Example A

Find the Fourier Series Coef., C_k :

$$\underline{x(t) = \cos(2t + \pi/4)}$$

Fundamental frequency:
$$w_o = 2 = \frac{2\pi}{T_o} \Rightarrow T_o = \pi$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jw_o t k}$$

$$= \cos(2t + \pi/4) \quad \text{using Euler's}$$

$$= \tfrac{1}{2} e^{j(2t+\pi/4)} + \tfrac{1}{2} e^{-j(2t+\pi/4)}$$

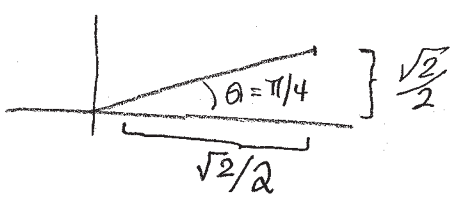
$$= \tfrac{1}{2} e^{j\pi/4} \underbrace{\left(e^{j2t}\right)}_{k=1} + \tfrac{1}{2} e^{-j\pi/4} \underbrace{\left(e^{-j2t}\right)}_{k=-1}$$

$\Rightarrow$ $C_k \neq 0$ for $k=1$ & $k=-1$ or $|k| = 1$

$$\Rightarrow \quad k = 1 \qquad C_k = C_1 = \tfrac{1}{2} e^{j\pi/4} \Rightarrow \boxed{|C_1| = \tfrac{1}{2} \quad \theta_{C_1} = \pi/4} \text{ Polar}$$

$$\text{or} \quad \tfrac{1}{2}\left(\tfrac{\sqrt{2}}{2}\right)[1 + j] = \boxed{\tfrac{\sqrt{2}}{4}(1 + j)} \text{ Rectangular}$$

remember:



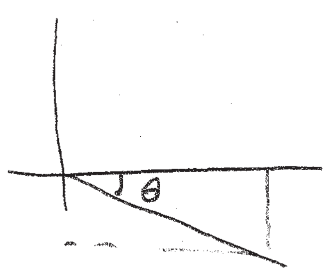
$$k = -1 \qquad C_k = C_{-1} = \tfrac{1}{2} e^{-j\pi/4}$$

same as above but negative angle $\Rightarrow$ Negative Im. part

$$\boxed{|C_{-1}| = \tfrac{1}{2} \quad \theta_{C_{-1}} = -\pi/4}$$

or

$$\boxed{\tfrac{\sqrt{2}}{4}(1 - j)}$$



Example B Find the Fourier Series Coef. of

$$x(t) = \sin^2 t$$

Note that w0 is 2 since we have sin^2t

Note $\sin t = \frac{1}{2j} e^{jt} - \frac{1}{2j} e^{-jt}$

Remember:

$$\frac{1}{2j} = -j/2$$

$$(j)^2 = -1 \;,\; (j)^3 = (-j)$$

$$(-j)^2 = (-1)^2 j^2 = -1$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j w_0 t k}$$

$$= \sin^2 t = \left[\frac{1}{2j} e^{jt} - \frac{1}{2j} e^{-jt}\right]^2$$

$$= \left(\frac{1}{2j}\right)^2 \left(e^{jt}\right)^2 + \left(\frac{-1}{2j}\right)^2 \left(e^{-jt}\right)^2 - 2\left(\frac{1}{2j}\right) e^{jt} \cdot \left(\frac{+1}{2j}\right) e^{-jt}$$

$$= \frac{(-j)^2}{4} e^{j2t} + \frac{j^2}{4} e^{-j2t} - \frac{2}{4(j \cdot j)} \cancel{e^{jt} \cdot e^{-jt}}_{1}$$

$$= \underbrace{\frac{-1}{4} e^{j2t}}_{k=1} + \underbrace{\frac{-1}{4} e^{-j2t}}_{k=-1} + \underbrace{\frac{+1}{2}}_{k=0}$$

$$\Rightarrow \underbrace{C_1 = -1/4}_{\text{Real}} \qquad \underbrace{C_{-1} = -1/4}_{\text{Real}} \qquad \underbrace{C_0 = +1/2}_{\text{DC value}}$$

$\theta_{C_1} = 0$ $\qquad$ $\theta_{C_{-1}} = 0$ $\qquad$ Never has an phase angle

$$\Rightarrow C_k = 0 \quad |k| \neq 1, 0$$

Example 6

Find C_k ; what is ω_0 ?

$$x(t) = \cos 4t + \sin 6t$$

Using Euler's formula:

$$x(t) = \tfrac{1}{2} e^{j4t} + \tfrac{1}{2} e^{-j4t} + \frac{1}{2j} e^{j6t} - \frac{1}{2j} e^{-j6t}$$

$$= \sum_{k=-\infty}^{\infty} C_k e^{-j\omega_0 t k}$$

Note: We have to find The fundamenta frequency (ω_0)

Consider $e^{j\omega_0 t k} \equiv e^{j4t} \;\&\; e^{j6t} \rightarrow e^{j2t}$ $\begin{cases} k=2 & e^{j4t} \\ k=3 & e^{j6t} \end{cases}$

$$\Rightarrow \omega_0 = 2 = \frac{2\pi}{T_0} \Rightarrow T_0 = \pi$$

$$\Rightarrow x(t) = \sum_{k=-\infty}^{\infty} C_k e^{-j2t(k)} = \tfrac{1}{2} \underset{k=2}{e^{j2t(2)}} + \tfrac{1}{2} \underset{k=-2}{e^{-j2t(2)}} + \frac{1}{2j} \underset{k=3}{e^{j2t(3)}} - \frac{1}{2j} \underset{k=-3}{e^{-j2t(3)}}$$

$C_2 = +\tfrac{1}{2} = \tfrac{1}{2}$

$C_{-2} = \tfrac{1}{2} = \tfrac{1}{2}$

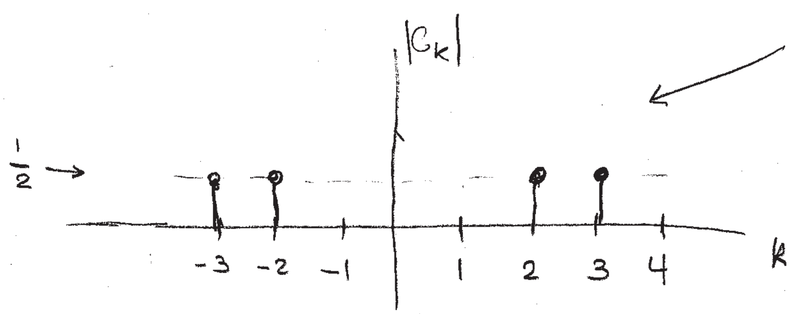
$C_{+3} = \frac{1}{2j} = \tfrac{1}{2} e^{-j\pi/2}$

$C_{-3} = \frac{-1}{2j} = \tfrac{1}{2} e^{+j\pi/2}$

$C_k = 0 \quad |k| \neq 2 \;\&\; 3$

Fourier Coefficients of $x(t)$

Has no DC value

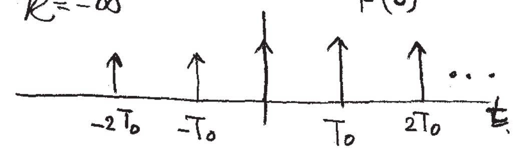


we plot the magnitude of $X(t)$'s freq. spectrum

Example D

Assume $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$

This is an impulse train



Find Fourier series Coefficients of $x(t)$.

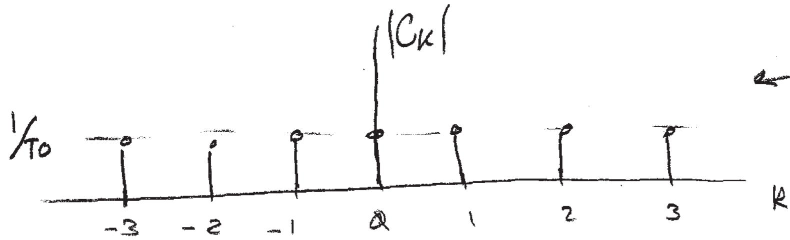
$$P(t) = \sum_{k=-\infty}^{\infty} C_k e^{jw_0 t k}$$ but what is $C_k = ?$

$$C_k = \frac{1}{T_0}\int_{T_0} P(t) e^{-jw_0 t}\, dt$$ ∍ note $P(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$ has value $\equiv \delta(t)$ at every k

$$= \frac{1}{T_0}\int_{T_0} \delta(t) e^{-jw_0 t}\, dt$$ using $\int f(t)\delta(t - t_0)\, dt$ to

$$\Rightarrow \frac{1}{T_0} e^{-jw_0(0)} = \frac{1}{T_0}$$

$$\Rightarrow C_k = \frac{1}{T_0} \quad \forall k$$ (For all ks)



mag. of the freq. spectrum

→ what happens as T_0 changes?
→ what is the DC value?

Look at Table 4.3 (P. 170 3rd Ed)
Case 7. In this case $X_0 = 1$