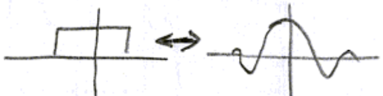
Chapter 12

CAMPAD 22-139 100 sheets

(chapter 4 & 5)
CT signals

CT – Fourier Trans.

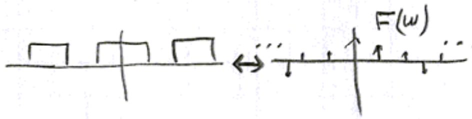
non Periodic
$g(t) \longleftrightarrow G(\omega)$



Contin. Contin.

(P. 236)
Periodic
$f(t) = \sum_{n=-\infty}^{\infty} g(t - nT_0)$

$F(\omega) =$
$\sum \omega_0 \, G(n\omega_0) \, \delta(\omega - n\omega_0)$



Train of pulses

Freq. spectrum with discrete freq. components

(chapter 12)
Discrete signal

DT – Fourier Transform
signal is sampled
$(-\infty, +\infty)$

nonperiodic
(A known set)
$X_0[n] \longleftrightarrow X_0(\Omega)$
DTFT

$X[n] = X[N+n]$
periodic
(N-period)
$X[n] \longleftrightarrow X(\Omega)$
DTFT

Convolution Eq. (12.19)

$\underbrace{X_0[n]}_{\text{value of } X[n] \text{ over one period}} \longrightarrow \underbrace{X_0(\Omega)}_{\text{DTFT of } X_0[n]} \longrightarrow \underbrace{X(\Omega)}_{\text{periodic}(\Omega)} \longleftarrow \underbrace{X[n]}$

$X_0(\Omega)$: Cont. Freq. period $= 2\pi$

$X(\Omega)$: Train of pulses

$X[n]$: periodic sequence using discrete impulses

Note:
both $X(\Omega)$ & $X(\omega)$ & $X_0(\Omega)$ produce continuous functions of freq.

only Finite set of DT samples is known
[*]

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[*]

Note 1: N is not period!

Note 2: we examine the function through a windowing Function

only $n = 0, \dots, N-1$ samples of a continuous signal are known (N-samples)

↓

we can only approximate Fourier Transform (as performed in a digital computer)

↓

Discrete Fourier transform (DFT)

$x[n] \longleftrightarrow X[k]$

N discrete-frequency samples

↓

$k = 0 \cdots N-1$

$$X[k] = DFT\{x[n]\} = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi kn}{N}}$$

$$x[n] = IDFT\{X[k]\} = \frac{1}{N}\sum_{n=0}^{N-1} X[k]\, e^{+j\frac{2\pi kn}{N}}$$

$n = 0, \dots, N-1$

↑

N discrete-time samples

↓

computation methods

(Direct)
N^2 complex multipications
Point-by-point

[Indirect]
Fast Fourier Transform (Collection of eff. alg.) ($N=2^m$)

← complexity of ($N \log N$)

← complexity of ($\frac{N}{2} \log_2 N$)

Decomposition in time / frequency (Sec 12.5)

Read: Fast Fourier Transform