Examples of Laplace Transform - Answers are provided at the end.

1. Find the integrals $\int_0^\infty 1 \times e^{-t} dt$, $\int_0^\infty 1 \times e^{-2t} dt$ and $\int_0^\infty 1 \times e^{-5t} dt$. Show that they all satisfy $\int_0^\infty 1 \times e^{-st} dt = \dfrac{1}{s}$,
2. By direct integration, find the Laplace Transforms of
   (a) 1
   (b) $5t$
   (c) e^{-6t}
   (d) $\sin t$
3. Using tables, find the Laplace Transforms of
   (a) $t^2 + 1$
   (b) $\cos 4t$
   (c) $\sinh 3t$
   (d) $7e^t - 5t^2$
   (e) $9e^{5t} + 5 \sin 3t$
   (f) $5e^{5t} \cos 6t$
   (g) $7 \sinh t - 3 \cosh t$
   (h) te^t
4. Using tables, find inverse Laplace Transforms of
   (a) $\dfrac{1}{s^6}$
   (b) $\dfrac{s+1}{s^2+1}$
   (c) $\dfrac{1}{s-3}$
   (d) $\dfrac{4}{s^2} - \dfrac{8}{s^2+4}$
5. By means of Partial Fractions, find the inverse Laplace Transforms of
   (a) $\dfrac{3s-1}{s^2-s}$
   (b) $\dfrac{6s^2+7s+6}{s^2(s+2)}$

(c) $\dfrac{1+s+s^2}{s\left(s^2+1\right)}$

(d) $\dfrac{8s^2+s+18}{(s+1)\left(s^2+4\right)}$

(e) $\dfrac{s^2+5s+5}{s^3+4s^2+5s}$

(f) $\dfrac{7s^2+37s+70}{(s+1)\left(s^2+6s+13\right)}$

6. Find the Inverse Laplace Transform of

$$\overline{f}(s)=\frac{5s^6+45s^5+139s^4+195s^3+282s^2+96s+80}{s^2\left(s+1\right)\left(s^2+1\right)\left(s^2+4s+20\right)}$$

Hint, $\overline{f}(s)$ may be written as

$$\overline{f}(s)=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+1}+\frac{Ds+E}{s^2+1}+\frac{Fs+G}{s^2+4s+20}$$

where $D=-1$, $E=3$, $F=1$ and $G=22$.

Answers

1. 1, $\frac{1}{2}$, $\frac{1}{5}$.
2. $1/s$; $5/s^2$; $1/(s+6)$; $1/(s^2+1)$
3. $2/s^3+1/s$; $s/\left(16+s^2\right)$; $3/\left(s^2-9\right)$; $7/\left(s-1\right)-10/s^3$; $9/(s-5)+15/(s^2+9)$; $(5s-25)/(s^2+10s+61)$, $(7-3s)/(s^2-1)$; $1/(s-1)^2$
4. $t^5/120$; $\sin t+\cos t$; e^{3t} ; $4t-4\sin 2t$
5. $1+2e^{t}$; $2+3t+4e^{-2t}$; $1+\sin t$; $5e^{-t}+3\cos 2t-\sin 2t$; $1+e^{-2t}\sin t$; $5e^{-t}+2e^{-3t}\cos 2t-\frac{1}{2}e^{-3t}\sin 2t$
6. $4t+5e^{-t}+3\sin t-\cos t+e^{-2t}\left(\cos 4t+5\sin 2t\right)$