + j\omega C')\, \widetilde{V}(z).$$

Transmission Line Equation in Phasor Form

Derivation of Wave Equations

$$-\frac{d\widetilde{V}(z)}{dz} = (R' + j\omega L')\,\widetilde{I}(z),$$
$$-\frac{d\widetilde{I}(z)}{dz} = (G' + j\omega C')\,\widetilde{V}(z).$$

Transmission Line Equation
First Order Coupled Equations!
WE WANT UNCOUPLED FORM!

Combining the two equations leads to:

$$\frac{d^2\widetilde{V}(z)}{dz^2} - (R' + j\omega L')(G' + j\omega C')\,\widetilde{V}(z) = 0,$$

$$\frac{d^2\widetilde{V}(z)}{dz^2} - \gamma^2\,\widetilde{V}(z) = 0, \qquad (2.21)$$

Second-order differential equation

Wave Equations for Transmission Line

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\,. \qquad (2.22)$$

Impedance and Shunt Admittance of the line

complex propagation constant

attenuation constant

Phase constant

$$\gamma = \alpha + j\beta$$

$$\alpha = \mathfrak{Re}(\gamma) = \mathfrak{Re}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(Np/m)}, \qquad (2.25a)$$

$$\beta = \mathfrak{Im}(\gamma) = \mathfrak{Im}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(rad/m)}. \qquad (2.25b)$$

Pay Attention to UNITS!

Solution of Wave Equations (cont.)

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\,\tilde{V}(z) = 0, \qquad (2.21)$$

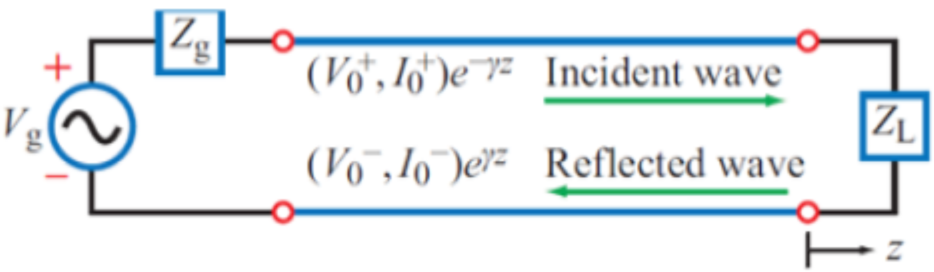
$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\,\tilde{I}(z) = 0. \qquad (2.23)$$

Proposed form of solution:

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad \text{(A)}.$$

So What does V+ and V- Represent?



Characteristic Impedance of the Line (ohm)

$$\frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-},$$

Using:

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\,\tilde{I}(z),$$

It follows that:

$$\tilde{I}(z) = \frac{\gamma}{R' + j\omega L'}\left[V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right]$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \qquad (\Omega), \qquad (2.29)$$

Pay att. To Direction

Make sure you know how we got this!

Solution of Wave Equations (cont.)

So, V(z) and I(z) have two parts:
But what are Vo+ and Vo- ?

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad (\text{V}),$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad (\text{A}).$$

In general (each component has Magnitude and Phase):

$$V_0^+ = |V_0^+| e^{j\phi^+},$$

$$V_0^- = |V_0^-| e^{j\phi^-}.$$

Refer to Notes

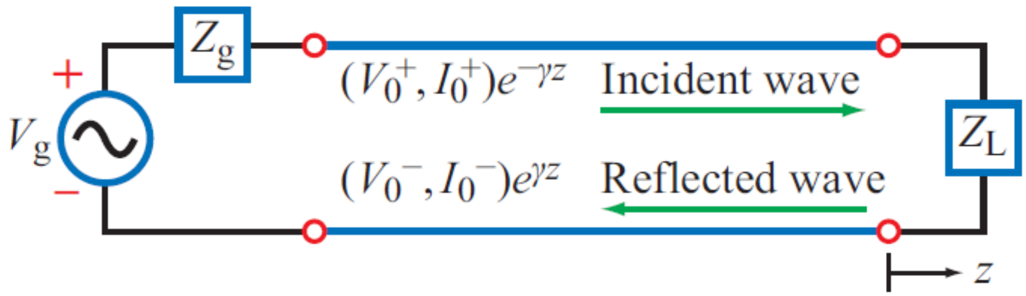
$$\begin{aligned}
\upsilon(z,t) &= \mathfrak{Re}(\tilde{V}(z) e^{j\omega t}) \\
&= \mathfrak{Re}\left[\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) e^{j\omega t}\right] \\
&= \mathfrak{Re}[|V_0^+| e^{j\phi^+} e^{j\omega t} e^{-(\alpha + j\beta)z} \\
&\quad + |V_0^-| e^{j\phi^-} e^{j\omega t} e^{(\alpha + j\beta)z}] \\
&= |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) \\
&\quad + |V_0^-| e^{\alpha z} \cos(\omega t + \beta z + \phi^-)
\end{aligned}$$

← We are interested in Sinusoidal Steady-state Condition

wave along +*z* because coefficients of *t* and *z* have opposite signs

wave along –*z* because coefficients of *t* and *z* have the same sign

Solution of Wave Equations (cont.)



The presence of two waves on the line propagating in opposite directions produces a ***standing wave****.*

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad \text{(A)}.$$

Applet for standing wave:
http://www.physics.smu.edu/~olness/www/05fall1320/applet/pipe-waves.html

Example

- Verify the solution to the wave equation for voltage in phasor form:

$$\frac{d^2\widetilde{V}(z)}{dz^2} - \gamma^2\,\widetilde{V}(z) = 0, \qquad (2.21)$$

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\widetilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad \text{(A)}.$$

Note:

$$V_0^+ = |V_0^+|e^{j\phi^+},$$

$$V_0^- = |V_0^-|e^{j\phi^-}.$$

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\,. \qquad (2.22)$$

$$\gamma = \alpha + j\beta$$

Example 2-1: Air Line

Assume the following waves:

$$V(z,t) = 10\cos(2\pi \cdot 700 \cdot 10^6 - 20z + 5)$$

$$I(z,t) = 0.2\cos(2\pi \cdot 700 \cdot 10^6 - 20z + 5)$$

Assume having perfect dielectric insolator and the wire have perfect conductivity with no loss

Draw the transmission line model and Find C' and L'

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad (\Omega),$$

$$\beta = \Im\mathfrak{m}(\gamma) = \Im\mathfrak{m}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(rad/m)}.$$

With $R' = G' = 0$, Eqs. (2.25b) and (2.29) reduce to

$$\beta = \Im\mathfrak{m}\left[\sqrt{(j\omega L')(j\omega C')}\right] = \Im\mathfrak{m}\left(j\omega\sqrt{L'C'}\right) = \omega\sqrt{L'C'},$$

$$Z_0 = \sqrt{\frac{j\omega L'}{j\omega C'}} = \sqrt{\frac{L'}{C'}}.$$

The ratio of β to Z_0 is

$$\frac{\beta}{Z_0} = \omega C',$$

or

$$C' = \frac{\beta}{\omega Z_0} = \frac{20}{2\pi \times 7 \times 10^8 \times 50} = 9.09 \times 10^{-11} \text{ (F/m)} = 90.9 \text{ (pF/m)}.$$

From $Z_0 = \sqrt{L'/C'}$, it follows that

$$L' = Z_0^2 C' = (50)^2 \times 90.9 \times 10^{-12} = 2.27 \times 10^{-7} \text{ (H/m)} = 227 \text{ (nH/m)}.$$

Section 2

Transmission Line Characteristics

- Line characterization
  - Propagation Constant (function of frequency)
  - Impedance (function of frequency)
    - Lossy or Losless
- If lossless (low ohmic losses)
  - Very high conductivity for the insulator
  - Negligible conductivity for the dielectric

Lossless Transmission Line

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\,.$$

If $\quad R' \ll \omega L'$ and $G' \ll \omega C'$

Then:

$$\gamma = \alpha + j\beta = j\omega\sqrt{L'C'}\,, \qquad (2.44)$$

$$\alpha = 0 \qquad \text{(lossless line)},$$
$$\beta = \omega\sqrt{L'C'} \qquad \text{(lossless line)}. \qquad (2.45)$$

What is Zo? $\quad Z_0 = \sqrt{\dfrac{(R' + j\omega L')}{(G' + j\omega C')}}$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{L'C'}}\,,$$

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}\,.$$

$$\beta = \omega\sqrt{\mu\varepsilon} \qquad \text{(rad/m)}, \qquad (2.49)$$
$$u_p = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \text{(m/s)}, \qquad (2.50)$$

Non-dispersive line:
All frequency components have the same speed!

Example

- Assume Lossless TL;
- Relative permittivity is 4
- C' =10 pF/m
  - Find phase velocity
  - Find L'
  - Find Zo

```
>> u=4*pi*1e-7;
>> e=8.854e-12

e =

  8.8540e-012

>> up=sqrt(1/(u*e*4))

up =

  1.4990e+008

>> L=1/10e-12 * 1/(up*up)

L =

  4.4505e-006

>> Zo=sqrt(L/10e-12)

Zo =

  667.1211
```

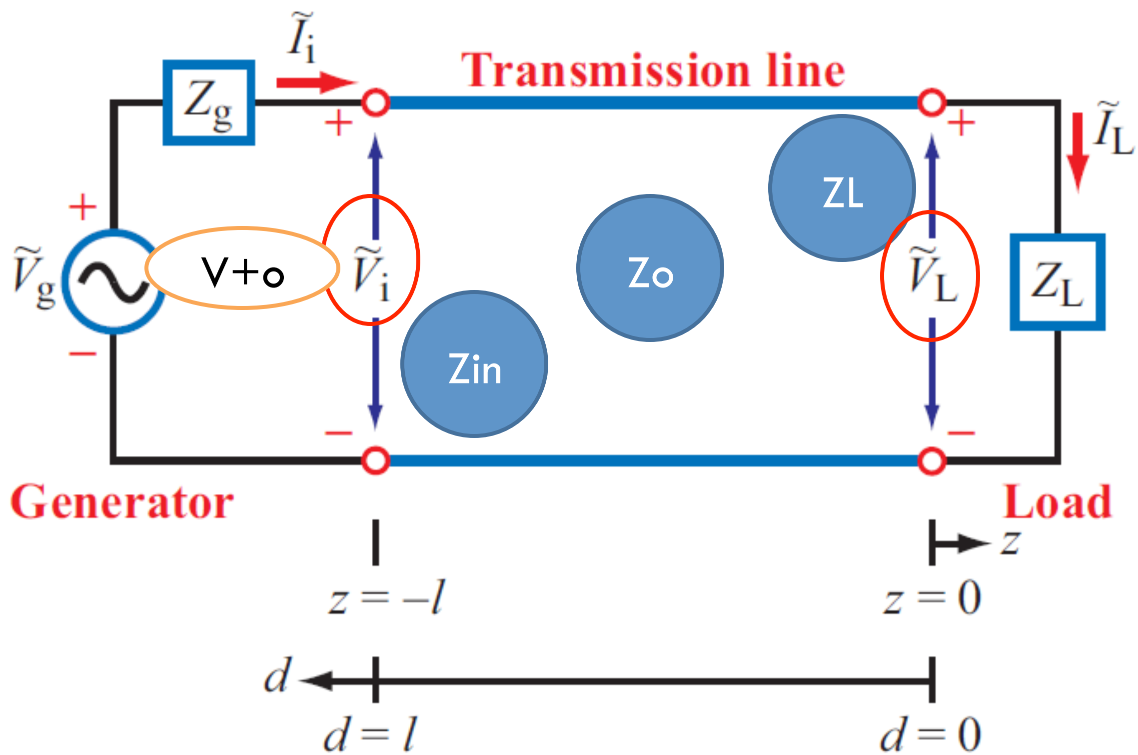
Notes-1

Table 2-2: Characteristic parameters of transmission lines.

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\dfrac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless $(R' = G' = 0)$	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \left(60/\sqrt{\varepsilon_r}\right)\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \left(120/\sqrt{\varepsilon_r}\right) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \simeq \left(120/\sqrt{\varepsilon_r}\right)\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \left(120\pi/\sqrt{\varepsilon_r}\right)(h/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.

The Big Idea....



What is the voltage/current magnitude at different points of the line in the presence of load??

Voltage Reflection Coefficient

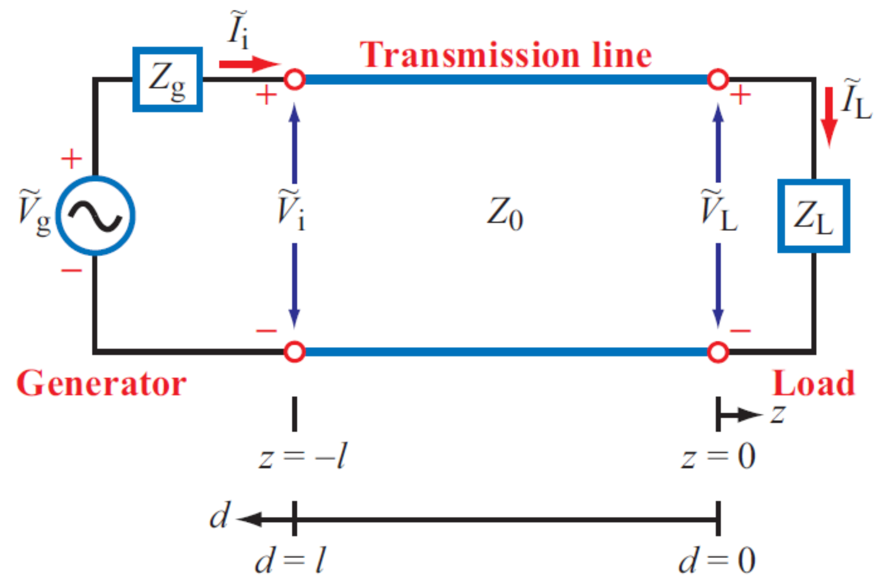
Consider looking from the Load point of view

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad \text{(V)},$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad \text{(A)}.$$

$$\tilde{V}_L = \tilde{V}(z{=}0) = V_0^+ + V_0^-,$$

$$\tilde{I}_L = \tilde{I}(z{=}0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}.$$



At the load (z = 0):

$$Z_L = \frac{\tilde{V}_L}{\tilde{I}_L}$$

$$Z_L = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0.$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1} \quad \text{(dimensionless)}, \quad (2.59)$$

Reflection coefficient

The smaller the better!

$$z_L = \frac{Z_L}{Z_0}$$

Normalized load impedance

Expressing wave in phasor form:

- Remember:

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad \text{(A)}.$$

- If lossless
  - no attenuation constant

$$\tilde{V}(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{j\beta z}).$$

$$V_0^- = \Gamma V_0^+$$

All of these wave representations
are **along** the
Transmission Line

Special Line Conditions (Lossless)

- Matching line
  - $Z_L = Z_o \rightarrow \Gamma = 0$; Vref=0
- Open Circuit
  - Z_L=INF $\rightarrow \Gamma = 1$; Vref=Vinc
- Short Circuit
  - $Z_L = 0 \rightarrow \Gamma = -1$; Vref=-Vinc

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1} \quad \text{(dimensionless)}, \qquad (2.59)$$

Remember:
Everything is with respect
to the load so far!

$$\frac{I_0^-}{I_0^+} = -\frac{V_0^-}{V_0^+} = -\Gamma. \qquad (2.61)$$

Notes

Voltage Reflection Coefficient

$\Gamma = |\Gamma| e^{j\theta_r}$

Reflection Coefficient $\Gamma = |\Gamma| e^{j\theta_r}$

Load	$\lvert\Gamma\rvert$	θ_r
Z_0, $Z_L = (r + jx)Z_0$	$\left[\dfrac{(r-1)^2 + x^2}{(r+1)^2 + x^2}\right]^{1/2}$	$\tan^{-1}\left(\dfrac{x}{r-1}\right) - \tan^{-1}\left(\dfrac{x}{r+1}\right)$
Z_0, Z_0	0 (no reflection)	irrelevant
Z_0, (short)	1	$\pm 180°$ (phase opposition)
Z_0, (open)	1	0 (in-phase)
Z_0, $jX = j\omega L$	1	$\pm 180° - 2\tan^{-1} x$
Z_0, $jX = \dfrac{-j}{\omega C}$	1	$\pm 180° + 2\tan^{-1} x$

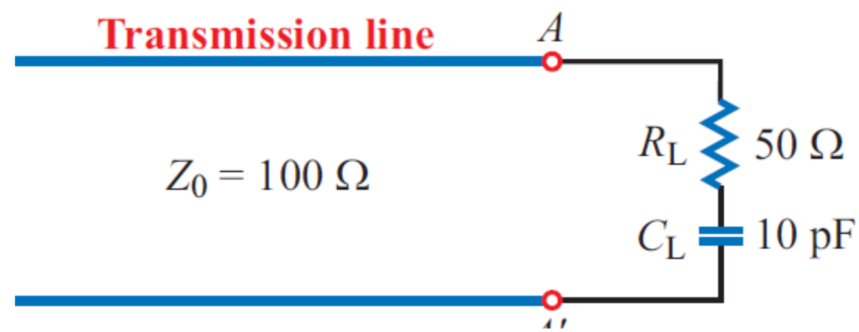
Pay attention!

Normalized load impedance

$$z_L = Z_L/Z_0 = (R + jX)/Z_0 = r + jx$$

Example

A 100-Ω transmission line is connected to a load consisting of a 50-Ω resistor in series with a 10-pF capacitor. Find the reflection coefficient at the load for a 100-MHz signal.



Notes

Standing Waves

Finding Voltage Magnitude

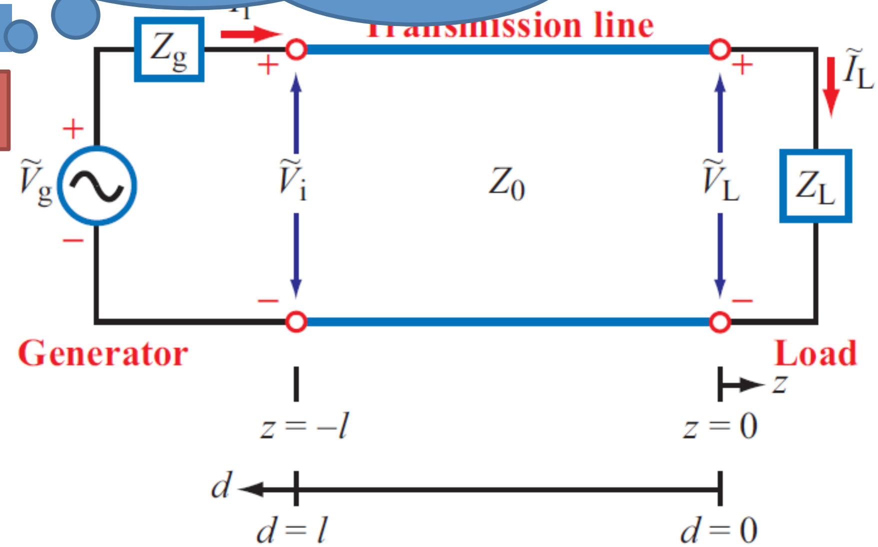
We are interested to know what happens to the magnitude of the |V| as such interference is created!

$$V_0^- = \Gamma V_0^+$$

When lossless!

$$\tilde{V}(z) = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{j\beta z}).$$

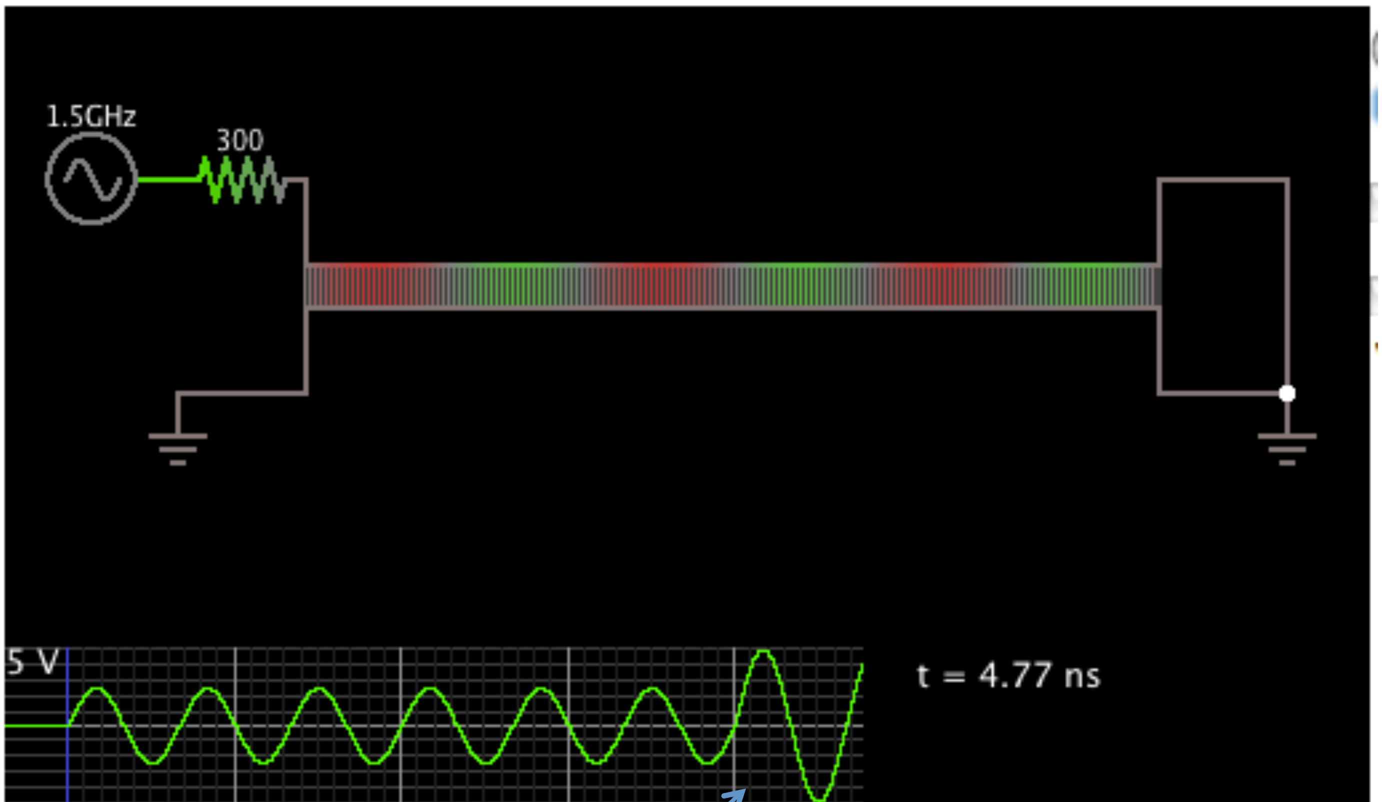


Remember: Standing wave is created due to interference between the traveling waves (incident & reflected)

Note: When there is no REFLECTION Coef. Of Ref. = 0 → No standing wave!

Standing Wave

http://www.falstad.com/circuit/e-tlstand.html



Due to standing wave the received wave at the load is now different

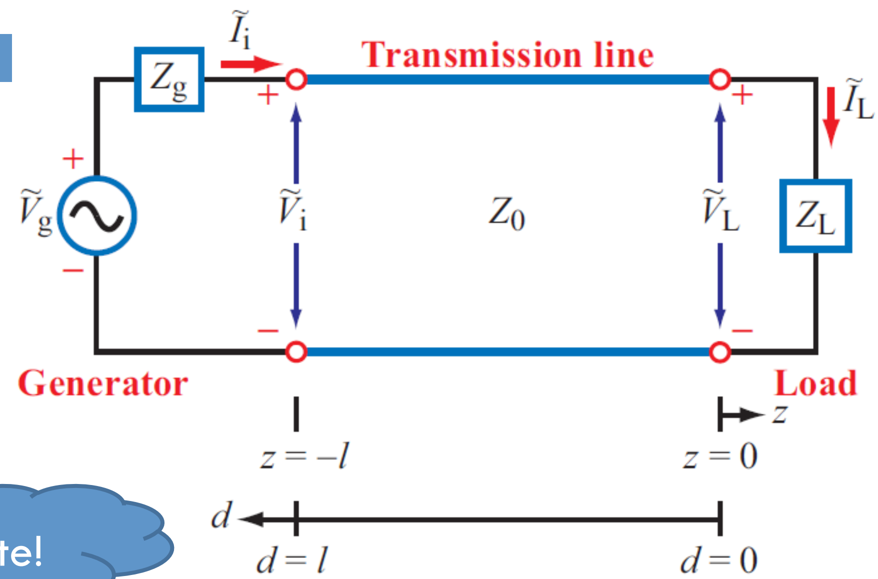
Standing Waves

Finding Voltage Magnitude

$$V_0^- = \Gamma V_0^+$$

$$\widetilde{V}(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\widetilde{I}(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{j\beta z}).$$



voltage magnitude due to interference

$$|\widetilde{V}(z)| = \left\{ \left[V_0^+ (e^{-j\beta z} + |\Gamma| e^{j\theta_r} e^{j\beta z}) \right] \cdot \left[(V_0^+)^* (e^{j\beta z} + |\Gamma| e^{-j\theta_r} e^{-j\beta z}) \right] \right\}^{1/2}$$

Conjugate!

$$= |V_0^+| \left[1 + |\Gamma|^2 + |\Gamma| (e^{j(2\beta z + \theta_r)} + e^{-j(2\beta z + \theta_r)}) \right]^{1/2}$$

$$= |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta z + \theta_r) \right]^{1/2},$$

This is standing wave!
Each position has a different value!

voltage magnitude

is the magnitude at the load?
What Z=-d

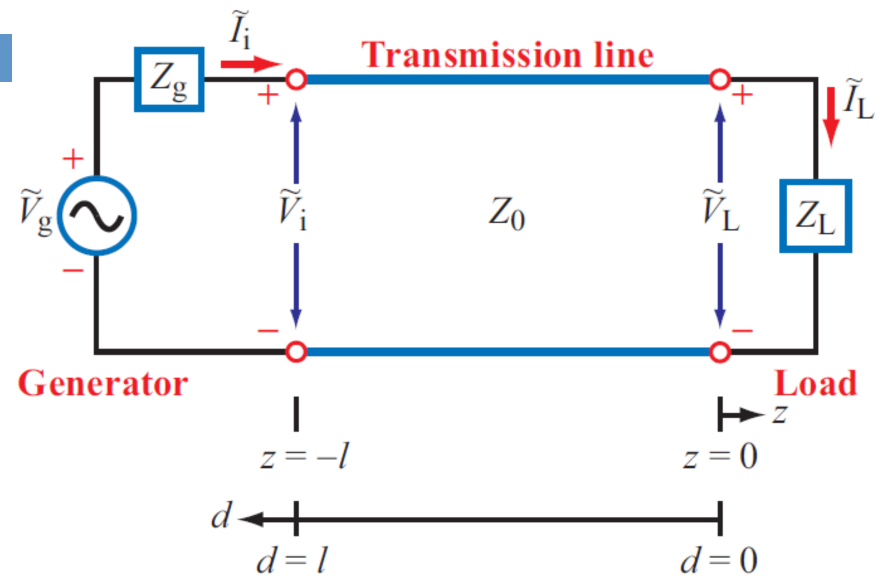
Standing Waves

Finding Voltage Magnitude

$$V_0^- = \Gamma V_0^+$$

$$\tilde{V}(z) = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{j\beta z}).$$



$$|\tilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}.$$

voltage magnitude at z= -d

$$|\tilde{I}(d)| = \frac{|V_0^+|}{Z_0}[1 + |\Gamma|^2 - 2|\Gamma|\cos(2\beta d - \theta_r)]^{1/2}.$$

current magnitude at the source

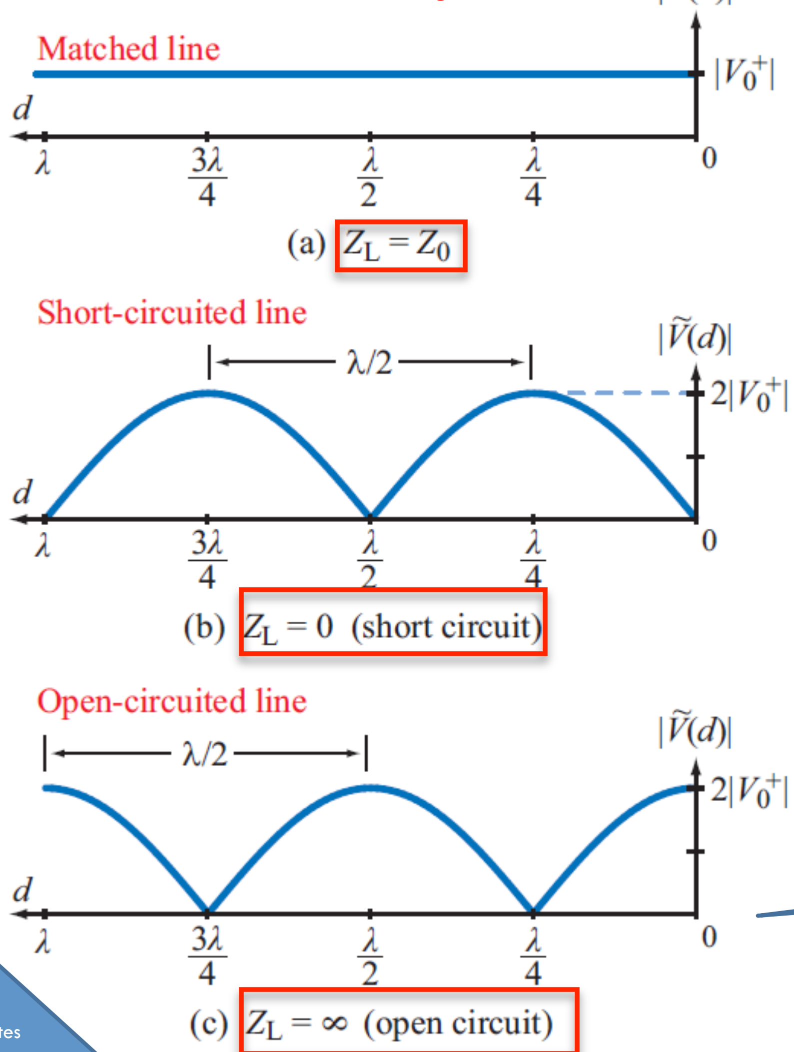
Let’s see how the magnitude looks like at different z values!

Remember max current occurs where minimum voltage occurs!

Standing Wave Patterns for 3 Types of Loads

(Matched, Open, Short)

No reflection, No standing wave



$$|\tilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta d - \theta_r)\right]^{1/2}.$$

- Matching line
  - $Z_L = Z_o$ → $\Gamma=0$; Vref=0
- Short Circuit
  - $Z_L=0$ → $\Gamma=-1$; Vref=-Vinc (angle −/+π)
- Open Circuit
  - Z_L=INF → $\Gamma=1$; Vref=Vinc (angle is 0)

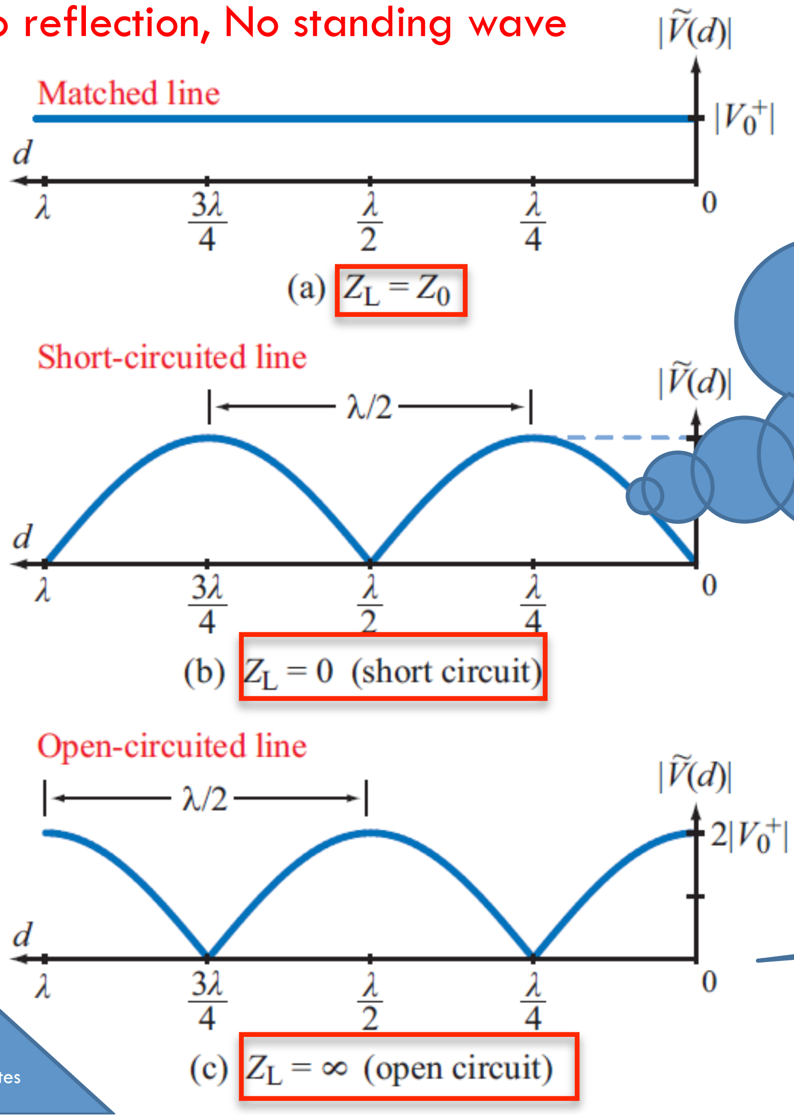
Remember max current occurs where minimum voltage occurs!

Notes

Standing Wave Patterns for 3 Types of Loads

(Matched, Open, Short)

No reflection, No standing wave



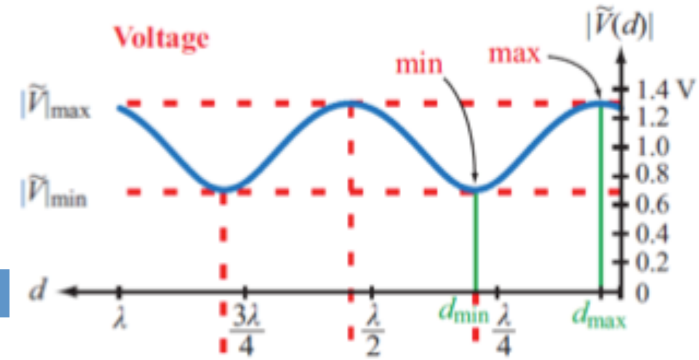
BUT WHEN DO MAX & MIN Voltages Occur?

=Vinc (angle is 0)

Remember max current occurs where minimum voltage occurs!

Notes

Standing Wave Pattern



- For Voltage:
  - Max occurs when cos( ) = 1 → $2\beta d_{max} - \theta_r = 2n\pi$
  - In this case n=0,1,2,…
  - NOTE that the FIRST & SECOND dmax are $\lambda/2$ apart
  - Thus, First MIN happens $\lambda/4$ after first dmax
  - And so on….

$$d_{max} = \frac{\theta_r + 2n\pi}{2\beta} = \frac{\theta_r\lambda}{4\pi} + \frac{n\lambda}{2}, \quad \begin{cases} n = 1, 2, \ldots & \text{if } \theta_r < 0, \\ n = 0, 1, 2, \ldots & \text{if } \theta_r \geq 0, \end{cases}$$

$$|\widetilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta d - \theta_r)\right]^{1/2}.$$

$$|\widetilde{I}(d)| = \frac{|V_0^+|}{Z_0} [1 + |\Gamma|^2 - 2|\Gamma| \cos(2\beta d - \theta_r)]^{1/2}.$$

Finding Maxima & Minima Of Voltage Magnitude

$$|\tilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta d - \theta_r) \right]^{1/2}.$$

$$|\tilde{V}|_{\min} = |V_0^+|[1 - |\Gamma|],$$

$$\text{when } (2\beta d_{\min} - \theta_r) = (2n+1)\pi$$

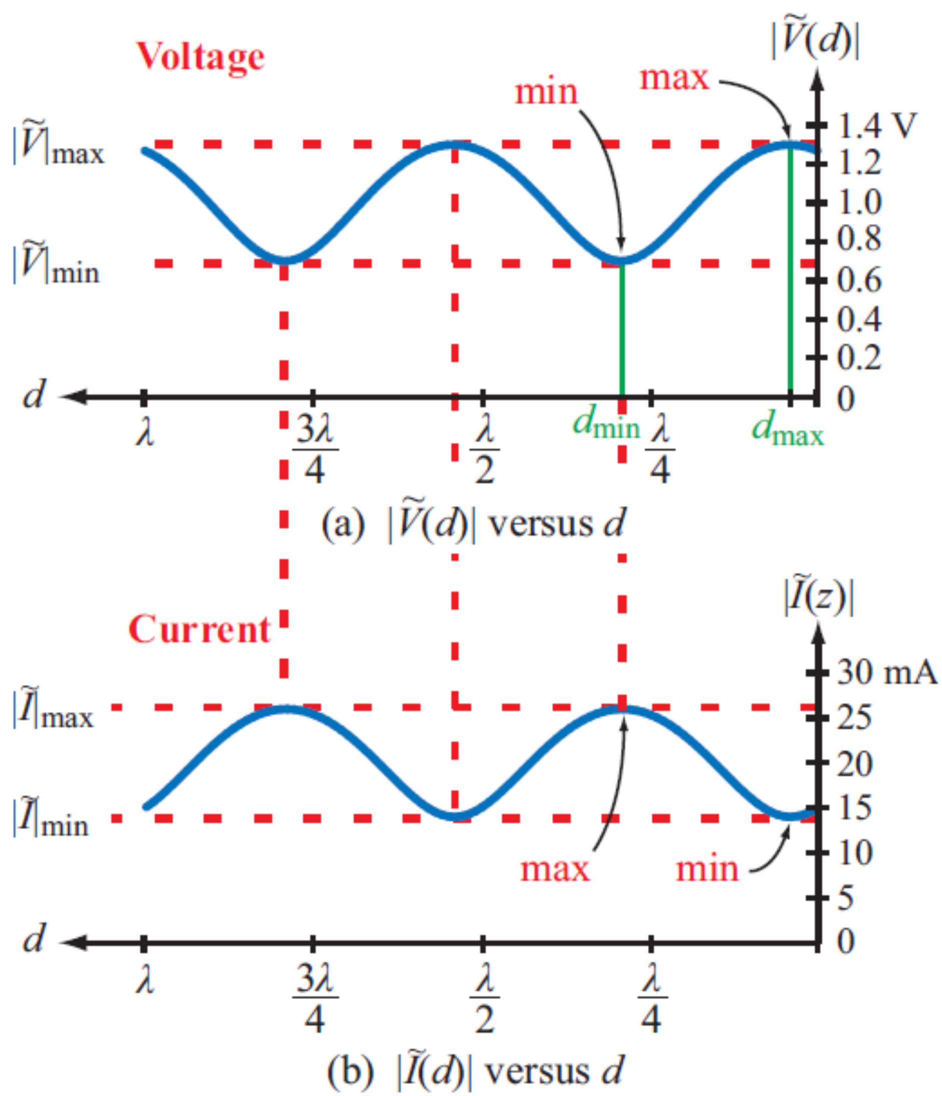
$$|\tilde{V}(d)| = |\tilde{V}|_{\max} = |V_0^+|[1 + |\Gamma|],$$

$$S = \frac{|\tilde{V}|_{\max}}{|\tilde{V}|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \text{(dimensionless)}$$

S = Voltage Standing Wave Ratio (VSWR)

For a matched load: $S = 1$

For a short, open, or purely reactive load: $S(open) = S(short) = \text{INF}$ where $|\Gamma| = 1$;



What is the Reflection Coefficient (Γd) at any point away from the load? (assume lossless line)

At a distance *d* from the load:

$$Z(d) = \frac{\tilde{V}(d)}{\tilde{I}(d)}$$

$$= \frac{V_0^+[e^{j\beta d} + \Gamma e^{-j\beta d}]}{V_0^+[e^{j\beta d} - \Gamma e^{-j\beta d}]} Z_0$$

$$= Z_0 \left[\frac{1 + \Gamma e^{-j2\beta d}}{1 - \Gamma e^{-j2\beta d}}\right]$$

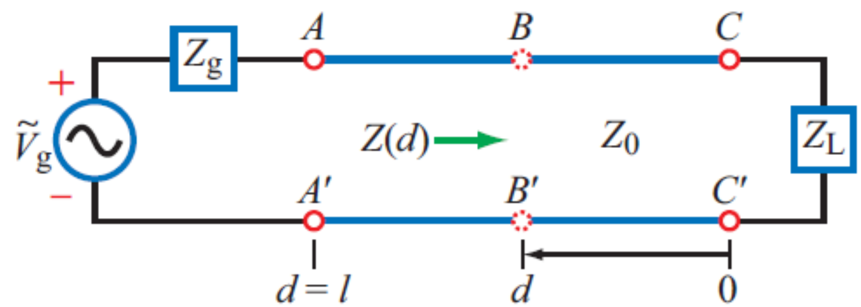
$$= Z_0 \left[\frac{1 + \Gamma_d}{1 - \Gamma_d}\right] \quad (\Omega),$$

Wave impedance

where we define

$$\Gamma_d = \Gamma e^{-j2\beta d} = |\Gamma| e^{j\theta_r} e^{-j2\beta d} = |\Gamma| e^{j(\theta_r - 2\beta d)}$$

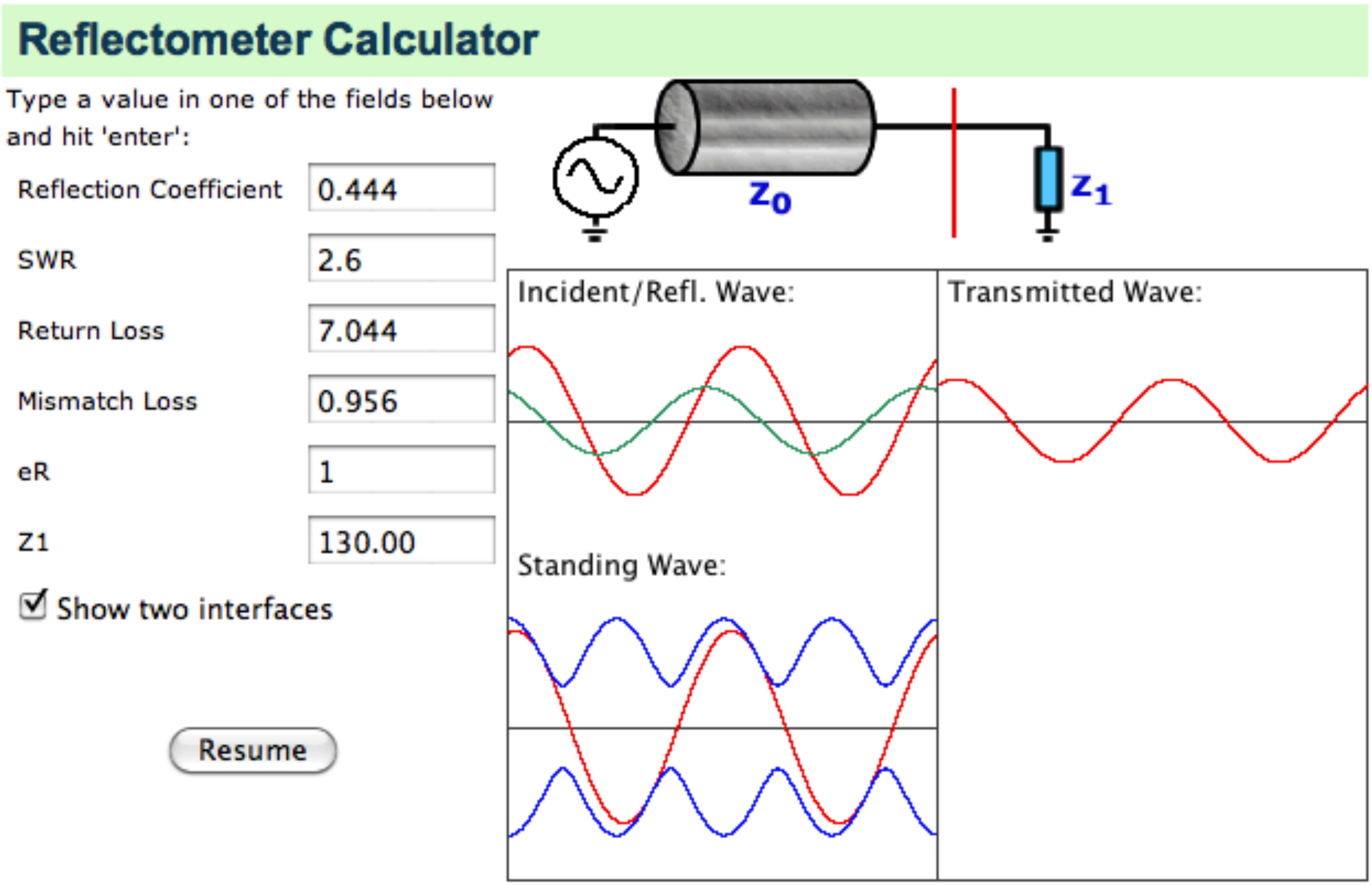
as the ***phase-shifted voltage reflection coefficient***,



(a) Actual circuit

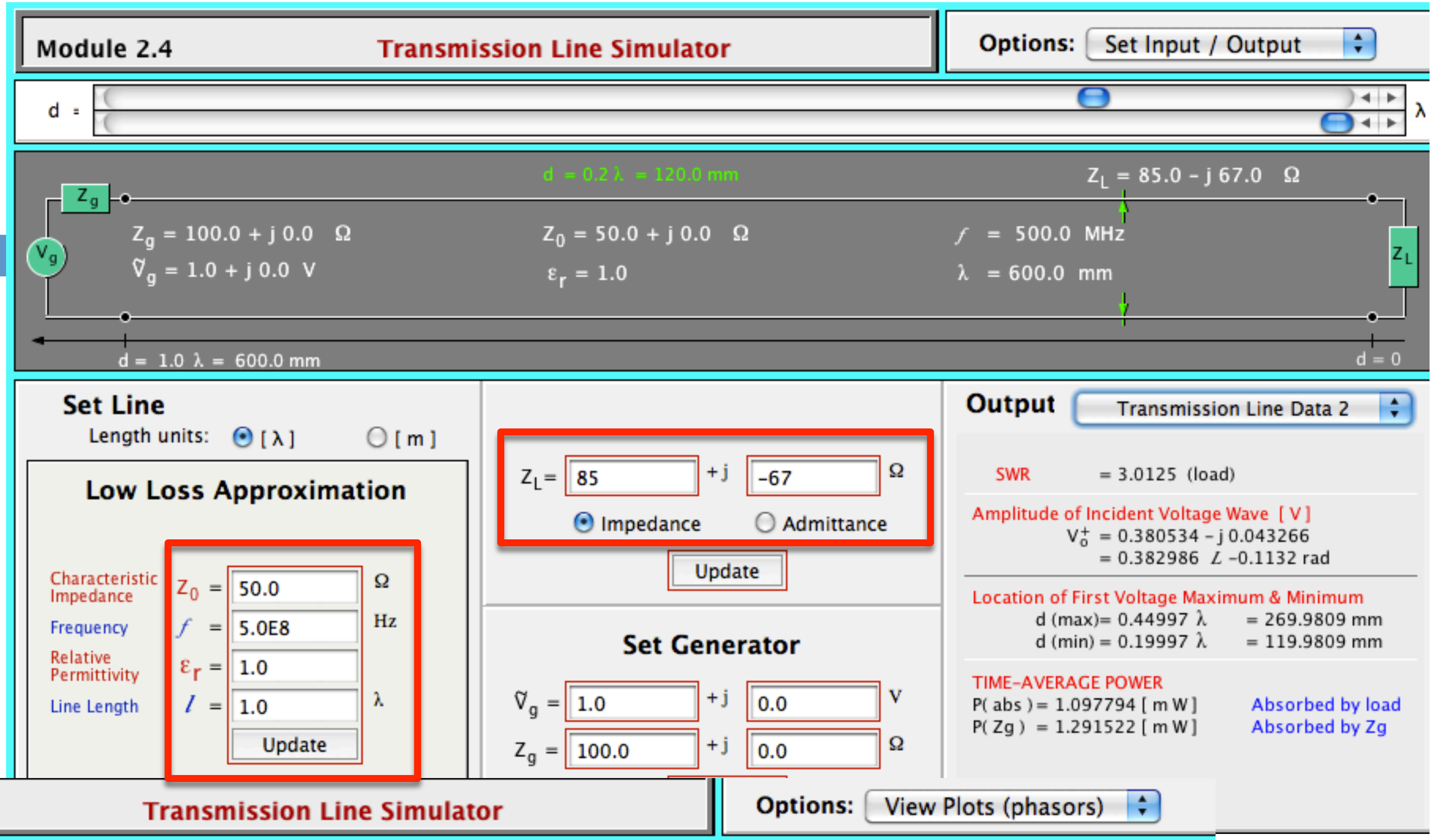
Example

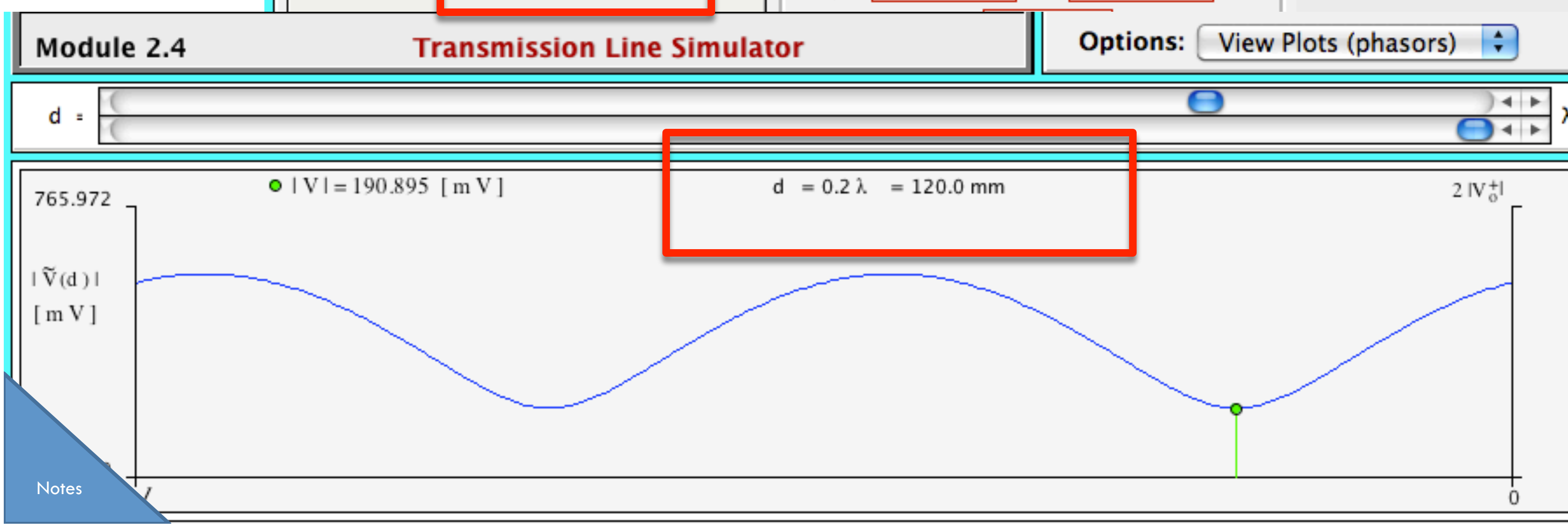
http://www.bessernet.com/Ereflecto/tutorialFrameset.htm



Notes

Example



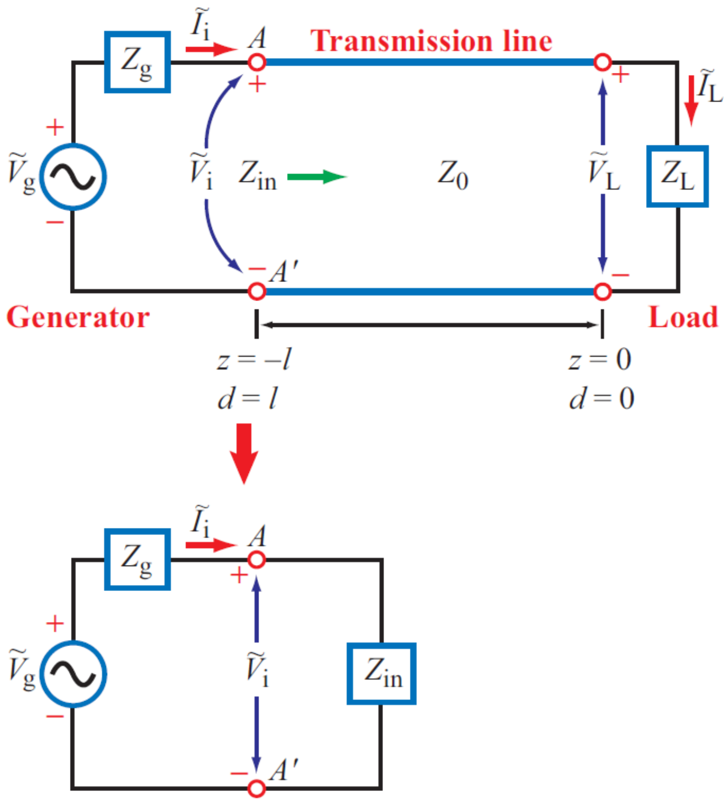


Notes

Input Impedance

Wave Impedance

$$Z_d = Z_0 \left[\frac{1+\Gamma_d}{1-\Gamma_d}\right]$$



$$Z_{in} = Z_0\left(\frac{z_L \cos\beta l + j\sin\beta l}{\cos\beta l + jz_L \sin\beta l}\right)$$

$$= Z_0\left(\frac{z_L + j\tan\beta l}{1 + jz_L \tan\beta l}\right). \quad (2.79)$$

What is input voltage?

$$\tilde{V}_i = \tilde{I}_i Z_{in} = \frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}}, \quad (2.80)$$

Simultaneously, from the standpoint of the transmission line, the voltage across it at the input of the line is given by Eq. (2.63a) with $z = -l$:

$$\tilde{V}_i = \tilde{V}(-l) = V_0^+[e^{j\beta l} + \Gamma e^{-j\beta l}]. \quad (2.81)$$

Equating Eq. (2.80) to Eq. (2.81) and then solving for V_0^+ leads to

At input, $d = l$:

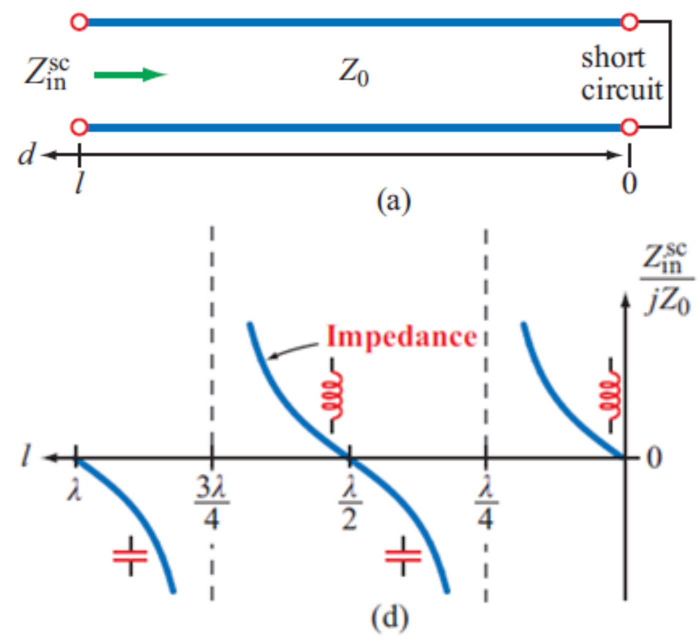
$$Z_{in} = Z(l) = Z_0\left[\frac{1+\Gamma_l}{1-\Gamma_l}\right].$$

$$V_0^+ = \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right). \quad (2.82)$$

$$\Gamma_l = \Gamma e^{-j2\beta l} = |\Gamma| e^{j(\theta_r - 2\beta l)}.$$

$$z_L = Z_L/Z_0 = (R + jX)/Z_0 = r + jx$$

Short-Circuited Line



$$Z_{in} = Z_0 \left(\frac{z_L \cos\beta l + j \sin\beta l}{\cos\beta l + j z_L \sin\beta l} \right)$$
$$= Z_0 \left(\frac{z_L + j \tan\beta l}{1 + j z_L \tan\beta l} \right). \quad (2.79)$$

ZL=0

$$j\omega L_{eq} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \geq 0$$

$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \leq 0$$

Input Impedance
Special Cases - Lossless

$$Z_{in} = Z_0 \left(\frac{z_L \cos\beta l + j \sin\beta l}{\cos\beta l + j z_L \sin\beta l} \right)$$
$$= Z_0 \left(\frac{z_L + j \tan\beta l}{1 + j z_L \tan\beta l} \right). \qquad (2.79)$$

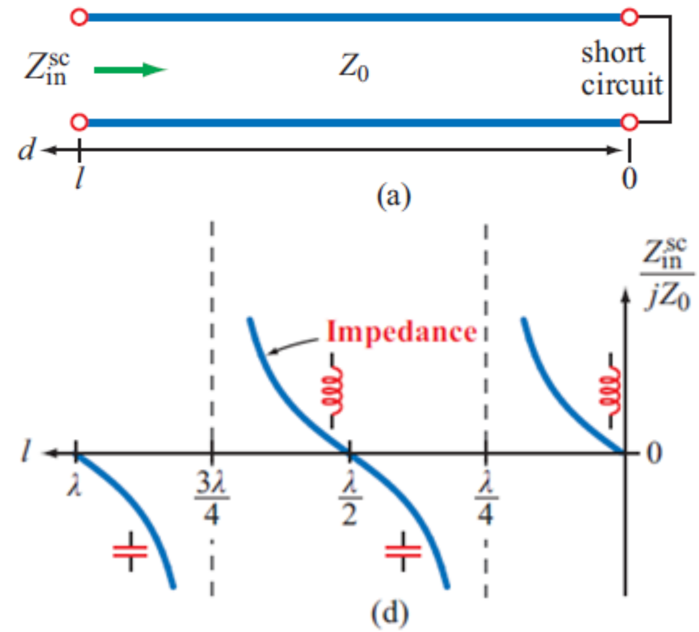
$$Z_{in}^{sc} = \frac{\tilde{V}_{sc}(l)}{\tilde{I}_{sc}(l)} = jZ_0 \tan\beta l.$$

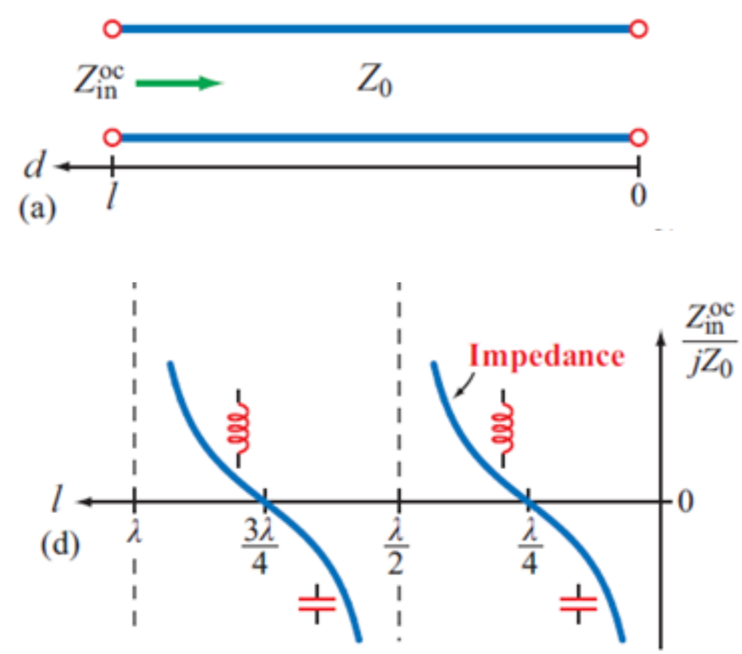
$$j\omega L_{eq} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \geq 0$$

$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \leq 0$$

$$Z_{in}^{oc} = \frac{\tilde{V}_{oc}(l)}{\tilde{I}_{oc}(l)} = -jZ_0 \cot\beta l.$$

What is Zin when matched?





Short-Circuit/Open-Circuit Method

- For a line of known length *l*, measurements of its input impedance, one when terminated in a short and another when terminated in an open, can be used to find its characteristic impedance Z_0 and electrical length βl

$$Z_{\text{in}}^{\text{sc}} = \frac{\widetilde{V}_{\text{sc}}(l)}{\widetilde{I}_{\text{sc}}(l)} = jZ_0 \tan \beta l.$$

$$Z_{\text{in}}^{\text{oc}} = \frac{\widetilde{V}_{\text{oc}}(l)}{\widetilde{I}_{\text{oc}}(l)} = -jZ_0 \cot \beta l.$$

$$Z_0 = +\sqrt{Z_{\text{in}}^{\text{sc}}\ Z_{\text{in}}^{\text{oc}}}\ ,$$

$$\tan \beta l = \sqrt{\frac{-Z_{\text{in}}^{\text{sc}}}{Z_{\text{in}}^{\text{oc}}}}\ .$$

Table 2-4: Properties of standing waves on a lossless transmission line.

Voltage Maximum	$\lvert\widetilde{V}\rvert_{max} = \lvert V_0^+\rvert[1 + \lvert\Gamma\rvert]$
Voltage Minimum	$\lvert\widetilde{V}\rvert_{min} = \lvert V_0^+\rvert[1 - \lvert\Gamma\rvert]$
Positions of voltage maxima (also positions of current minima)	$d_{max} = \frac{\theta_r\lambda}{4\pi} + \frac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$
Position of first maximum (also position of first current minimum)	$d_{max} = \begin{cases} \frac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \frac{\theta_r\lambda}{4\pi} + \frac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of current maxima)	$d_{min} = \frac{\theta_r\lambda}{4\pi} + \frac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$
Position of first minimum (also position of first current maximum)	$d_{min} = \frac{\lambda}{4}\left(1 + \frac{\theta_r}{\pi}\right)$
Input Impedance	$Z_{in} = Z_0\left(\frac{z_L + j\tan\beta l}{1 + jz_L\tan\beta l}\right) = Z_0\left(\frac{1+\Gamma_l}{1-\Gamma_l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{in} = Z_0\left(\frac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{in} = Z_0\left(\frac{1-\lvert\Gamma\rvert}{1+\lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{in}^{sc} = jZ_0\tan\beta l$
Z_{in} of open-circuited line	$Z_{in}^{oc} = -jZ_0\cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{in} = Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$	$Z_{in} = Z_0^2/Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of matched line	$Z_{in} = Z_0$
$\lvert V_0^+\rvert$ = amplitude of incident wave; $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians; $\Gamma_l = \Gamma e^{-j2\beta l}$.	

Example

- Check your notes!

Power Flow

- How much power is flowing and reflected?
  - Instantaneous $P(d,t) = v(d,t).i(d,t)$
    - Incident
    - Reflected
  - Average power: $Pav = Pav^i + Pav^r$
    - Time-domain Approach
    - Phasor-domain Approach (z and t independent)
      - $\frac{1}{2}\,\mathrm{Re}\{I^*(z)\,.\,V(z)\}$

$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{2Z_0}[1+\cos(2\omega t + 2\beta d + 2\phi^+)],$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2\frac{|V_0^+|^2}{2Z_0}[1+\cos(2\omega t - 2\beta d + 2\phi^+ + 2\theta_{\mathrm{r}})].$$

Instantaneous Power Flow

$$v(d,t) = \mathfrak{Re}[\widetilde{V} e^{j\omega t}]$$
$$= \mathfrak{Re}[|V_0^+| e^{j\phi^+} (e^{j\beta d} + |\Gamma| e^{j\theta_r} e^{-j\beta d}) e^{j\omega t}]$$
$$= |V_0^+| [\cos(\omega t + \beta d + \phi^+)$$
$$+ |\Gamma| \cos(\omega t - \beta d + \phi^+ + \theta_r)], \qquad (2.99a)$$

$$i(d,t) = \frac{|V_0^+|}{Z_0} [\cos(\omega t + \beta d + \phi^+)$$
$$- |\Gamma| \cos(\omega t - \beta d + \phi^+ + \theta_r)], \qquad (2.99b)$$

$$P(d,t) = v(d,t)\, i(d,t)$$
$$= |V_0^+| [\cos(\omega t + \beta d + \phi^+)$$
$$+ |\Gamma| \cos(\omega t - \beta d + \phi^+ + \theta_r)]$$
$$\times \frac{|V_0^+|}{Z_0} [\cos(\omega t + \beta d + \phi^+)$$
$$- |\Gamma| \cos(\omega t - \beta d + \phi^+ + \theta_r)]$$
$$= \frac{|V_0^+|^2}{Z_0} [\cos^2(\omega t + \beta d + \phi^+)$$
$$- |\Gamma|^2 \cos^2(\omega t - \beta d + \phi^+ + \theta_r)]$$

$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t + \beta d + \phi^+) \qquad \text{(W)},$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2 \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t - \beta d + \phi^+ + \theta_r)$$

Using the trigonometric identity

$$\boxed{\cos^2 x = \tfrac{1}{2}(1 + \cos 2x),}$$

the expressions in Eq. (2.101) can be rewritten as

$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{2Z_0} [1 + \cos(2\omega t + 2\beta d + 2\phi^+)],$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} [1 + \cos(2\omega t - 2\beta d + 2\phi^+ + 2\theta_r)].$$

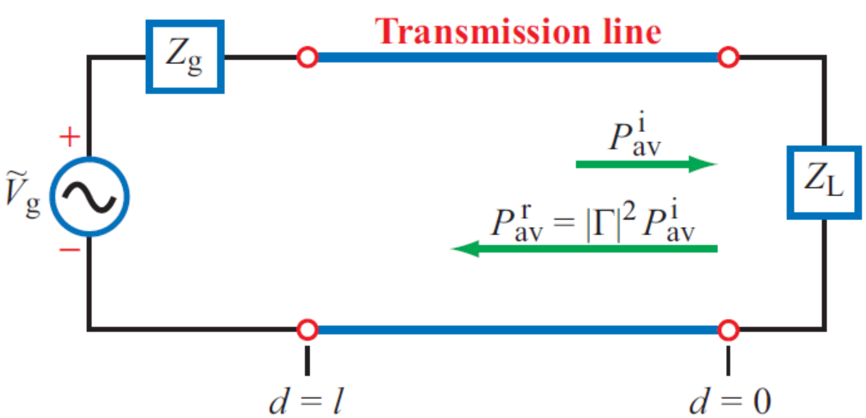
The power oscillates at twice the rate of the voltage or current.

Average Power
(Phasor Approach)

Avg Power: ½ Re{I(z) * V_(z)}

$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t + 2\beta d + 2\phi^+)]$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t - 2\beta d + 2\phi^+ + 2\theta_{\mathrm{r}})].$$



$$V_0^+ = \left(\frac{\widetilde{V}_{\mathrm{g}} Z_{\mathrm{in}}}{Z_{\mathrm{g}} + Z_{\mathrm{in}}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right).$$

Fraction of power reflected!

$$P^{\mathrm{i}}_{\mathrm{av}} = \frac{|V_0^+|^2}{2Z_0} \quad \text{(W)}, \qquad (2.104)$$

which is identical with the dc term of $P^{\mathrm{i}}(d,t)$ given by Eq. (2.102a). A similar treatment for the reflected wave gives

$$P^{\mathrm{r}}_{\mathrm{av}} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} = -|\Gamma|^2 P^{\mathrm{i}}_{\mathrm{av}}. \qquad (2.105)$$

The average reflected power is equal to the average incident power, diminished by a multiplicative factor of $|\Gamma|^2$.

Example

- Assume Zo=50 ohm, ZL=100+i50 ohm; What fraction of power is reflected?

$$P_{av}^{r} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} = -|\Gamma|^2 P_{av}^{i}. \quad (2.105)$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

```
>> x=(50+i*50)/(150+i*50)

x =

   0.4000 + 0.2000i

>> mag=abs(x)

mag =

    0.4472

>> angle=cart2pol(.4,.2)

angle =

    0.4636
```

```
angle =

    0.4636

>> radtodeg(.4636)

ans =

   26.5623
```

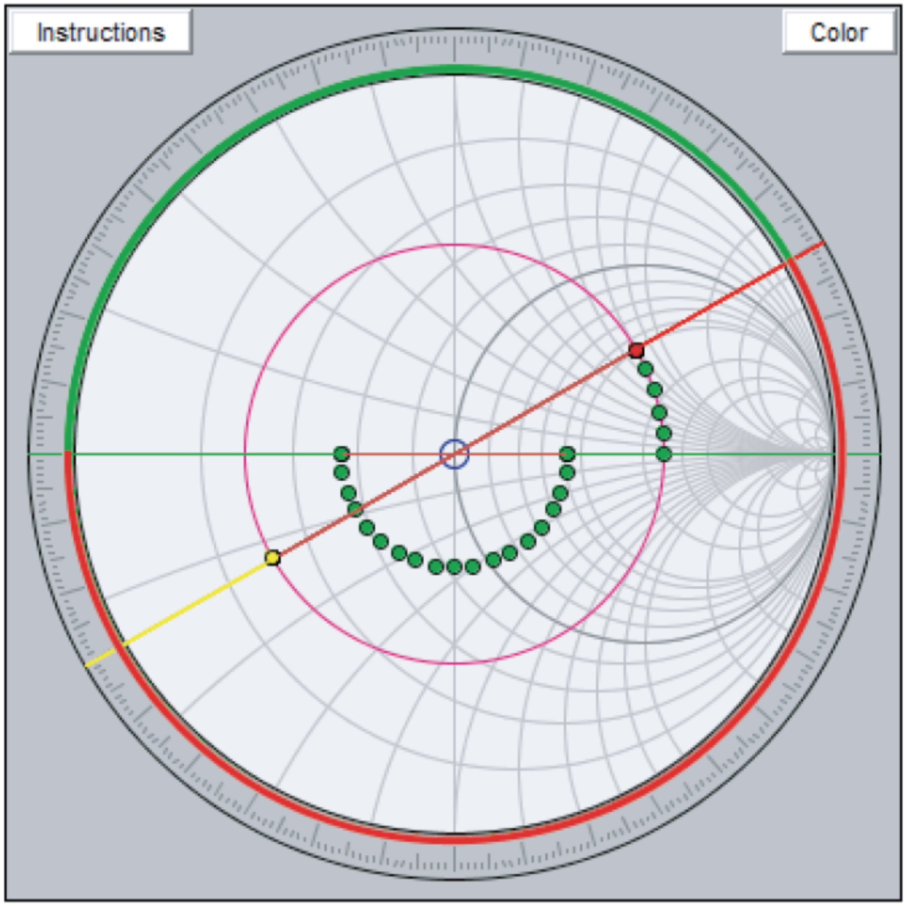
```
>> mag^2

ans =

    0.2000
```

20 percent! This is $|\Gamma|$^2

Notes

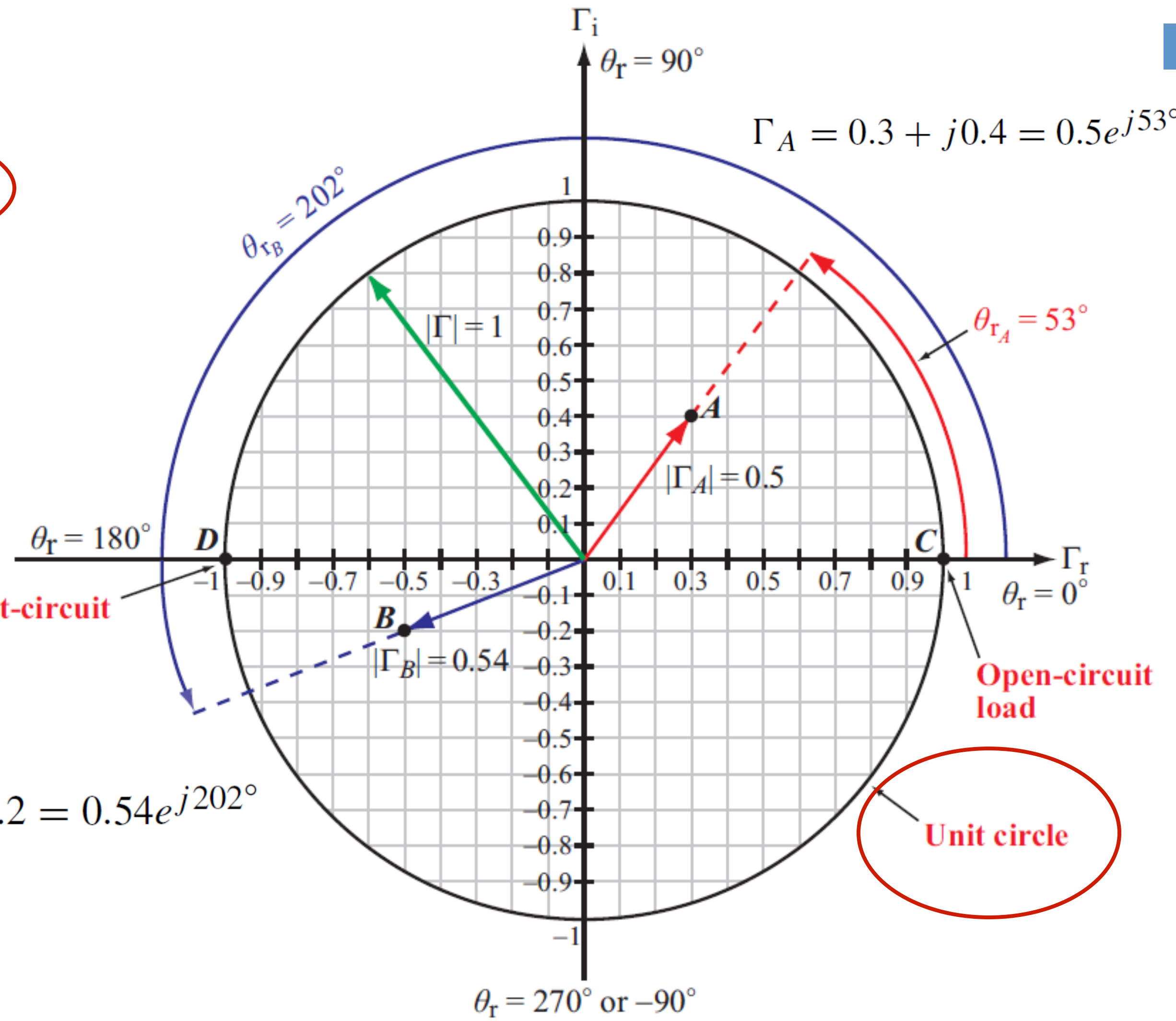
The Smith Chart

- Developed in 1939 by P. W. Smith as a graphical tool to analyze and design transmission-line circuits
- Today, it is used to characterize the performance of microwave circuits



Complex Plane

$$\Gamma = |\Gamma| e^{j\theta_r} = \Gamma_r + j\Gamma_i$$



$$\Gamma_B = -0.5 - j0.2 = 0.54e^{j202°}$$

Smith Chart Parametric Equations

$$\Gamma = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1+\Gamma}{1-\Gamma}. \qquad (2.112)$$

$$z_L = r_L + jx_L.$$

$$r_L + jx_L = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}$$

$$\Gamma = |\Gamma|e^{j\theta_r} = \Gamma_r + j\Gamma_i$$

$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

Equation for a circle

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2. \qquad (2.116)$$

Parameteric Equation!

For a given Coef. Of Reflection
various load combinations can be considered.
These combinations can be represented by
different circuits!
Smith Chart help us see these variations!

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2, \qquad (2.118)$$

Smith Chart Parametric Equations

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2. \qquad (2.116)$$

r_L circles

r_L circles are contained inside the unit circle

Each node on the chart will tell us about the load characteristics and coef. of ref. of the line!

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2, \qquad (2.118)$$

x_L circles

Imag. Part of ZL

Only parts of the x_L circles are contained within the unit circle

$$z_L = r_L + jx_L.$$

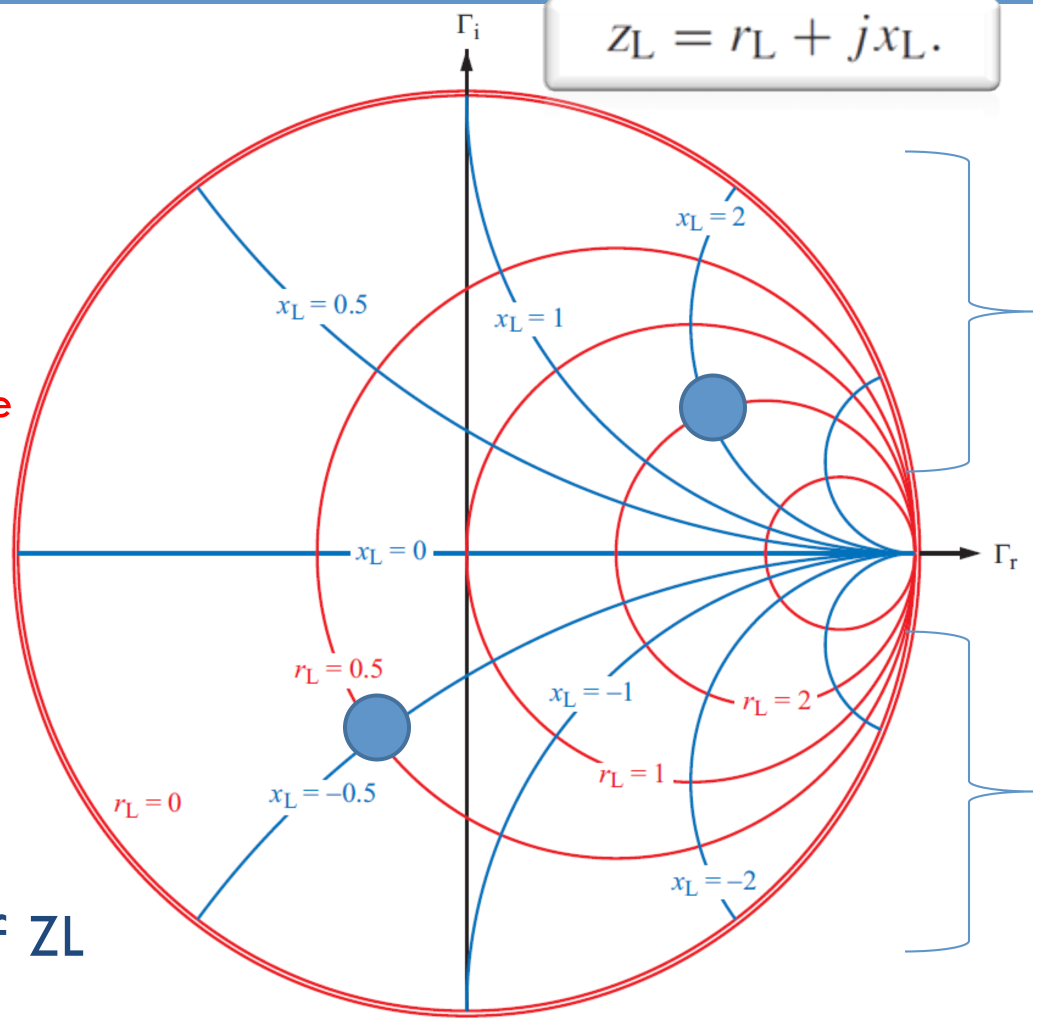
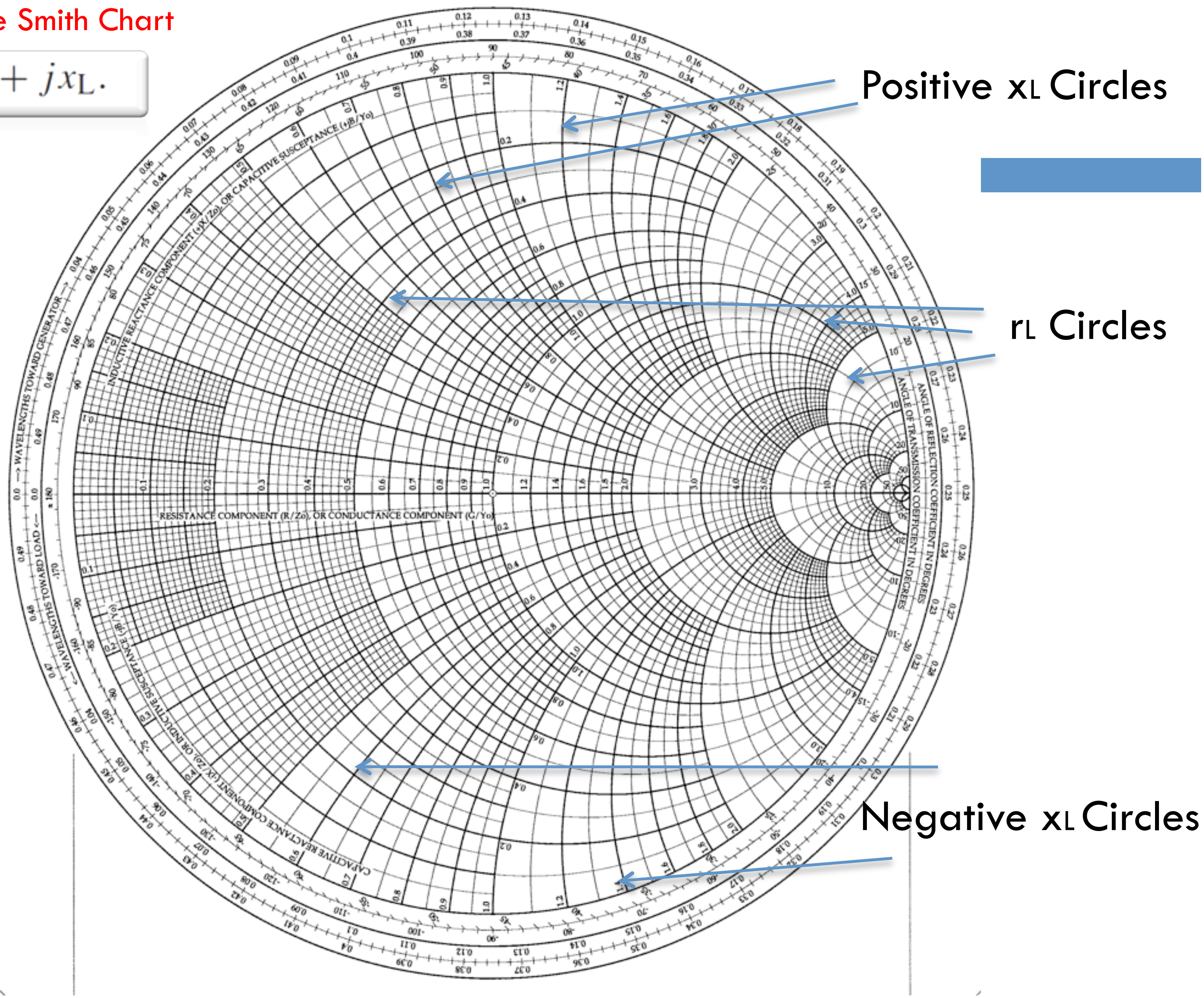


Figure 2-25: Families of r_L and x_L circles within the domain $|\Gamma| \leq 1$.

Complete Smith Chart

$$z_L = r_L + jx_L.$$



Basic Rules

- Given ZL find the coefficient of reflection (COR)
  - Find ZL on the chart (Pt. P) [1] – Normalized Load
  - Extend it and find the angle of COR [3]
  - Use ruler to measure find OP/OR ; OR is simply unity circle - This will be the magnitude of COR
- Find dmin and dmax
  - From the extended OP to
- Find VSWR (or S)
  - Draw a circle with radius of ZL (OP)
  - Find Pmin and Pmax=S along the circle (where |Vmin| and |Vmax| are)
- Input impedance Zd=Zin
  - Find S on the chart (OP)
  - Extend ZL all the way to hit a point on the outer circle
  - Then move away in the direction of WL TOWARD GENERATOR by $d=x\lambda$
  - Draw a line toward the center of the circle
    - The intersection of the S circle and this line will be the input load (Zin)

ZL/Zo
COR
dmin/dmax
SWR
zin & Zin
yin & Yin

Notes

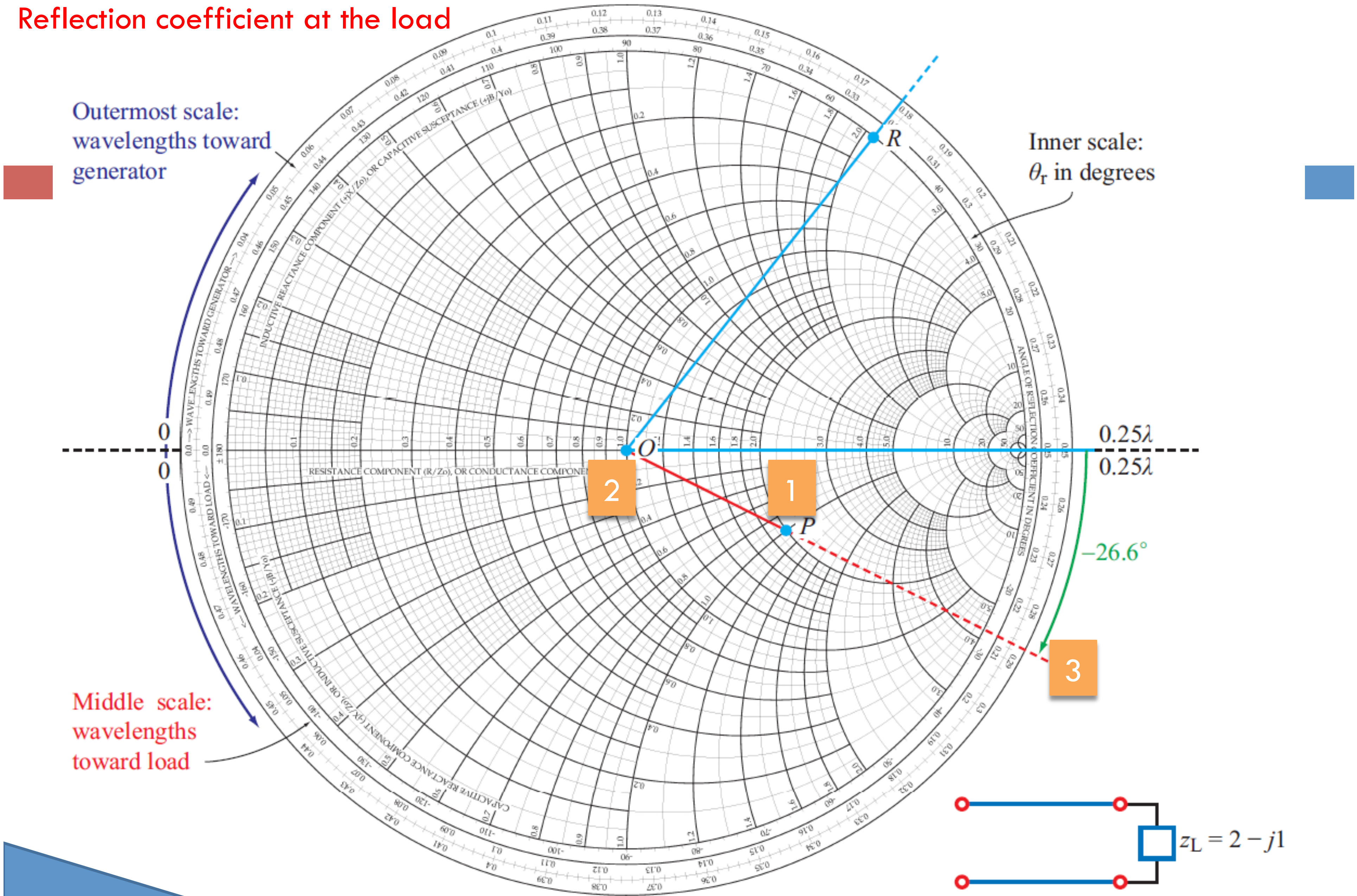
Basic Rules

- Input impedance Yd=Yin (admittance)
  - Once zin (normalized

ZL/Zo
COR
dmin/dmax
SWR
zin & Zin
yin & Yin

Notes

Reflection coefficient at the load



Example 1

...presents a normalized load impedance $z_L = 2 - j1$. The reflection coefficient has a magnitude ... $= -26.6°$. Point R is an arbitrary point on the $r_L = 0$ circle (which also is the $|\Gamma| = 1$ circle).

Input Impedance

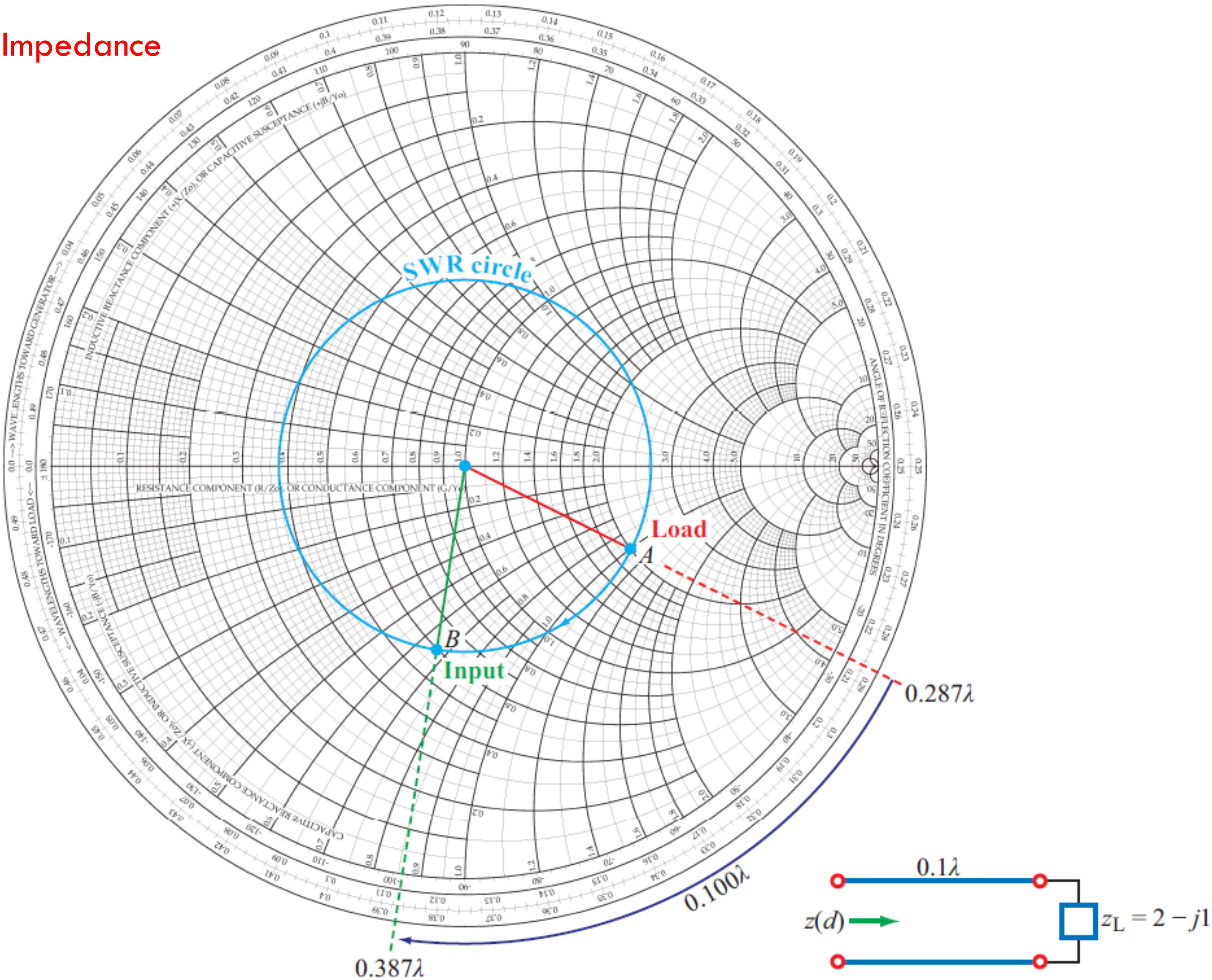


Figure 2-27: Point A represents a normalized load $z_L = 2 - j1$ at 0.287λ on the WTG scale. Point B represents the line input at $d = 0.1\lambda$ from the load. At B, $z(d) = 0.6 - j0.66$.

Maxima and Minima

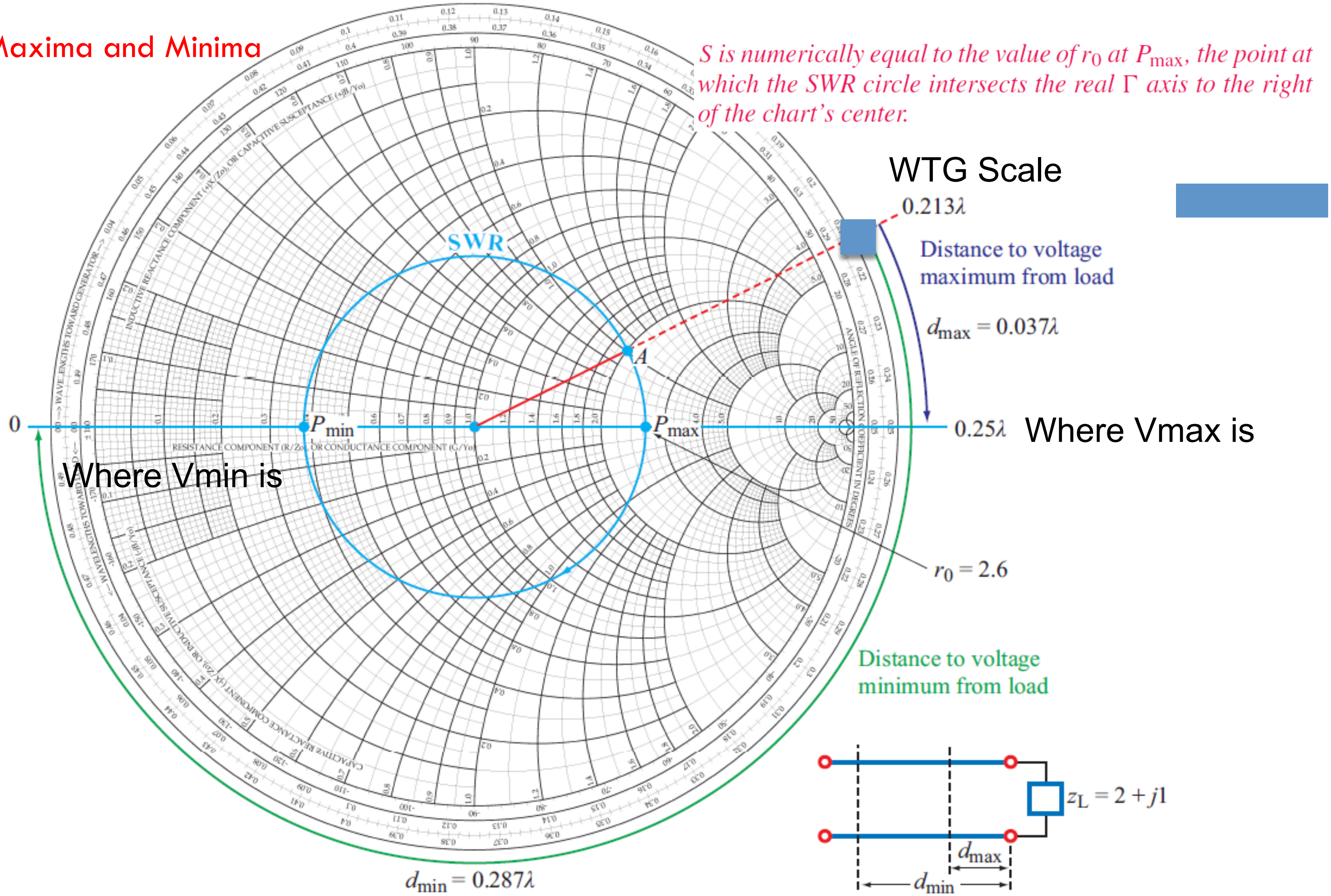
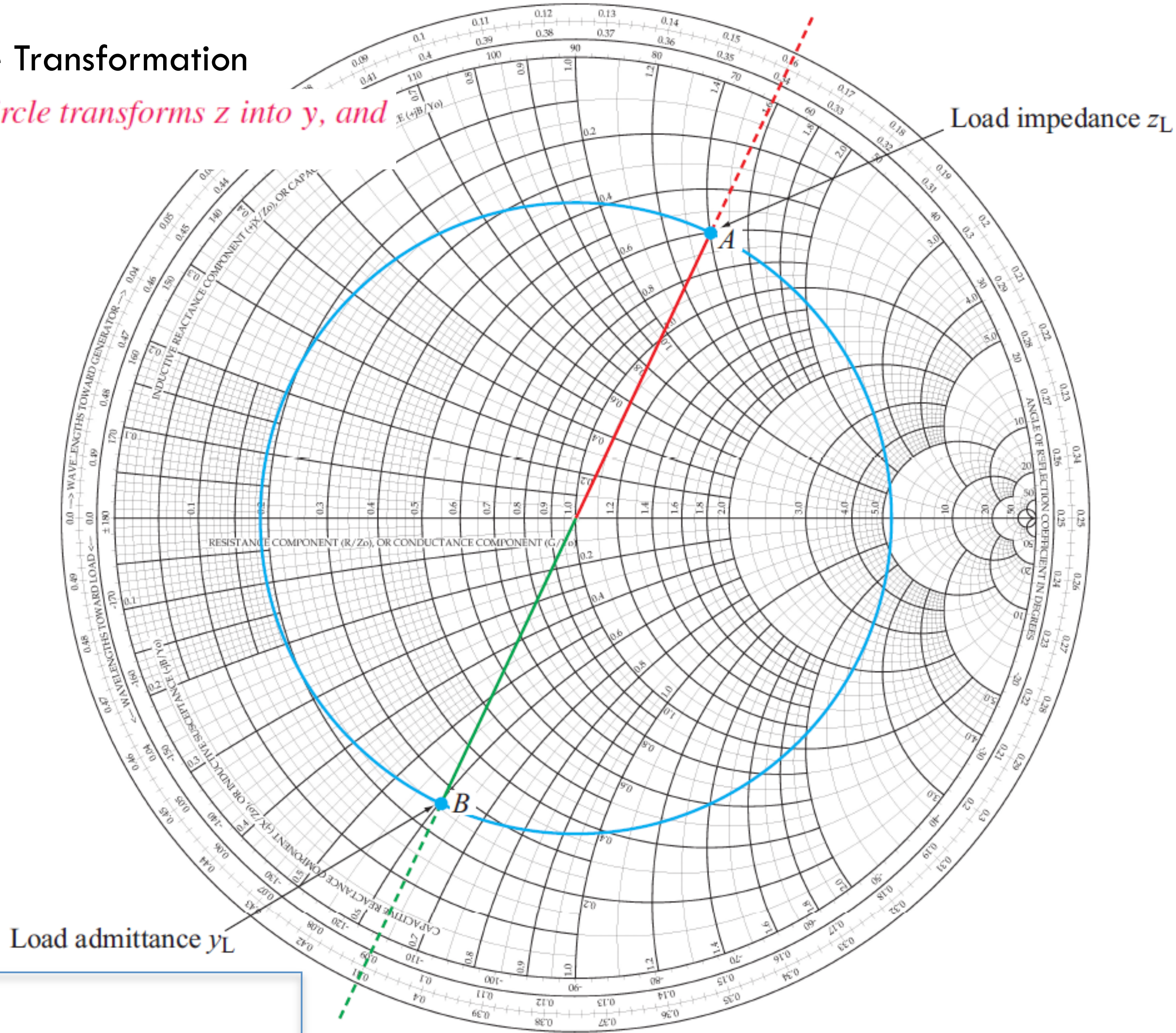


Figure 2-28: Point A represents a normalized load with $z_L = 2 + j1$. The standing wave ratio is $S = 2.6$ (at P_{max}), the distance between the load and the first voltage maximum is $d_{max} = (0.25 - 0.213)\lambda = 0.037\lambda$, and the distance between the load and the first voltage minimum is $d_{min} = (0.037 + 0.25)\lambda = 0.287\lambda$.

Impedance to Admittance Transformation

Rotation by λ/4 on the SWR circle transforms z into y, and vice versa.



Example 2-11: Smith Chart Calculations

(3.3)λ→(0.3)λ

A 50-Ω lossless transmission line of length 3.3λ is terminated by a load impedance $Z_L = (25 + j50)\ \Omega$.

$$z_L = \frac{Z_L}{Z_0} = \frac{25 + j50}{50} = 0.5 + j1$$

(a)

$$\Gamma = 0.62\angle 83^\circ$$

(b)

$$S = 4.26$$

(c)

$$d_{max} = (0.25 - 0.135)\lambda = 0.115\lambda$$

$$d_{min} = (0.5 - 0.135)\lambda = 0.365\lambda$$

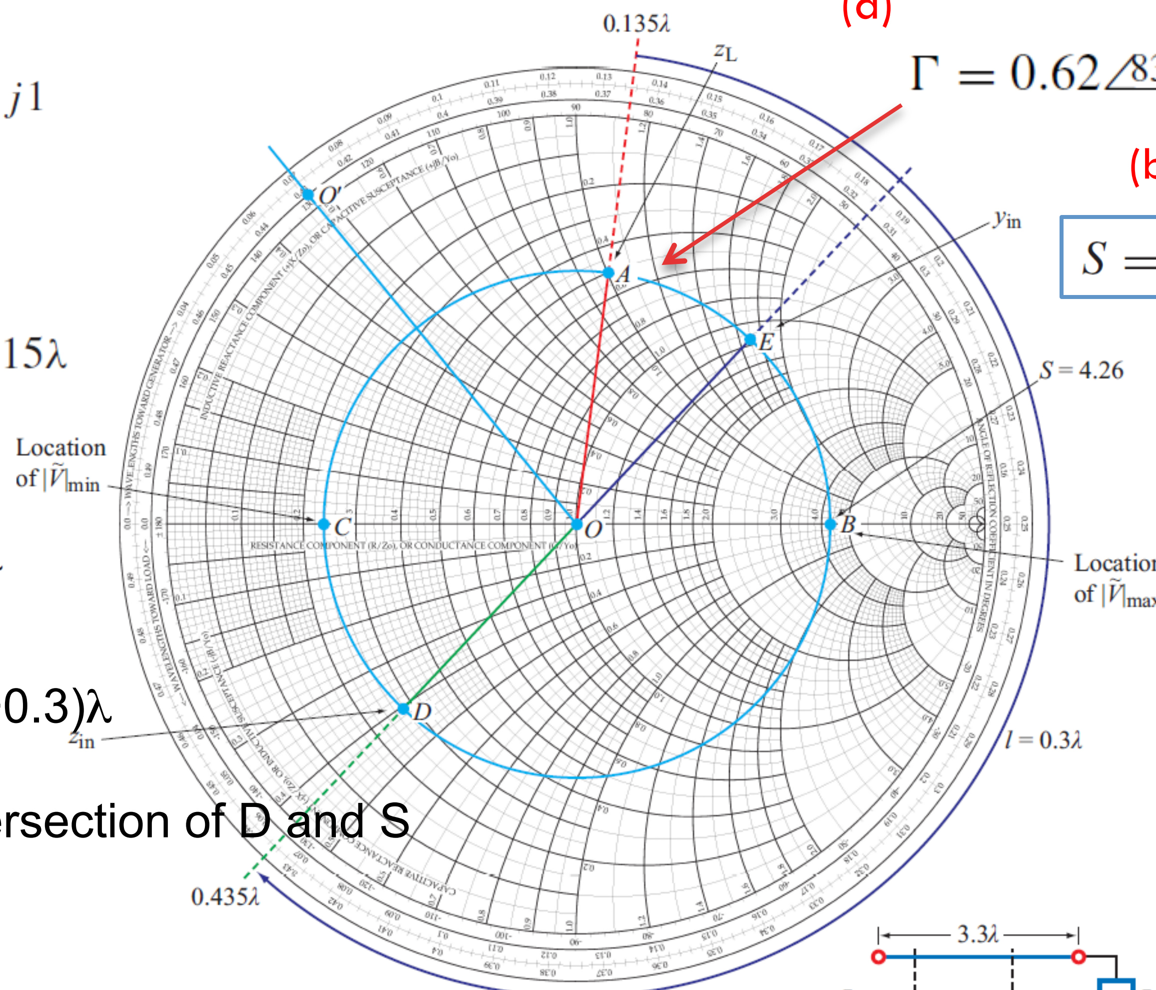
(d)

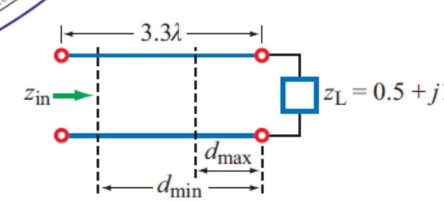
The generator is at (0.135+0.3)λ = .435λ –this is pt. D

→Zin normalized is the intersection of D and S

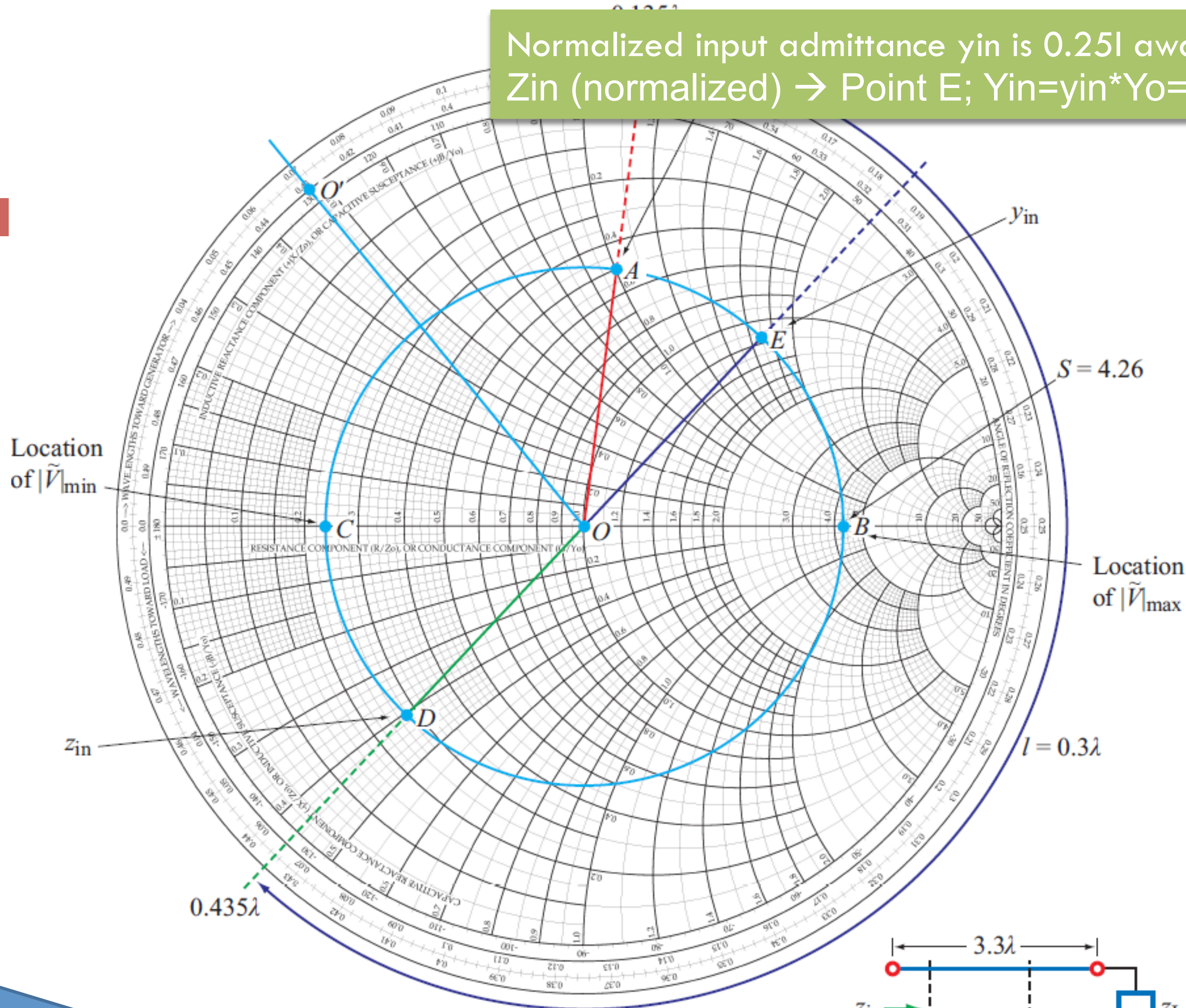
$$z_{in} = 0.28 - j0.40$$

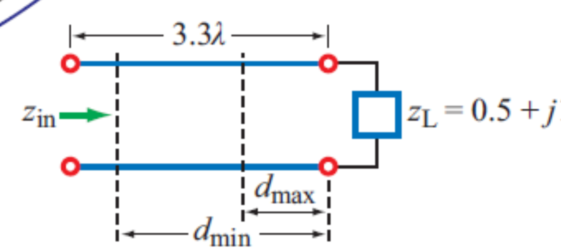
$$Z_{in} = z_{in} Z_0 = (0.28 - j0.40)50 = (14 - j20)\ \Omega$$





Normalized input admittance yin is 0.25l away from Zin (normalized) → Point E; Yin=yin*Yo=yin/Zo





Example 3

Example 2-12: Determining Z_L Using the Smith Chart

Given:

$S = 3$

$Z_0 = 50\ \Omega$

first voltage min @ 5 cm from load

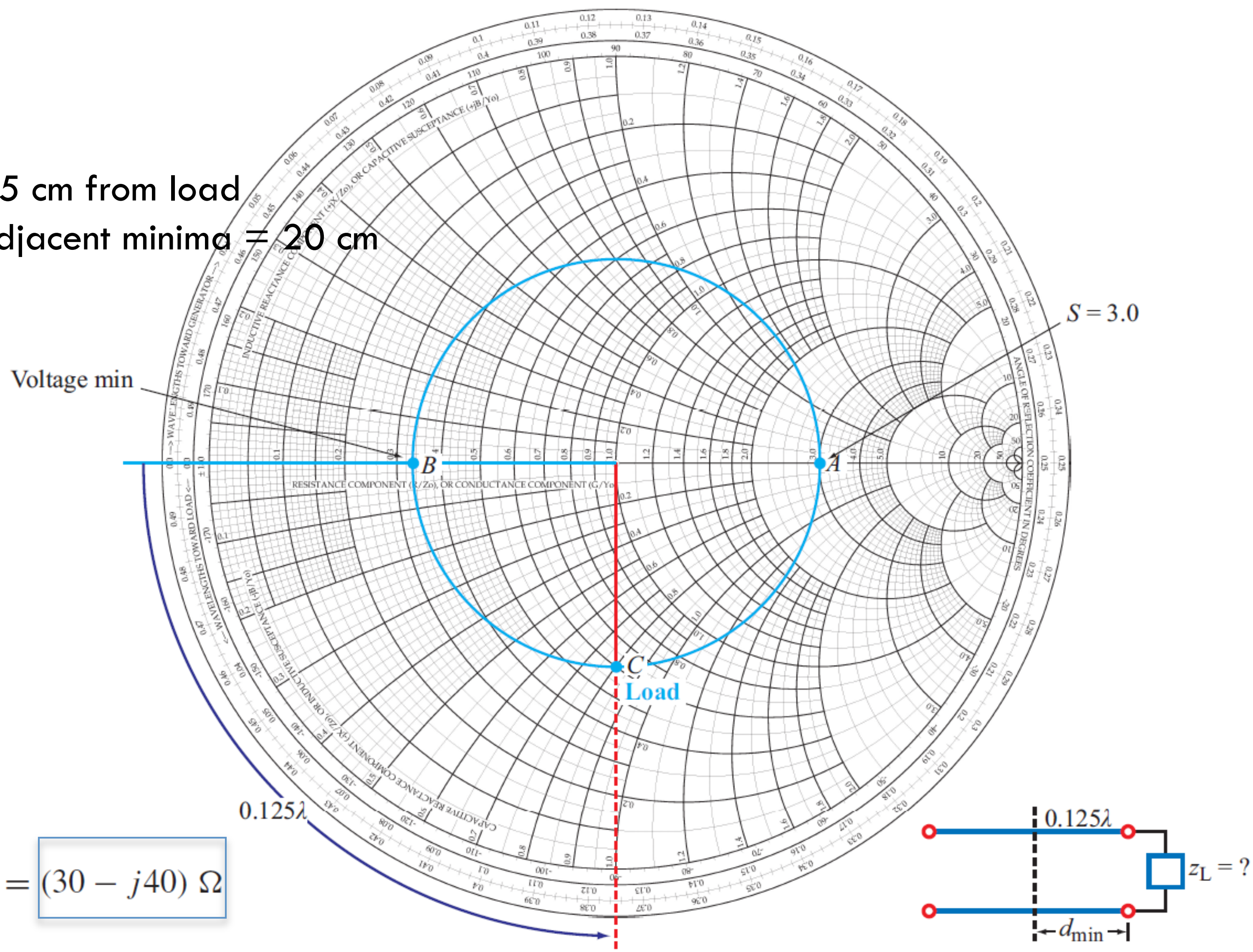
Distance between adjacent minima = 20 cm

Determine: Z_L

$$d_{min} = \frac{5}{40} = 0.125\lambda$$

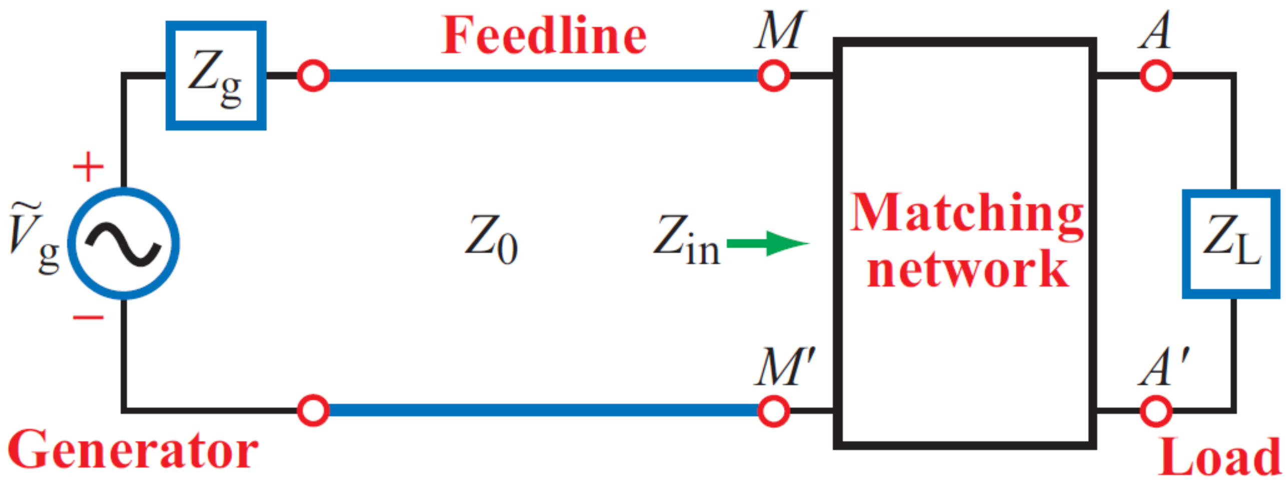
$$z_L = 0.6 - j0.8$$

$$Z_L = 50(0.6 - j0.8) = (30 - j40)\ \Omega$$

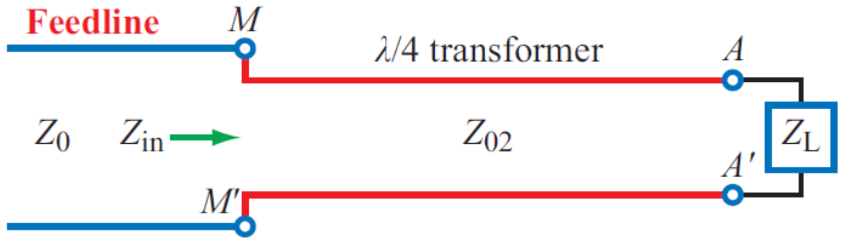


Matching Networks

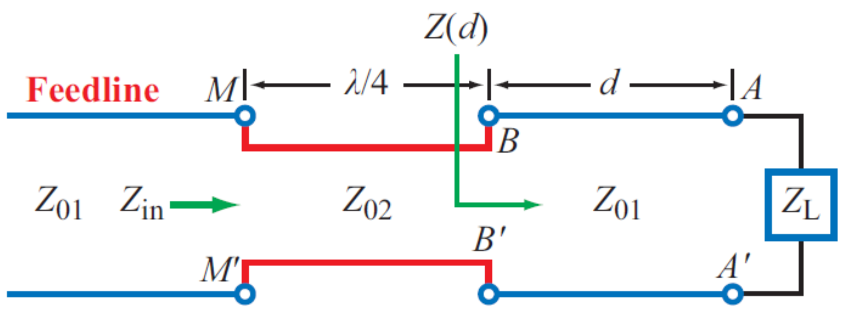
The purpose of the matching network is to eliminate reflections at terminals MM' for waves incident from the source. Even though multiple reflections may occur between AA' and MM', only a forward traveling wave exists on the feedline.



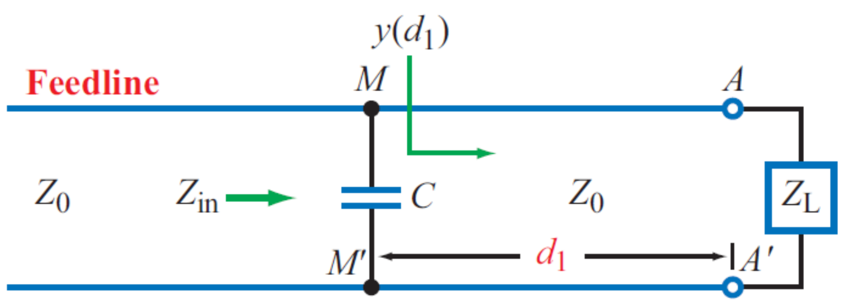
Examples of Matching Networks



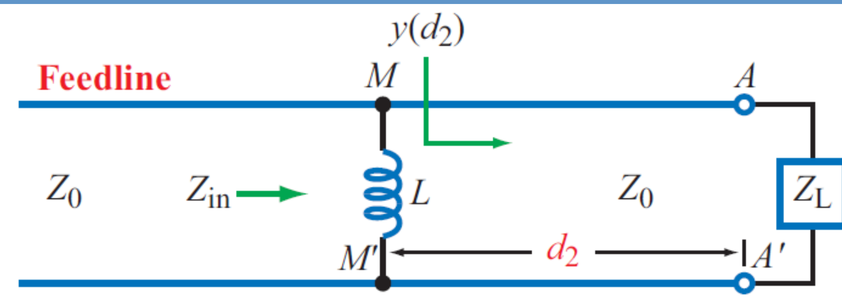
(a) In-series $\lambda/4$ transformer inserted at AA'



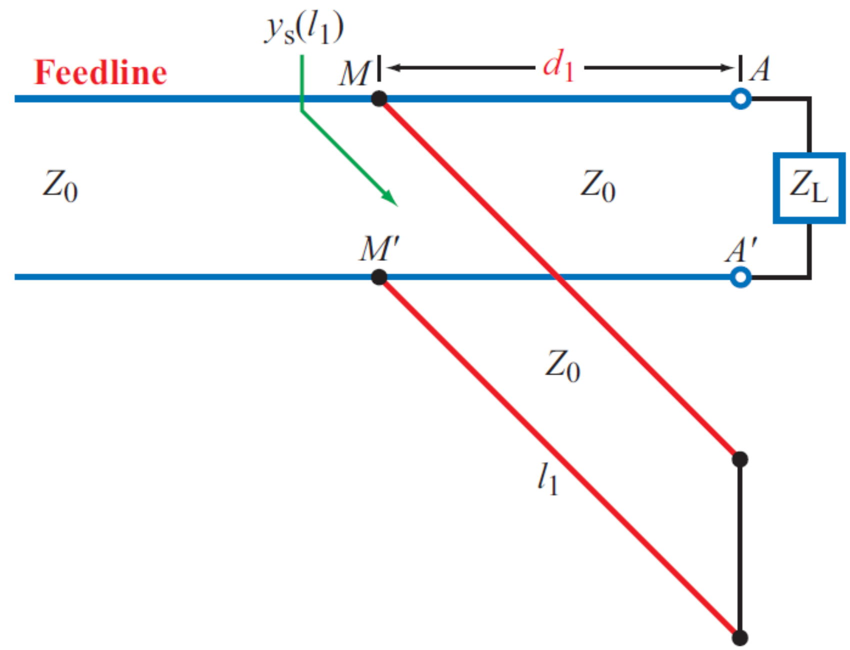
(b) In-series $\lambda/4$ transformer inserted at $d = d_{max}$ or $d = d_{min}$



(c) In-parallel insertion of capacitor at distance d_1



(d) In-parallel insertion of inductor at distance d_2



(e) In-parallel insertion of a short-circuited stub

Lumped-Element Matching

Choose d and Ys to achieve a match at MM’

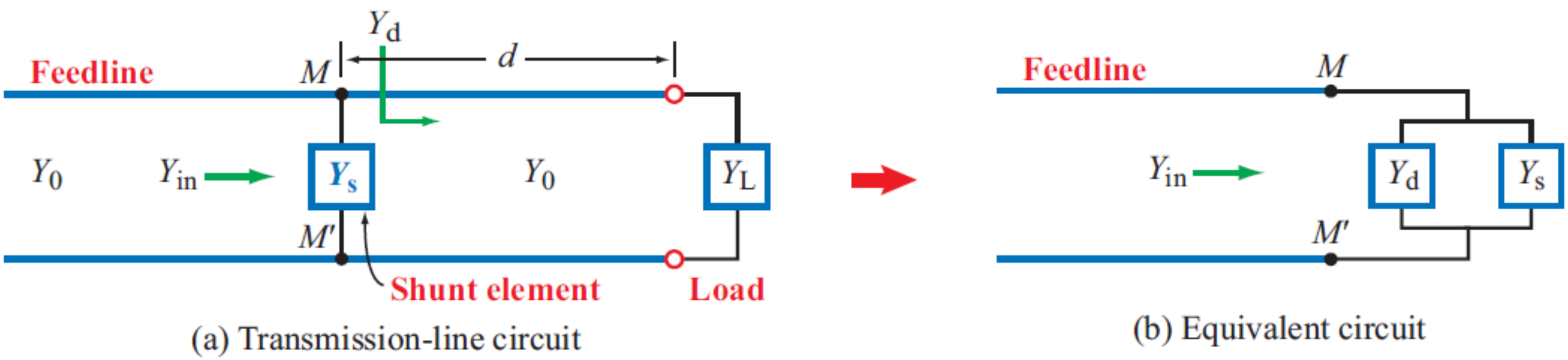


Figure 2-34: Inserting a reactive element with admittance Y_s at MM' modifies Y_d to Y_{in}.

$$Y_{in} = Y_d + Y_s$$

$$Y_{in} = (G_d + jB_d) + jB_s = G_d + j(B_d + B_s).$$

$$y_{in} = g_d + j(b_d + b_s). \qquad (2.140)$$

To achieve a matched condition at MM', it is necessary that $y_{in} = 1 + j0$, which translates into two specific conditions, namely

$$g_d = 1 \quad \text{(real-part condition)}, \qquad (2.141a)$$

$$b_s = -b_d \quad \text{(imaginary-part condition)}. \qquad (2.141b)$$

Example 2-13: Lumped Element

A load impedance $Z_L = 25 - j50\ \Omega$ is connected to a 50-Ω transmission line. Insert a shunt element to eliminate reflections towards the sending end of the line. Specify the insert location d (in wavelengths), the type of element, and its value, given that $f = 100$ MHz.

$$z_L = \frac{Z_L}{Z_0} = \frac{25 - j50}{50} = 0.5 - j1$$

$$y_L = 0.4 + j0.8$$

Solution for Point C (Fig. 2-36): At C,

$$y_d = 1 + j1.58,$$

which is located at 0.178λ on the WTG scale. The distance between points B and C is

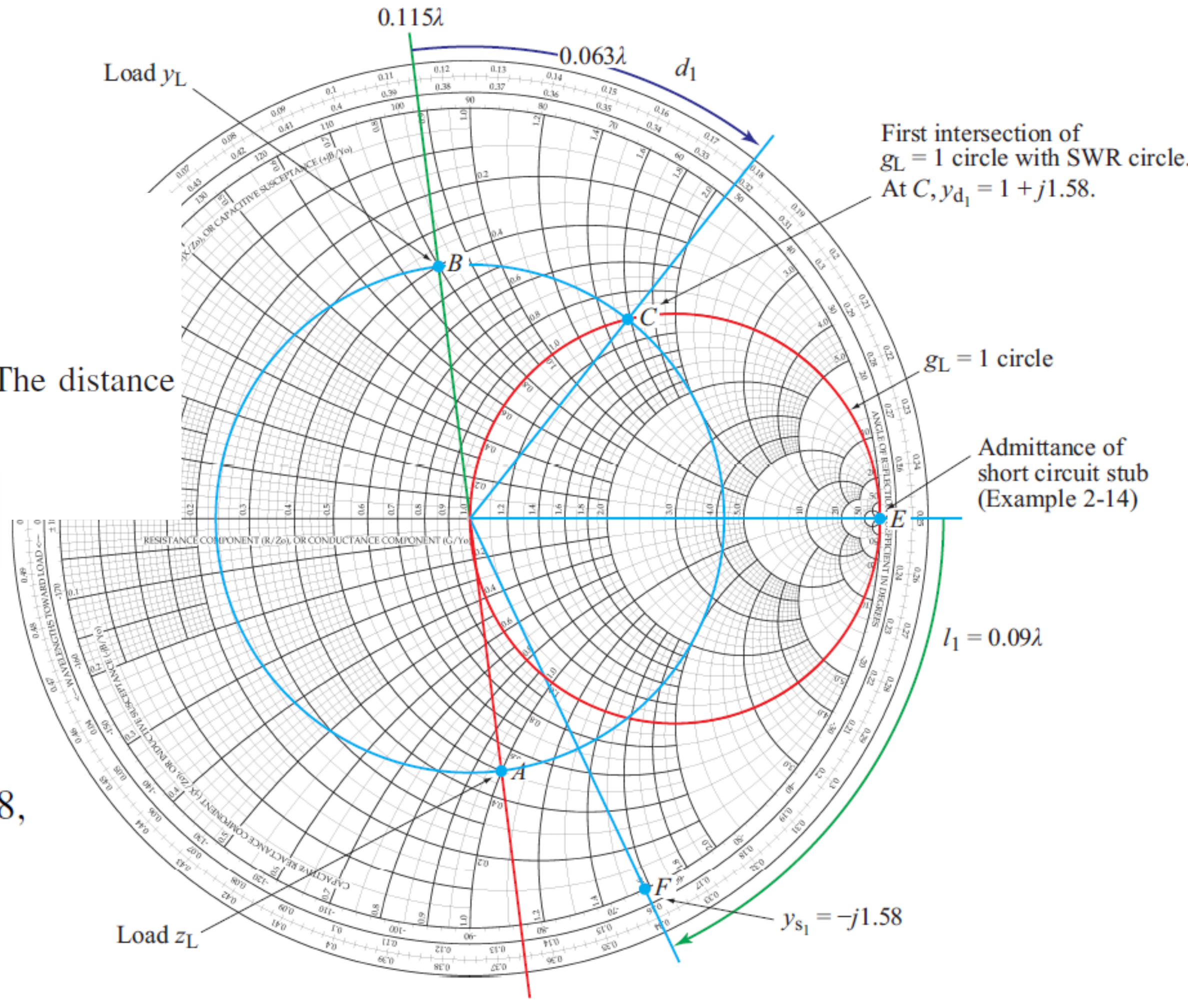
$$d_1 = (0.178 - 0.115)\lambda = 0.063\lambda.$$

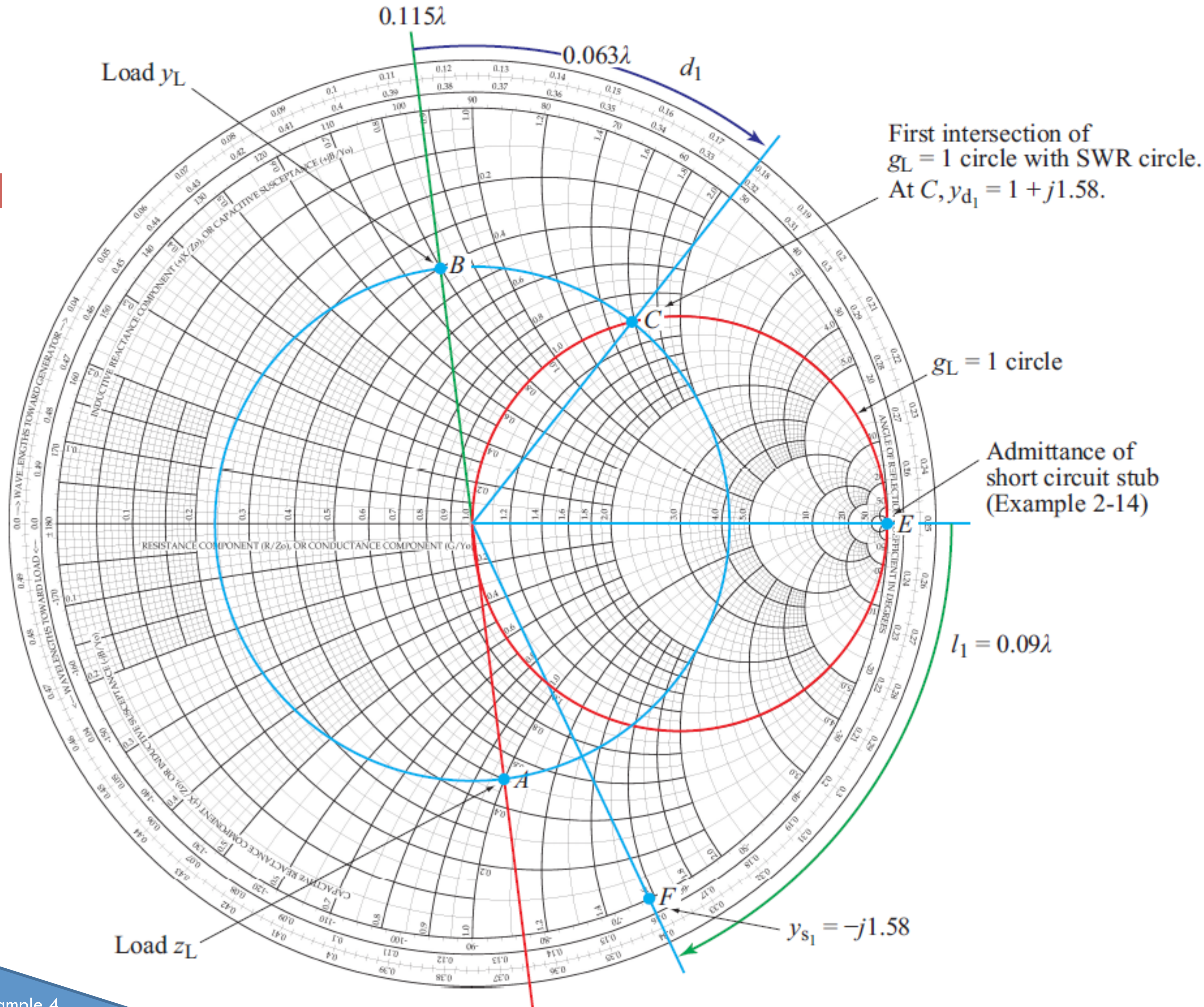
we need $y_{in} = 1 + j0$. Thus,

$$1 + j0 = y_s + 1 + j1.58,$$

or

$$y_s = -j1.58.$$





0.115λ
0.063λ
d1
Load yL
First intersection of
gL = 1 circle with SWR circle.
At C, yd1 = 1 + j1.58.
gL = 1 circle
Admittance of
short circuit stub
(Example 2-14)
l1 = 0.09λ
ys1 = −j1.58
Load zL
B
C
E
A
F

Example 4

Example 2-13: Lumped Element Cont.

A load impedance $Z_L = 25 - j50\ \Omega$ is connected to a 50-Ω transmission line. Insert a shunt element to eliminate reflections towards the sending end of the line. Specify the insert location d (in wavelengths), the type of element, and its value, given that $f = 100$ MHz.

$$z_L = \frac{Z_L}{Z_0} = \frac{25 - j50}{50} = 0.5 - j1$$

$$y_L = 0.4 + j0.8$$

Solution for Point C (Fig. 2-36): At C,

$$y_d = 1 + j1.58,$$

which is located at 0.178λ on the WTG scale. The distance between points B and C is

$$d_1 = (0.178 - 0.115)\lambda = 0.063\lambda.$$

we need $y_{in} = 1 + j0$. Thus,

$$1 + j0 = y_s + 1 + j1.58,$$

or

$$y_s = -j1.58.$$

The corresponding impedance of the lumped element is

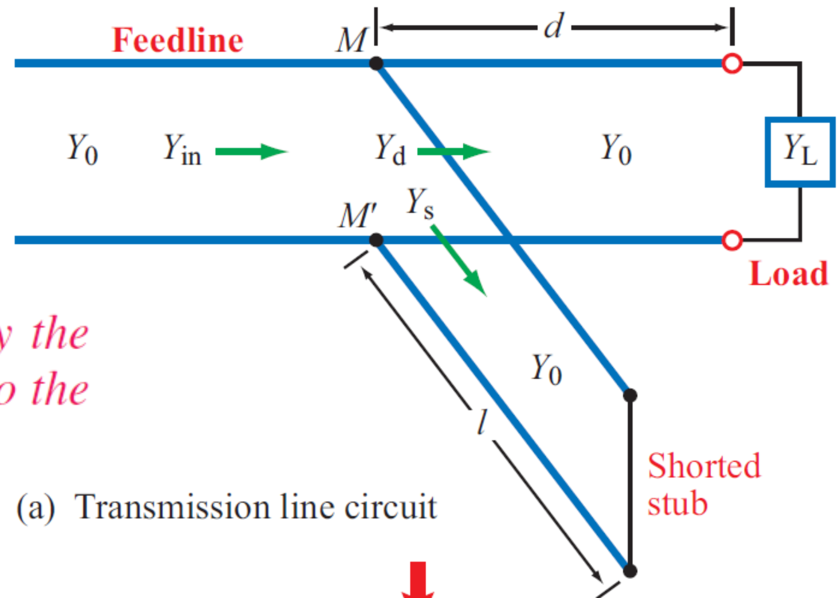
$$Z_{s_1} = \frac{1}{Y_{s_1}} = \frac{1}{y_{s_1} Y_0} = \frac{Z_0}{jb_{s_1}} = \frac{Z_0}{-j1.58} = \frac{jZ_0}{1.58} = j31.62\ \Omega.$$

Since the value of Z_{s_1} is positive, the element to be inserted should be an inductor and its value should be

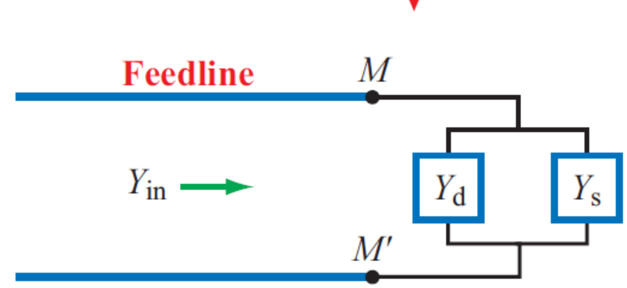
$$L = \frac{31.62}{\omega} = \frac{31.62}{2\pi \times 10^8} = 50\text{ nH}.$$

Single-Stub Matching

The required two degrees of freedom are provided by the length l of the stub and the distance d from the load to the stub position.



(a) Transmission line circuit



(b) Equivalent circuit

Example 2-14: Single-Stub Matching

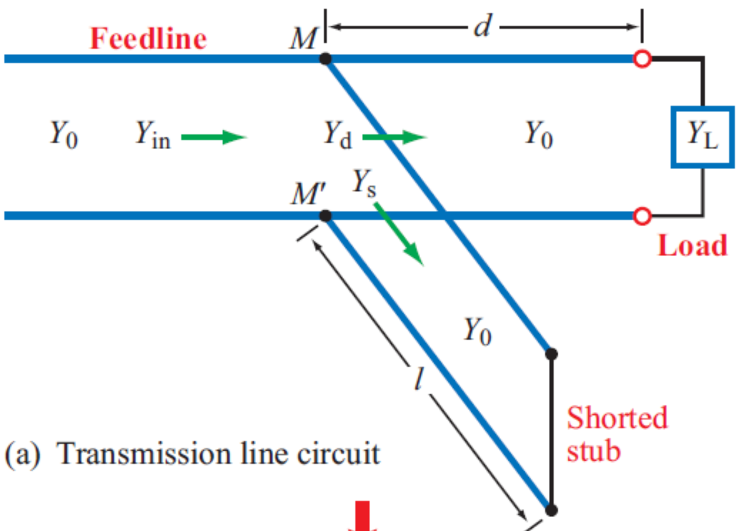
Repeat Example 2-13, but use a shorted stub (instead of a lumped element) to match the load impedance $Z_L = (25 - j50)\ \Omega$ to the 50-Ω transmission line.

Solution: In Example 2-13, we demonstrated that the load can be matched to the line via either of two solutions:

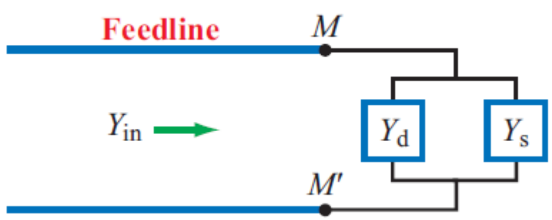
(1) $d_1 = 0.063\lambda$, and $y_{s_1} = jb_{s_1} = -j1.58$,

(2) $d_2 = 0.207\lambda$, and $y_{s_2} = jb_{s_2} = j1.58$.

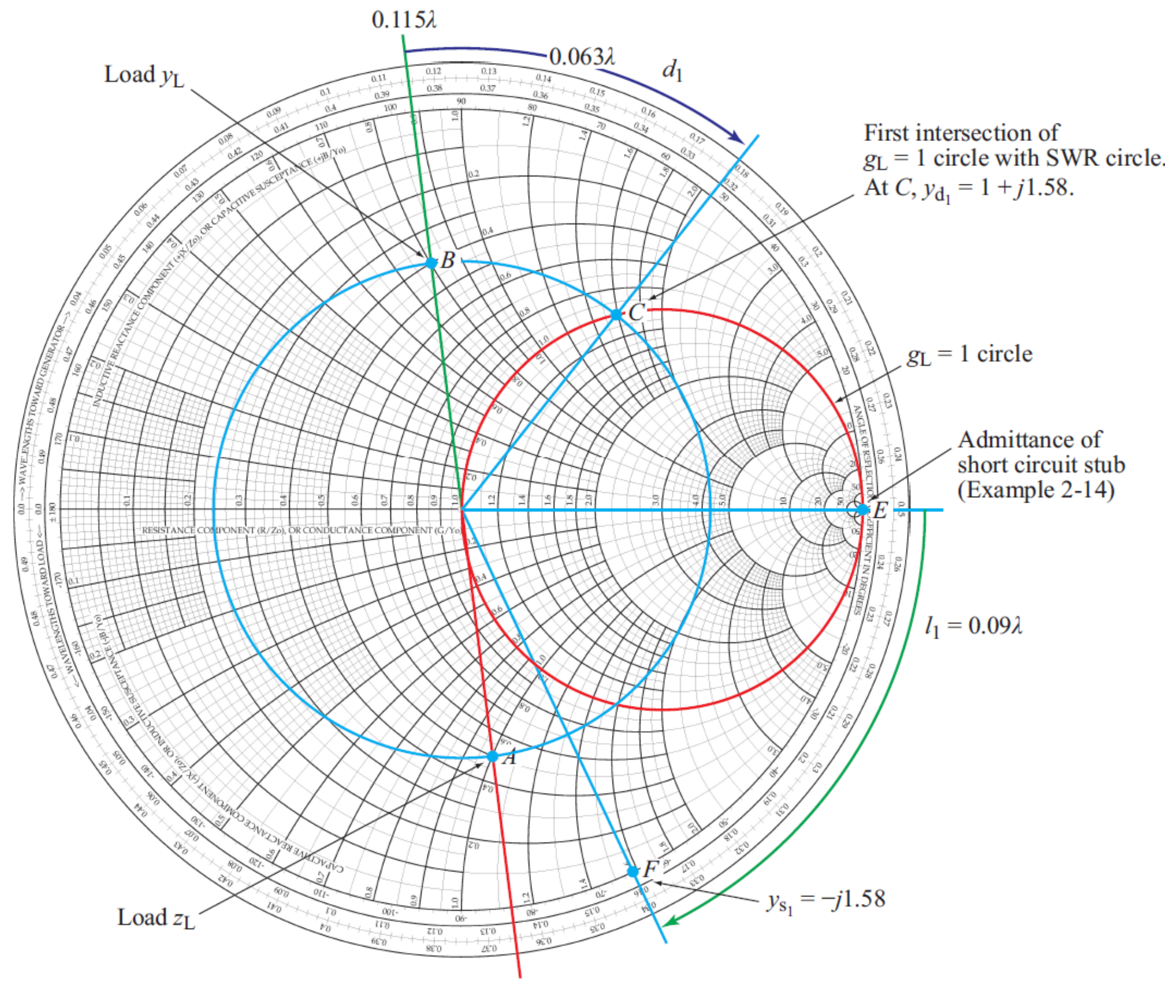
$$l_1 = (0.34 - 0.25)\lambda = 0.09\lambda$$



(a) Transmission line circuit

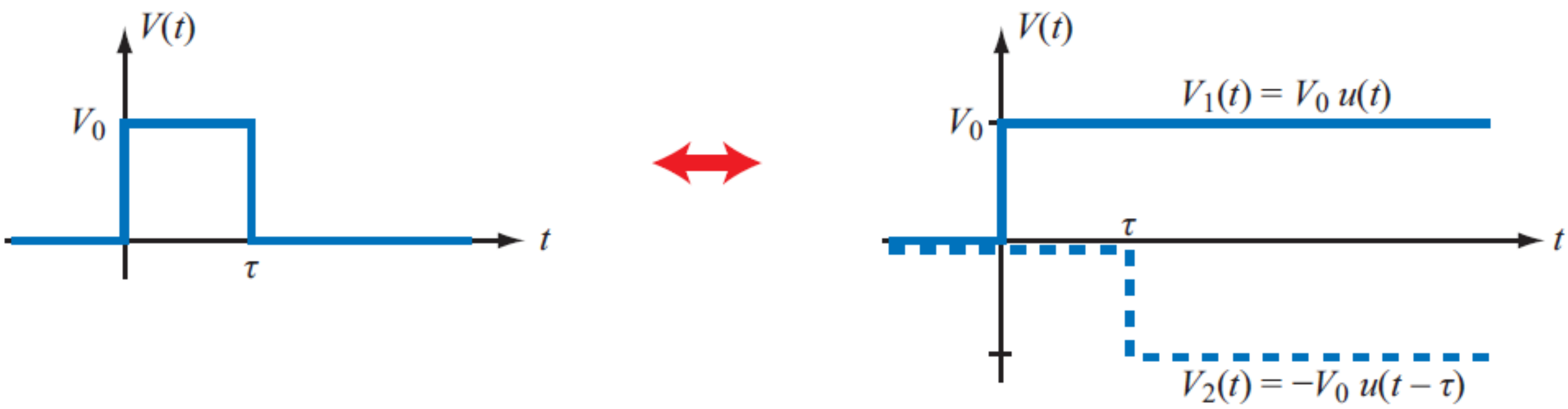


(b) Equivalent circuit



Transients

The ***transient response*** *of a voltage pulse on a transmission line is a time record of its back and forth travel between the sending and receiving ends of the line, taking into account all the multiple reflections (echoes) at both ends.*

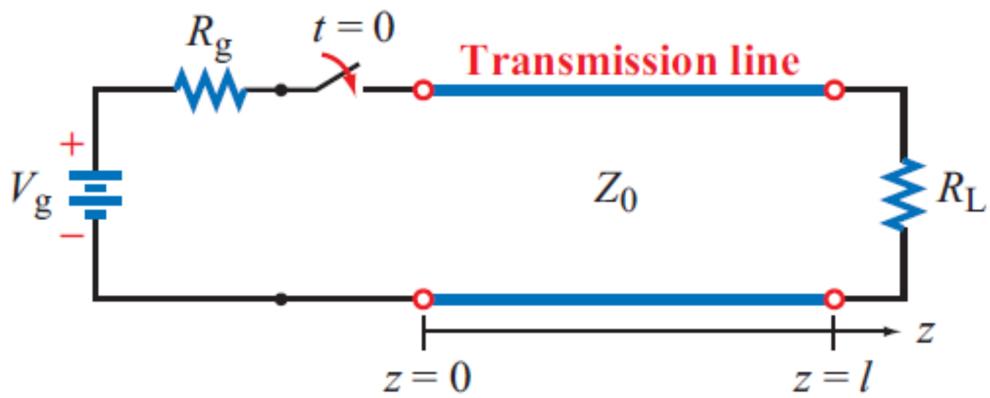


(a) Pulse of duration τ

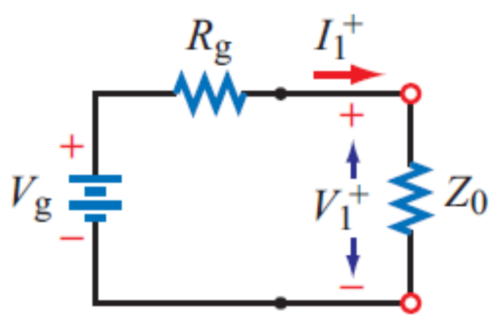
(b) $V(t) = V_1(t) + V_2(t)$

Rectangular pulse is equivalent to the sum of two step functions

Transient Response



(a) Transmission-line circuit



(b) Equivalent circuit at $t = 0^+$

Initial current and voltage

$$I_1^+ = \frac{V_g}{R_g + Z_0},$$

$$V_1^+ = I_1^+ Z_0 = \frac{V_g Z_0}{R_g + Z_0}$$

Reflection at the load

$$V_1^- = \Gamma_L V_1^+,$$

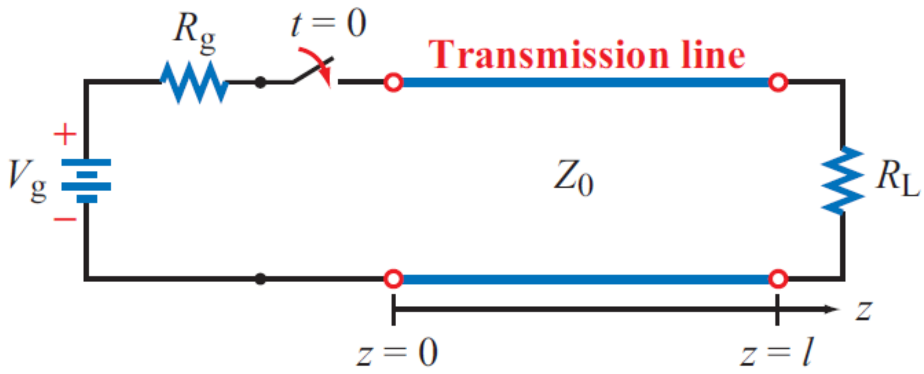
Load reflection coefficient $$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

Second transient

$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+$$

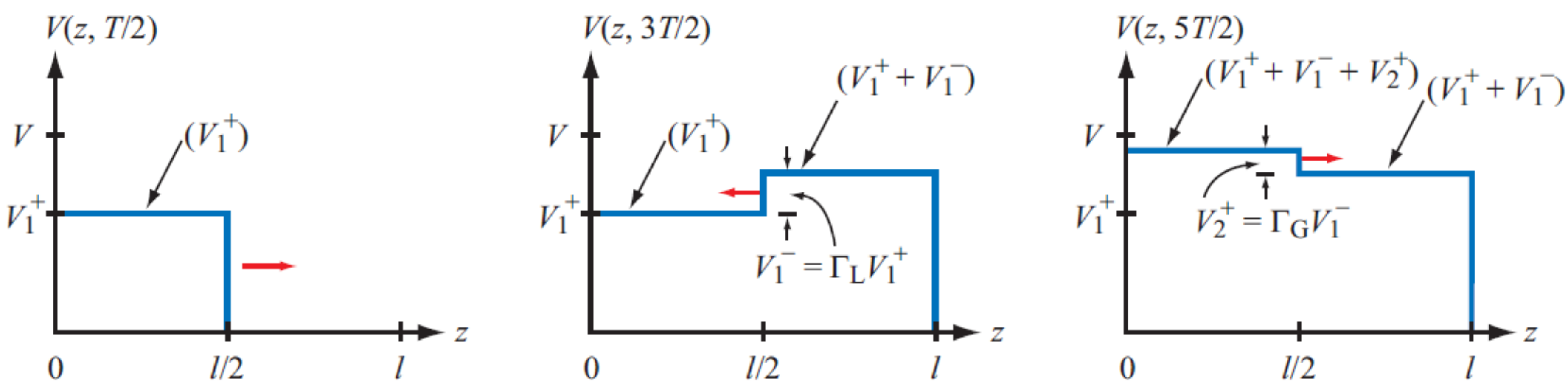
Generator reflection coefficient $$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

Voltage Wave



(a) Transmission-line circuit

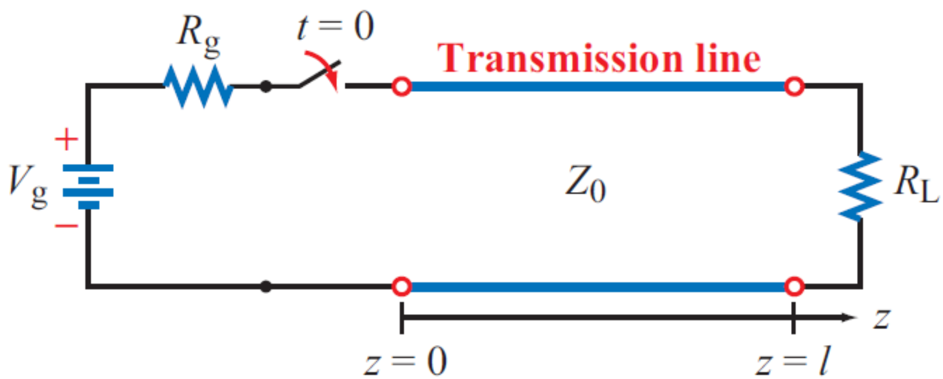
$T = l/u_p$ is the time it takes the wave to travel the full length of the line



(a) $V(z)$ at $t = T/2$ (b) $V(z)$ at $t = 3T/2$ (c) $V(z)$ at $t = 5T/2$

$R_g = 4Z_0$ and $R_L = 2Z_0$. The corresponding reflection coefficients are $\Gamma_L = 1/3$ and $\Gamma_g = 3/5$.

Steady State Response



(a) Transmission-line circuit

$$V_\infty = \frac{V_g R_L}{R_g + R_L}$$

The multiple-reflection process continues indefinitely, and the ultimate value that $V(z, t)$ reaches as t approaches $+\infty$ is the same at all locations on the transmission line.

$$I_\infty = \frac{V_\infty}{R_L} = \frac{V_g}{R_g + R_L}$$

Bounce Diagrams