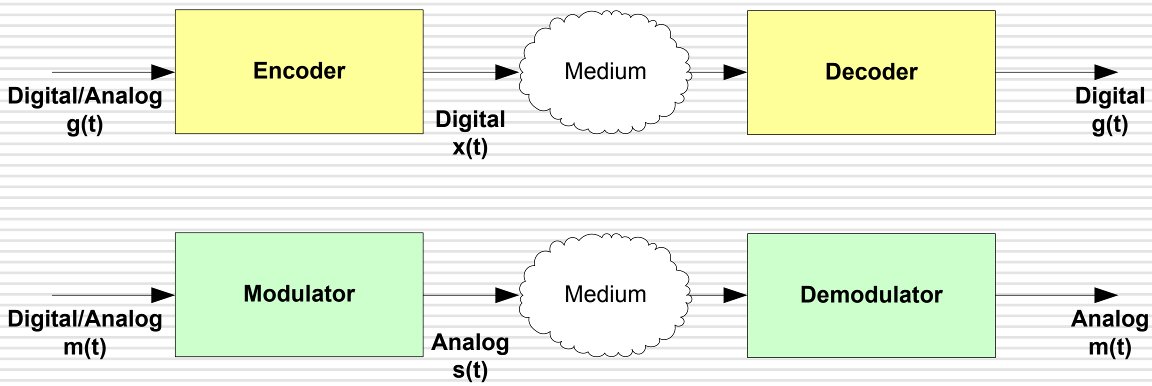
Signal Modulation

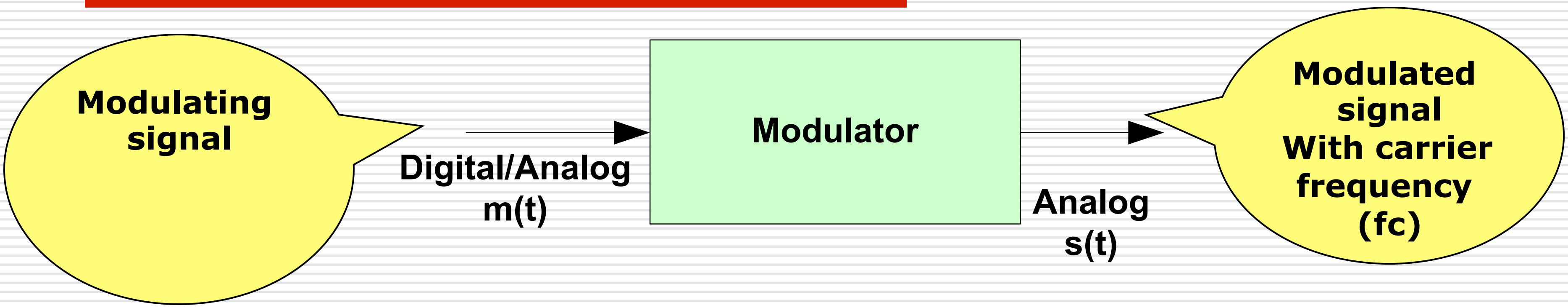
Updated: 10/29/2014

Typical Modulation and Coding Schemes in Wireless



Original Data	Signal Transmitted: Digital	Signal Transmitted: Analog
Digital		ASK FSK/BFSK/MFSK PSK/BPSK/MPSK
Analog	PCM PAM DM	AM PM FM

What is Modulation or Encoding?



Changing signal characteristics including
-Phase
-Amplitude
-Frequency

Depending on the medium, signal range, and data Properties different encoding techniques can be used

Reasons for Choosing Enc Modulation Techniques

Original Data \ Signal Transmitted	Digital	Analog
Digital		ASK FSK/BFSK/MFSK PSK/BPSK/MPSK
Analog	PCM PAM DM	AM PM FM

- Digital data, digital signal
  - Less complex equipments
  - Less expensive than digital-to-analog modulation equipment
- Analog data, digital signal
  - Permits use of modern digital transmission and switching equipment
  - Requires conversion to analog prior to wireless transmission

Original Data \ Signal Transmitted	Digital	Analog
Digital		ASK FSK/BFSK/MFSK PSK/BPSK/MPSK
Analog	PCM PAM DM	AM PM FM

Reasons for Choosing Encoding Techniques

- Digital data, analog signal ← Used in Wireless!
  - Some transmission media will only propagate analog signals
  - E.g., optical fiber and unguided media
- Analog data, analog signal
  - Analog data in electrical form can be transmitted easily and cheaply
  - Done with voice transmission over voice-grade lines

A- Refer to notes!

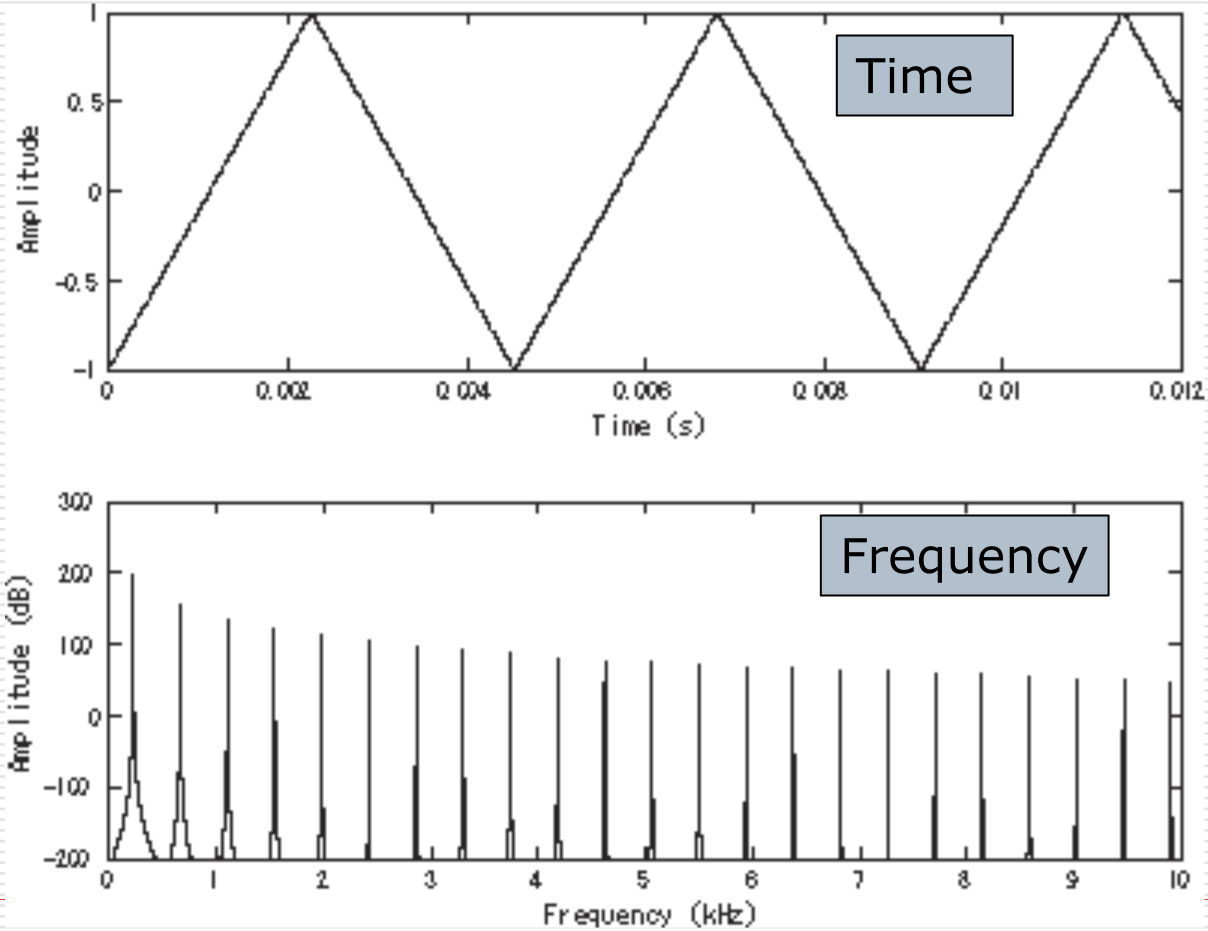
Signal Encoding Criteria

- What determines how successful a receiver will be in interpreting an incoming signal?
  - Signal-to-noise ratio
  - Data rate
  - Bandwidth

1- $R \propto BER$
2- $SNR \propto 1/BER$
3- $BW \propto R$

- An increase in data rate <u>increases</u> bit error rate
- An increase in SNR <u>decreases</u> bit error rate
- An increase in bandwidth allows an <u>increase</u> in data rate

Signal Spectrum

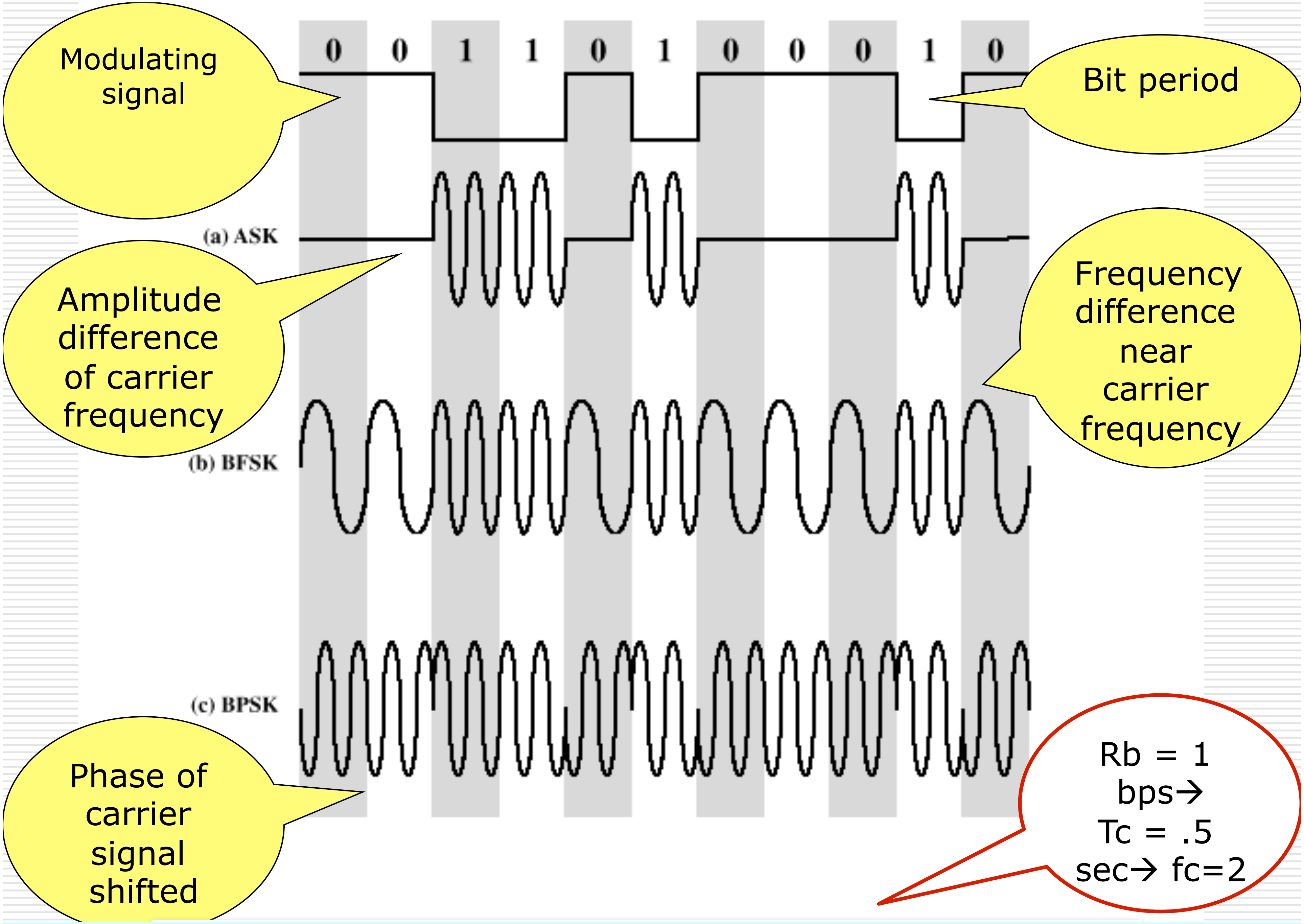


Basic Modulation Techniques
- Digital data to analog signal

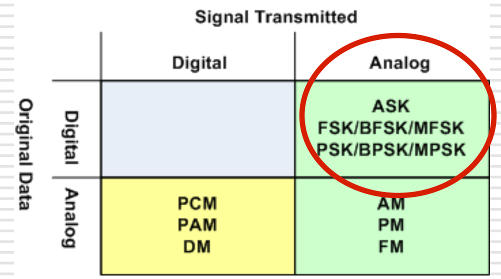
Original Data \ Signal Transmitted	Digital	Analog
Digital		ASK FSK/BFSK/MFSK PSK/BPSK/MPSK
Analog	PCM PAM DM	AM PM FM

- Applications
  - Public telephone (300-3400 Hz)
  - Modems and microwave signals
- Modulation Techniques
  - Amplitude-shift keying (ASK)
    - Amplitude difference of carrier frequency
  - Frequency-shift keying (FSK)
    - Frequency difference near carrier frequency
  - Phase-shift keying (PSK)
    - Phase of carrier signal shifted

Refer to notes!



Question: If data rate is 1 what is the frequency of the carrier in ASK?

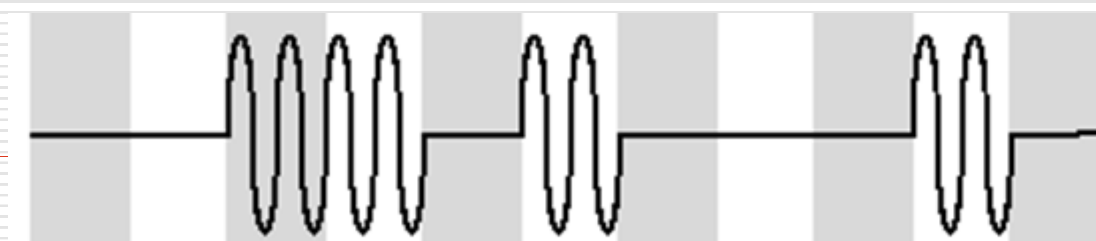


Amplitude-Shift Keying

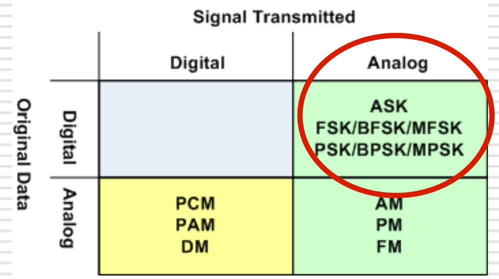
- One binary digit represented by presence of carrier, at constant amplitude
- Other binary digit represented by absence of carrier

$$s(t) = \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ 0 & \text{binary } 0 \end{cases}$$

  - where the carrier signal is $A\cos(2\pi f_c t)$



Amplitude-Shift Keying



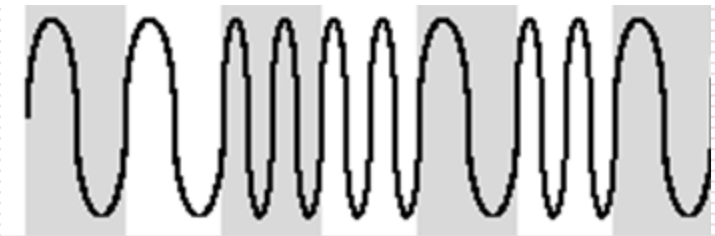
- Susceptible to sudden gain changes
- Inefficient modulation technique
- On voice-grade lines, used up to 1200 bps
- Used to transmit digital data over optical fiber

Binary Frequency-Shift Keying (BFSK)

- Two binary digits represented by two different frequencies near the carrier frequency

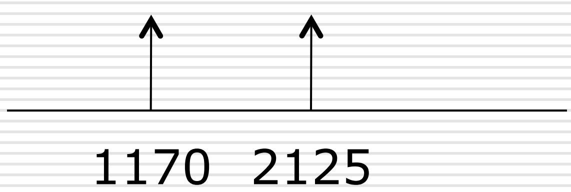
$$s(t) = \begin{cases} A\cos(2\pi f_1 t) & \text{binary } 1 \\ A\cos(2\pi f_2 t) & \text{binary } 0 \end{cases}$$

  - where f_1 and f_2 are offset from carrier frequency f_c by equal but opposite amounts



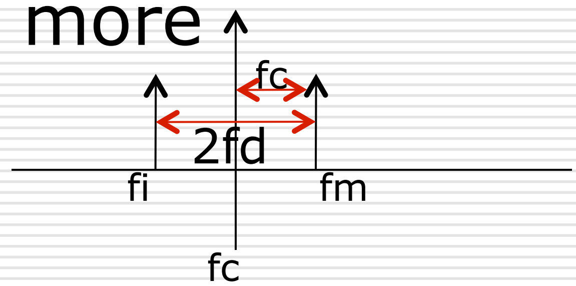
Binary Frequency-Shift Keying (BFSK)

- Less susceptible to error than ASK
- On voice-grade lines, used up to 1200bps
- Used for high-frequency (3 to 30 MHz) radio transmission
- Can be used at higher frequencies on LANs that use coaxial cable
- Different frequencies can be used to support FULL DUPLEX transmission
  - TX: 1070-1270 Hz
  - RX: 2025-2225 Hz (1200 Hz)



Multiple Frequency-Shift Keying (MFSK)

- More than two frequencies are used
- More bandwidth efficient but more susceptible to error



$$s_i(t) = A\cos 2\pi f_i t \qquad 1 \le i \le M$$

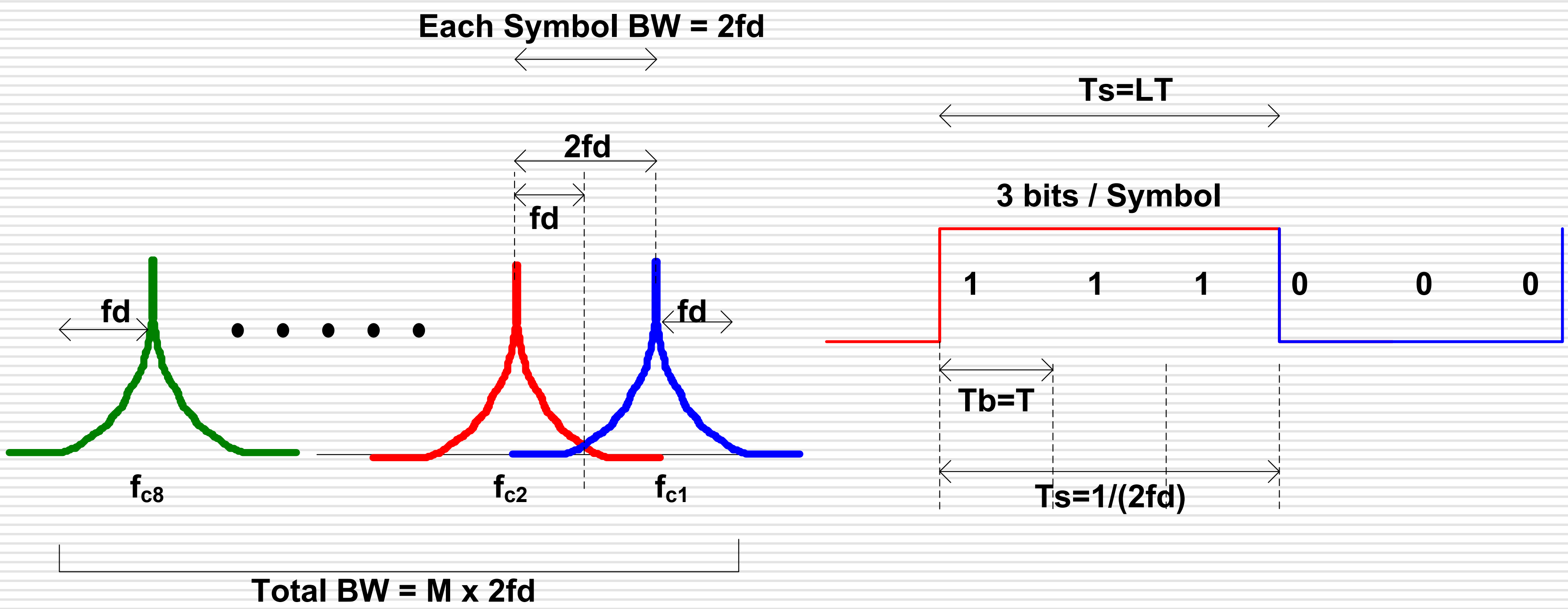
- $f_i = f_c + (2i - 1 - M)f_d$
- f_c = the carrier frequency
- f_d = the difference frequency (freq. separation)
- Symbols/ Levels → M = number of different signal elements = 2^L
- L = number of bits per signal element

Example:

- Assume fc = 250 Khz
- Required frequency separation is 25 KHz (fd=25KHz)
- 8 levels of signals (M=8)
- Use Multi-FSK
- Answer the following questions:
  1. What is the center frequency of the modulated signal?
  2. How many different frequencies do we need for this system?
  3. How many bits do we need to generate the data?
  4. What are M different frequencies?
  5. What is the period of each symbol (How many bits per symbol)?
  6. What is the BW of each symbol?
  7. What is the total BW required?
  8. What is the data rate (total BW/bit)?

Fc=250Khz / 8 diff. freq. / 3 bits/ f1 to f8:75K(000),125(001), 175(101) …/3 bits per symbol / (next slide)

Example:



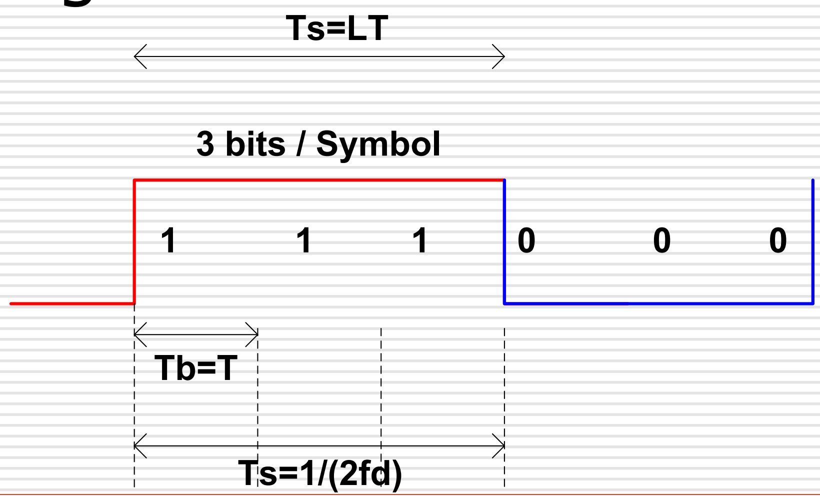
Minimum Frequency Separation:
2fd = 1/Ts

Multiple Frequency-Shift Keying (MFSK)

- To match data rate of input bit stream, each output signal element is held for:

$$T_s = LT \text{ seconds}$$

  - where T is the bit period (data rate = $1/T$)
- So, one signal element encodes L bits



Multiple Frequency-Shift Keying (MFSK)

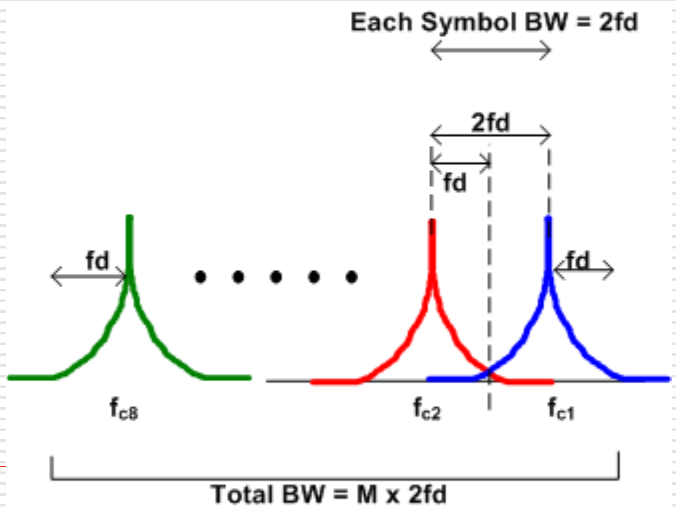
- Total bandwidth required

  (# of symbols x BW/symbol)=$2Mf_d$

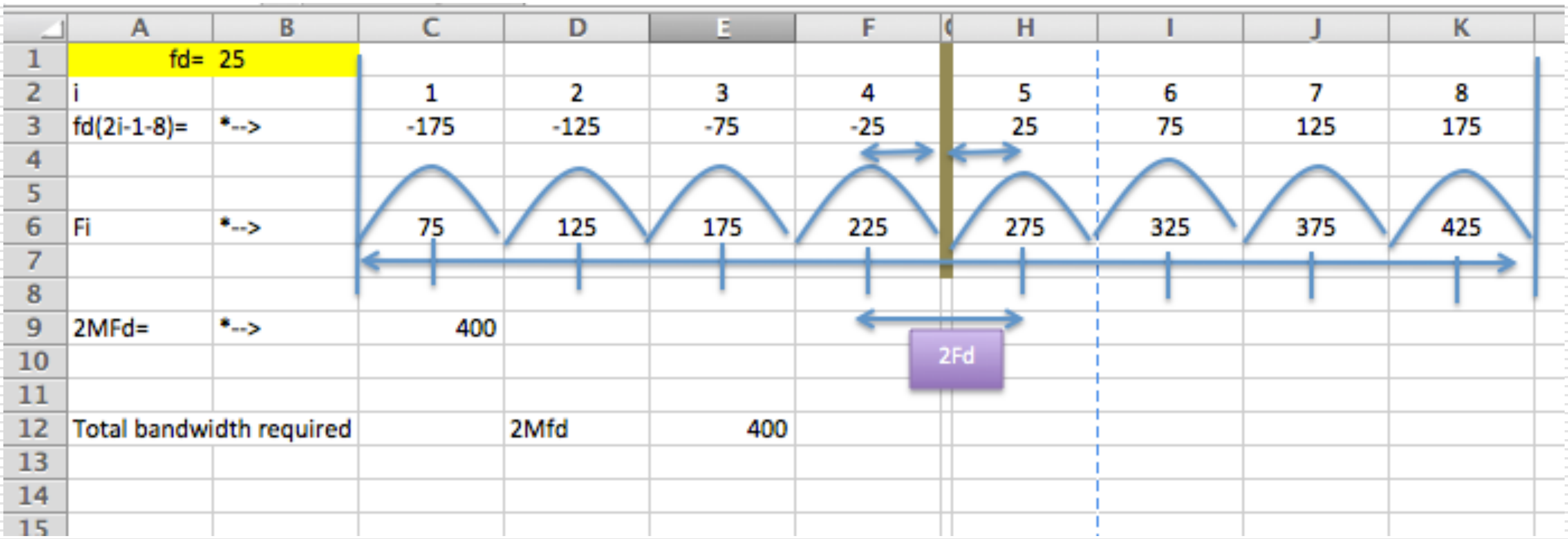
  Minimum frequency separation required BW/symbol $=2f_d=1/T_s$

- Therefore, modulator requires a bandwidth of

$$W_d=2^L/LT=M/T_s$$

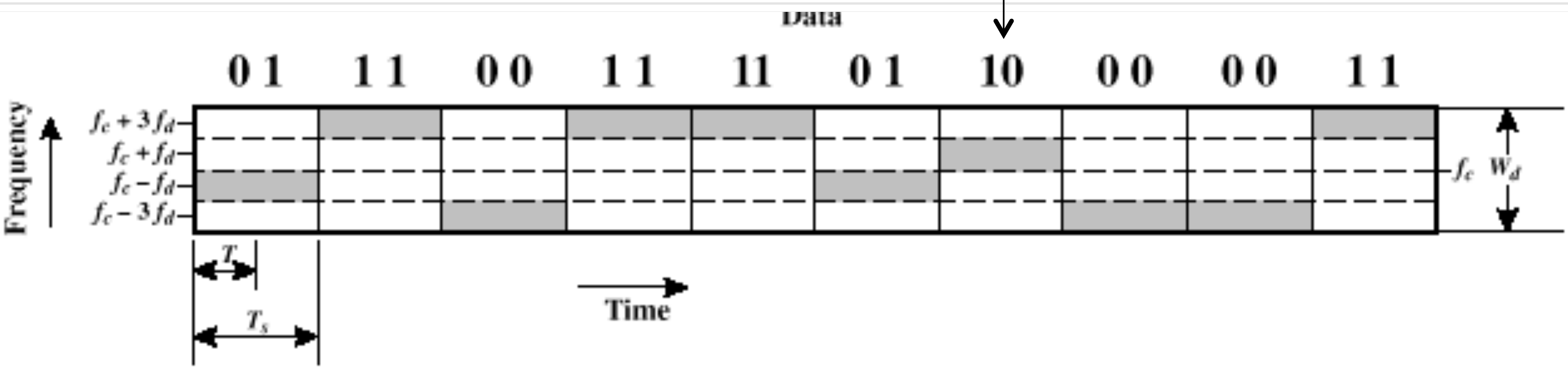


Multiple Frequency-Shift Keying (MFSK)



Multiple Frequency-Shift Keying (MFSK) - Example

Assume M=4 → 4 frequencies
20 bit stream: we send 2 bits per frequency
Note: Ts = 2Tb = Symbol period
Total BW = 2M.fd



Phase-Shift Keying (PSK)

- Two-level PSK (BPSK)
  - Uses two phases to represent binary digits

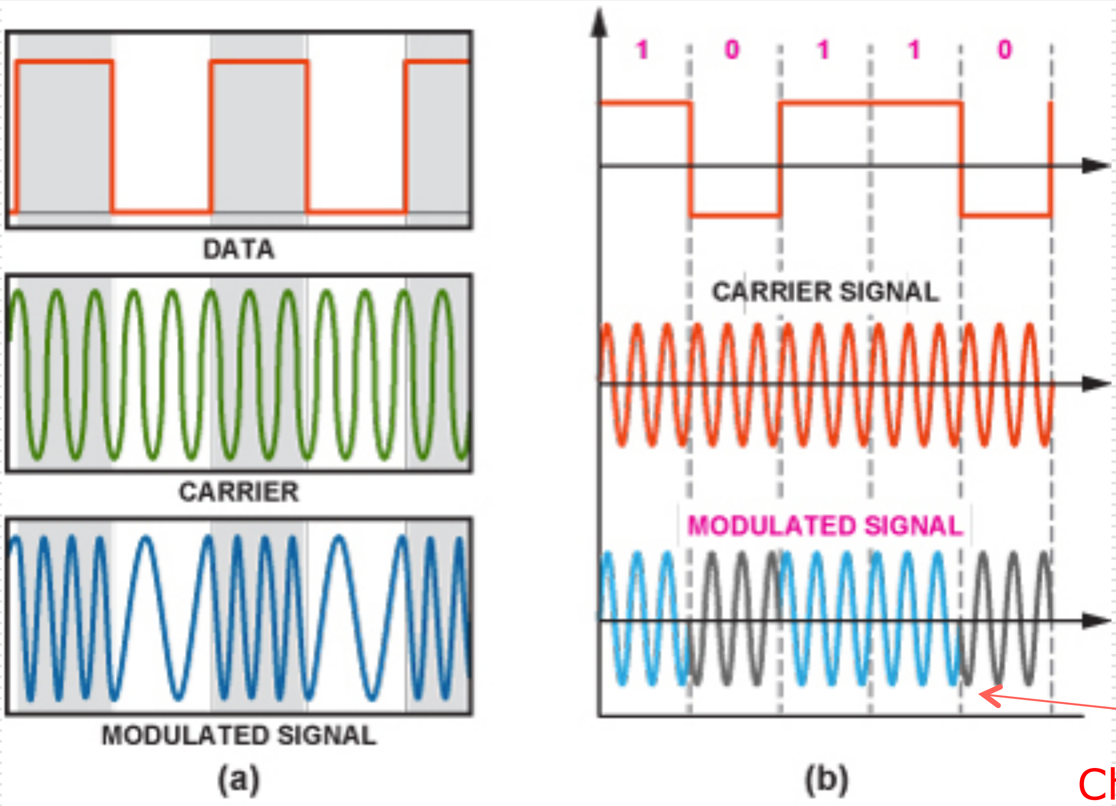
$$s(t) = \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ A\cos(2\pi f_c t + \pi) & \text{binary } 0 \end{cases}$$

$$= \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ -A\cos(2\pi f_c t) & \text{binary } 0 \end{cases}$$

In General:

$$s(t) = Ad(t)\cos(2\pi f_c t)$$

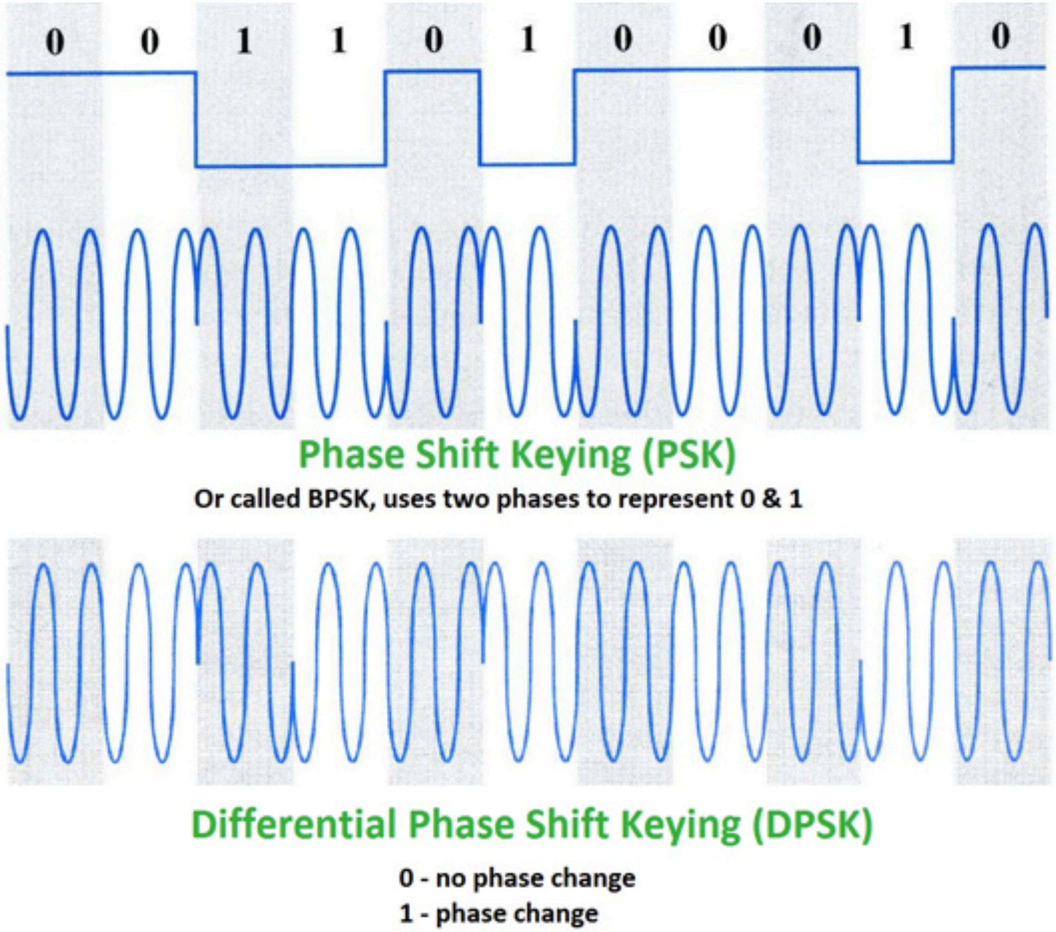
Phase-Shift Keying (PSK)



Change phase 180 deg.
When a ONE is transmitted

Phase-Shift Keying (PSK) - Variations

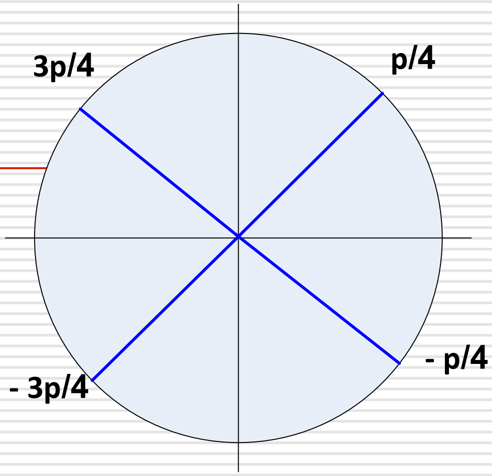
- Differential PSK (DPSK)
  - Phase shift with reference to previous bit
    - Binary 0 – signal burst of same phase as previous signal burst
    - Binary 1 – signal burst of opposite phase to previous signal burst



NRZ-L*: 0→ +V ; 1→ -V

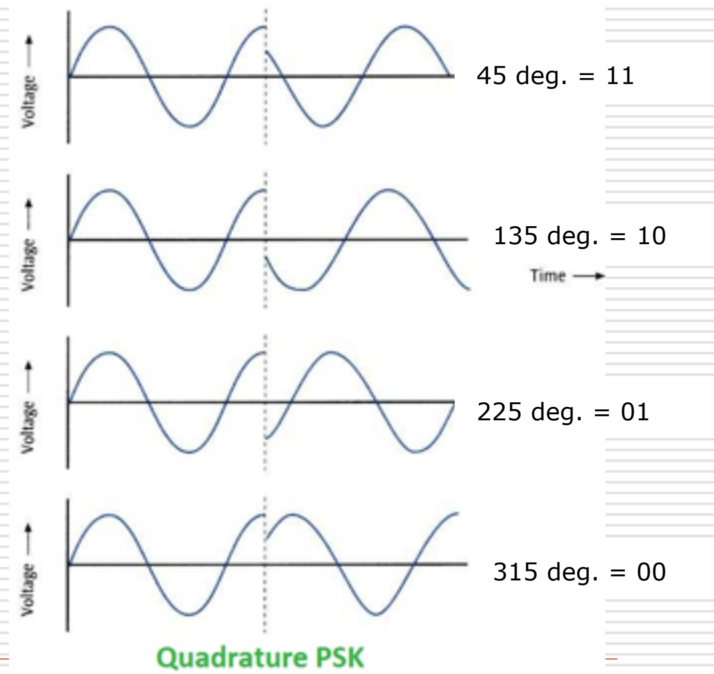
* Nonreturn zero, level

Four-level
Phase-Shift Keying (PSK)



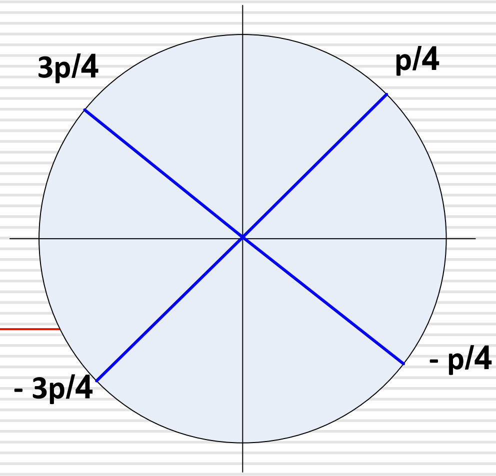
- Four-level PSK (QPSK)
  - Each element represents more than one bit

$$s(t) = \begin{cases} A\cos\left(2\pi f_c t + \frac{\pi}{4}\right) \\ A\cos\left(2\pi f_c t + \frac{3\pi}{4}\right) \\ A\cos\left(2\pi f_c t - \frac{3\pi}{4}\right) \\ A\cos\left(2\pi f_c t - \frac{\pi}{4}\right) \end{cases}$$

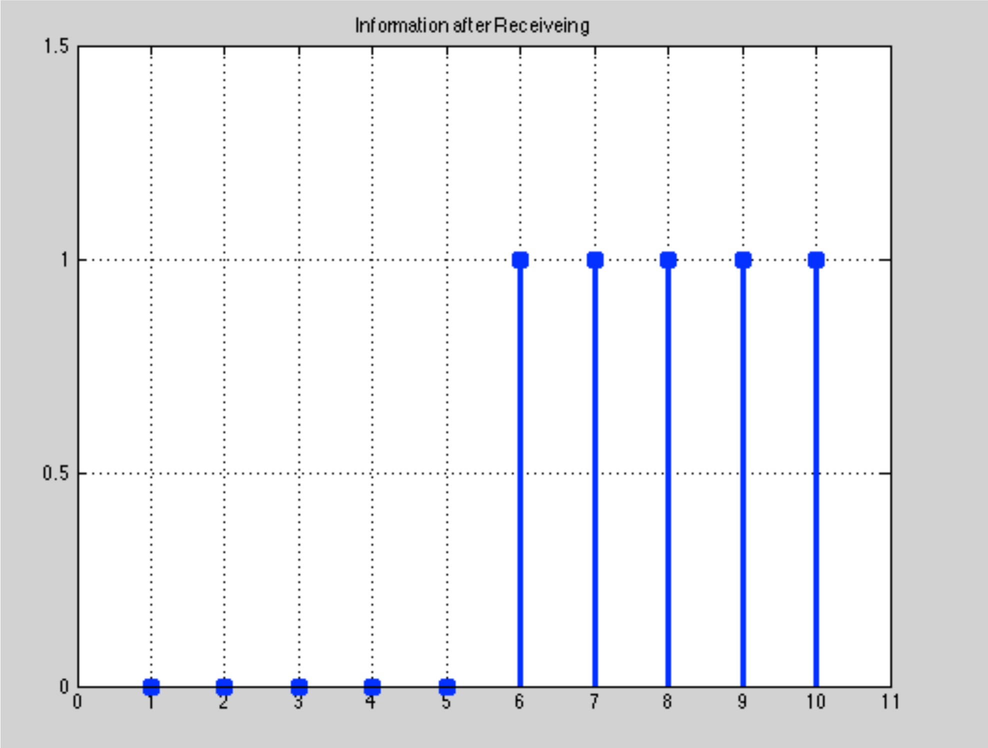


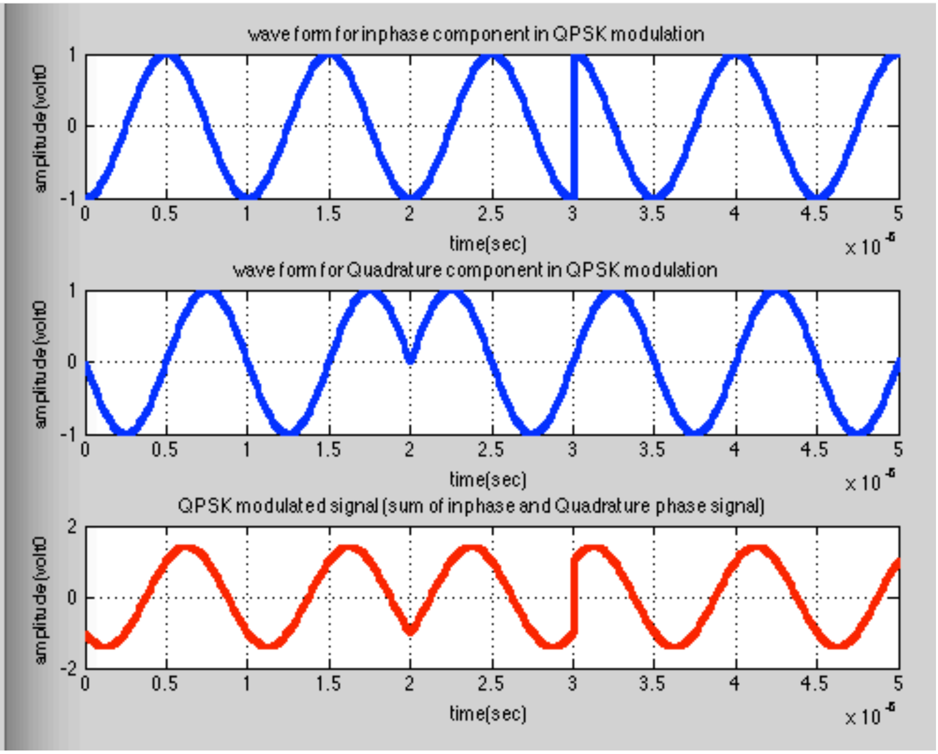
On bit changing at a time

Four-level Phase-Shift Keying (PSK)

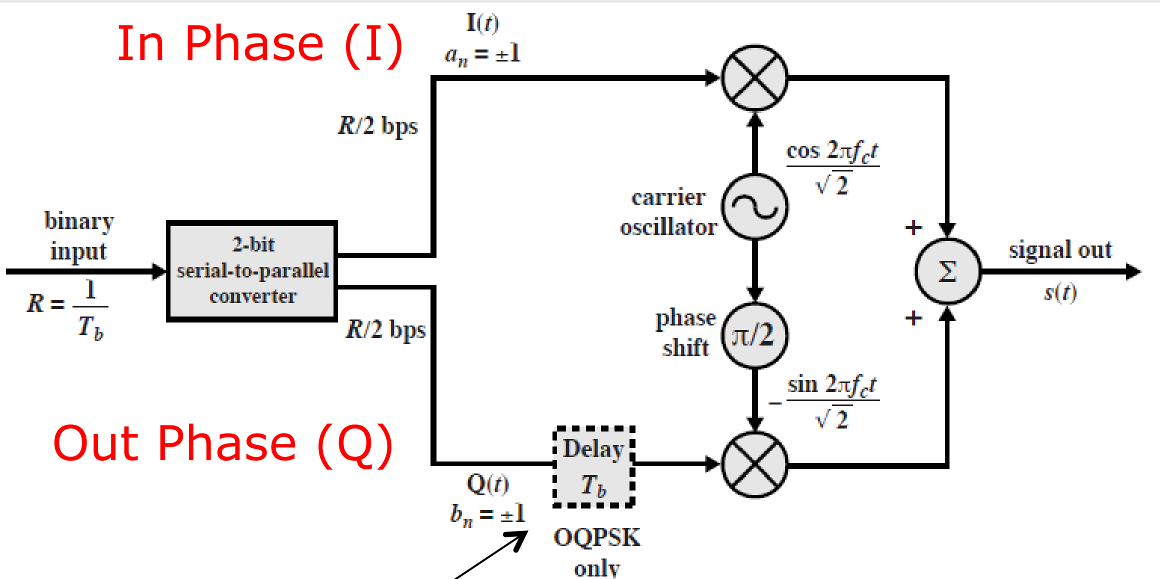


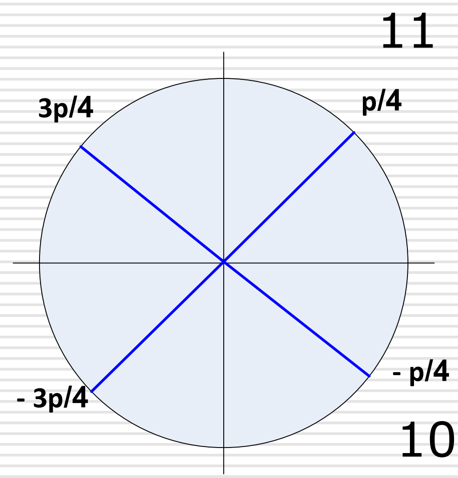
data=[0 0 0 0 0 1 1 1 1 1]; % information





QPSK and OQPSK Modulators





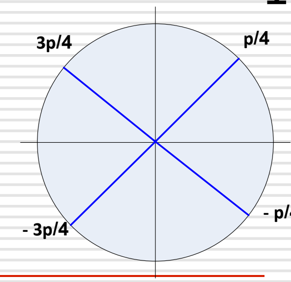
QPSK

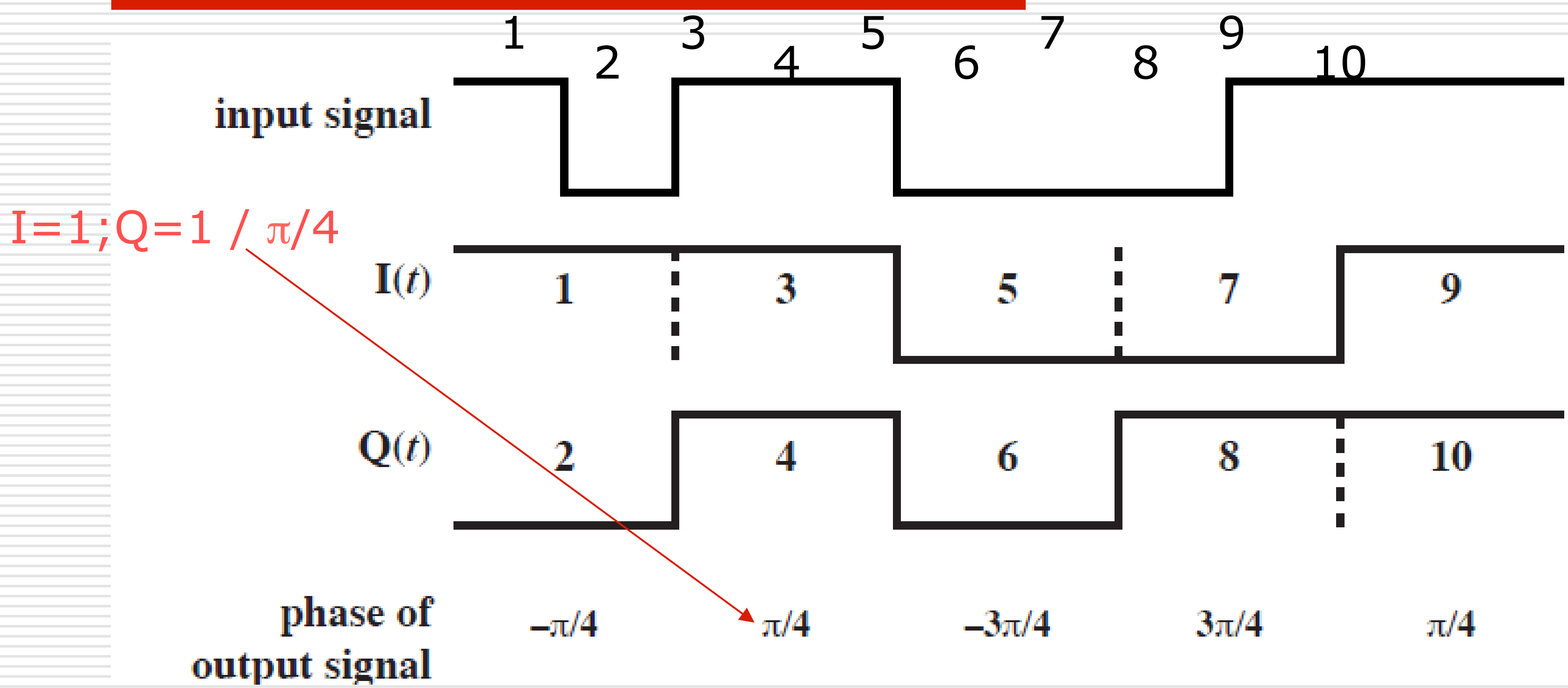
$$s(t) = \frac{1}{\sqrt{2}} I(t)\cos(2\pi f_c t) - \frac{1}{\sqrt{2}} Q(t)\sin(2\pi f_c t)$$

OQPSK

$$s(t) = \frac{1}{\sqrt{2}} I(t)\cos(2\pi f_c t) - \frac{1}{\sqrt{2}} Q(t - Tb)\sin(2\pi f_c t)$$

QPSK Modulator - Example

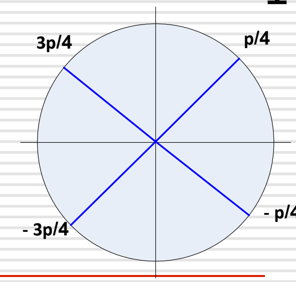


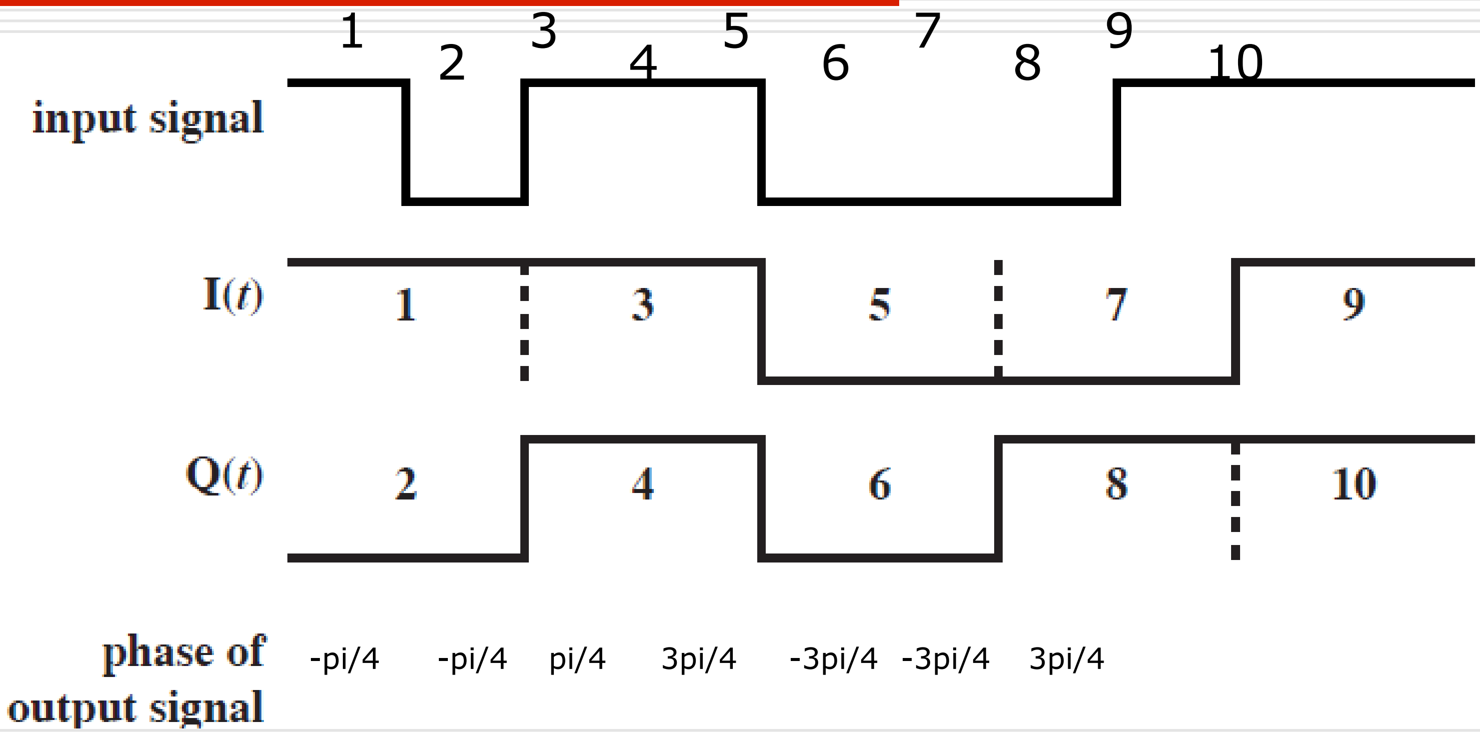


$$s(t) = \frac{1}{\sqrt{2}} I(t)\cos(2\pi f_c t) - \frac{1}{\sqrt{2}} Q(t)\sin(2\pi f_c t)$$

Phase change never exceeds 180 deg! →
Large phase shift is hard to implement in HW (imaging going from 00→ 11)

OQPSK Modulator - Example

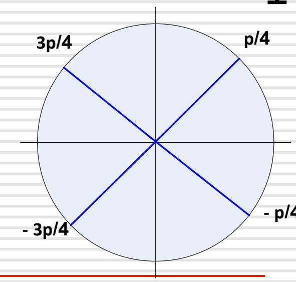




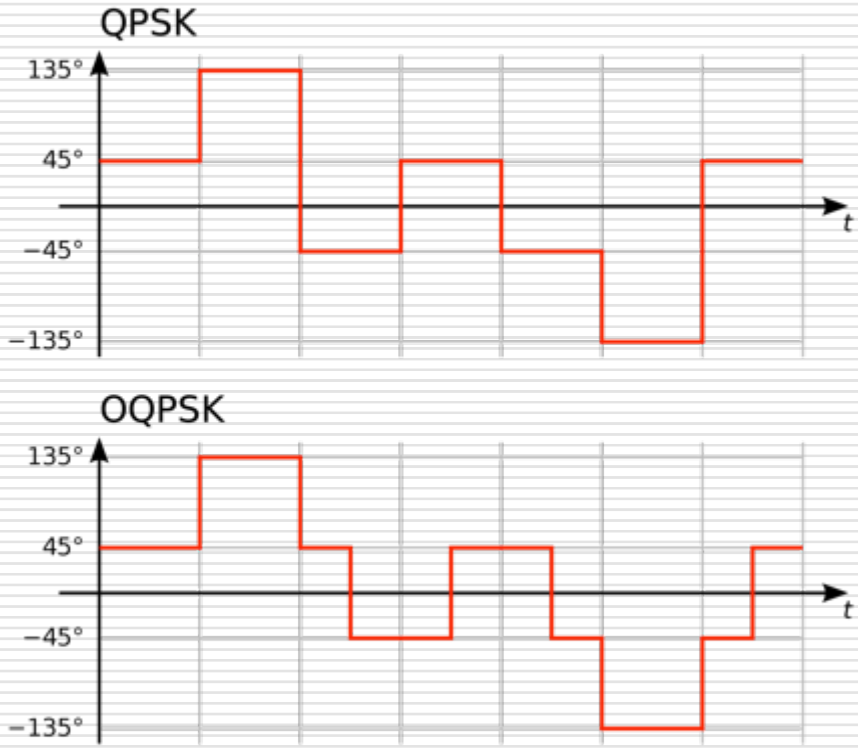
OQPSK

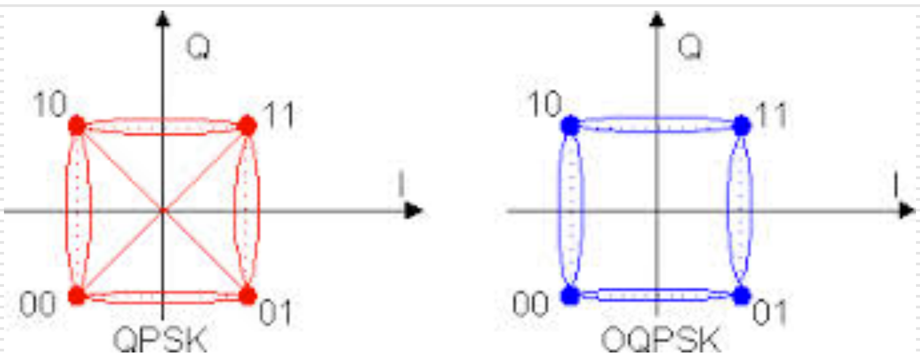
$$s(t) = \frac{1}{\sqrt{2}} I(t)\cos(2\pi f_c t) - \frac{1}{\sqrt{2}} Q(t - Tb)\sin(2\pi f_c t)$$

Phase change never exceeds 90 deg!



OQPSK Modulator - Example





OQPSK $$s(t) = \frac{1}{\sqrt{2}} I(t)\cos(2\pi f_c t) - \frac{1}{\sqrt{2}} Q(t - Tb)\sin(2\pi f_c t)$$

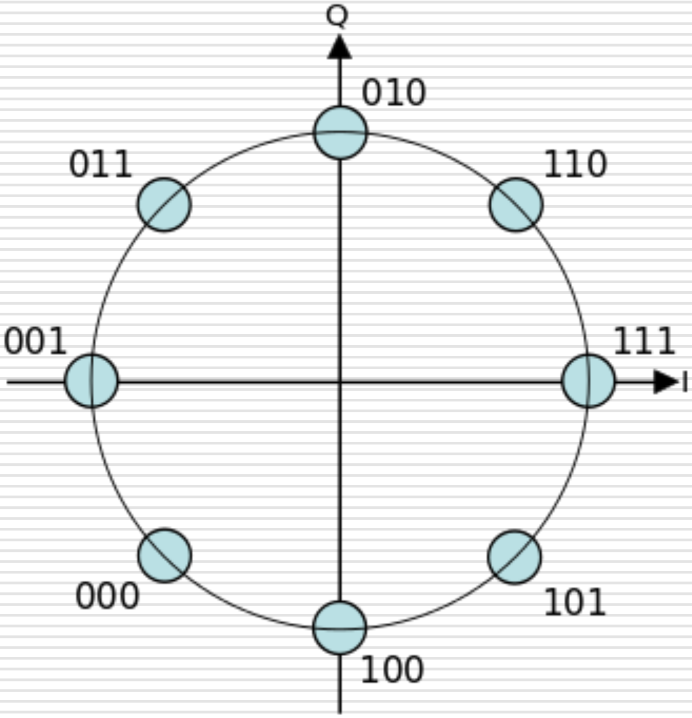
Phase change never exceeds 90 deg!

So what is the difference in performance? - QPSK and OQPSK Modulators

- Both have the same spectral characteristics and error performance
- Max. phase change for QPSK is 180 deg.
- Max. phase change for OQPSK is 90 deg.
  - Results in smaller sudden phase change → good for limiting non-linearity impact
  - Less non-linearity → less signal spread → less interference

Phase-Shift Keying (PSK)

- Multilevel-PSK
  - Using multiple phase angles with each angle having more than one amplitude, multiple signals elements can be achieved
  - Example: Standard 9600 baud used in Modem
    - 12 phase angles / four of them have 2 diff. amplitude levels → 16 levels



8-QPSK (same amplitude!)

Modulation Impact on Performance

- *Modulation rate*
- *Bit Error Ratio (rate)*
- *Bit Energy Level*
- *Bandwidth Efficiency*

Modulation Impact on Performance

$$D = \frac{R}{L} = \frac{R}{\log_2 M}$$

- D = modulation rate, baud
- R = data rate = 1/Tb, bps
- M = number of different signal elements = 2^L
- L = number of bits per signal element

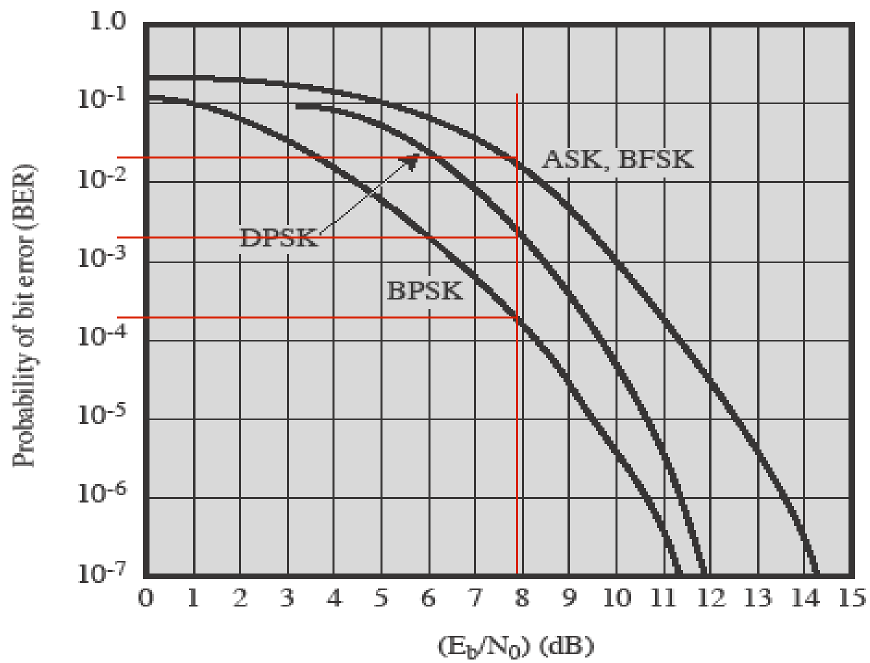
Example: We can support a data rate of 9600 bps
Using 2400 baud rate if M = 16, L = 4 using a complex modulation scheme!

This is how we can transmit more bits in the same medium!

Performance

Original Data \ Signal Transmitted	Digital	Analog
Digital		ASK FSK/BFSK/MFSK PSK/BPSK/MPSK
Analog	PCM DM	AM PM FM

- For the same signal energy BPSK can achieve the best performance in terms of BER
- Example:
  - Assume BER = 10^-7; SNR=12 dB; Find Bandwidth Efficiency (R/B_T).

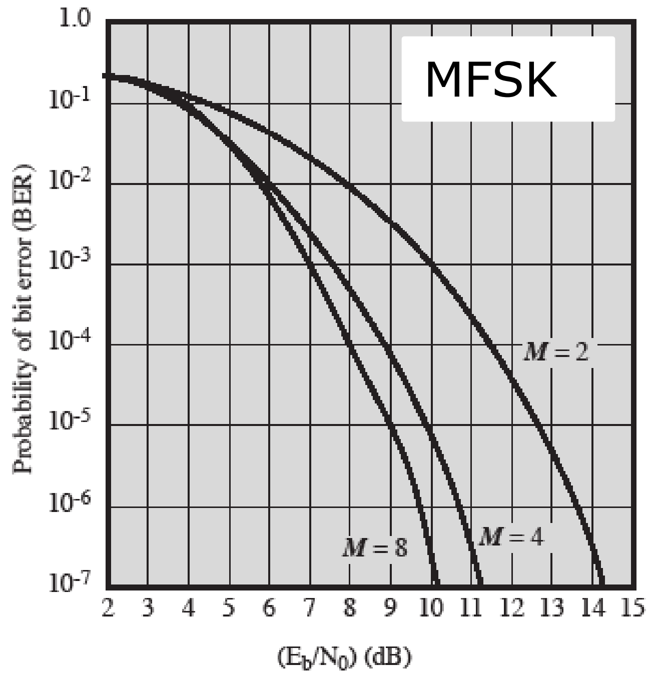


C- Refer to notes!

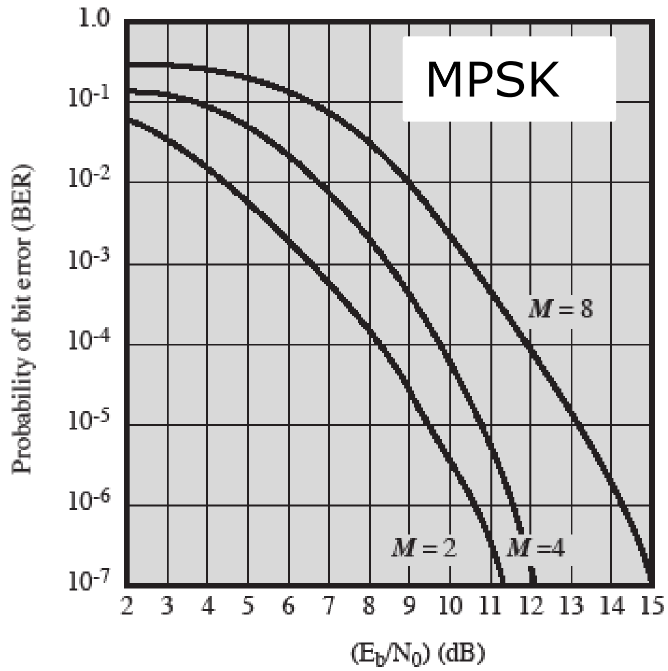
$Eb/No=(S/R)/No$; $N=NoB_T$; → $Eb/No(dB) = S/N(dB) – R/B_T\ (dB)$

Eb/No is the ratio of energy per bit to noise power density per hertz

Performance Comparison



(a) Multilevel FSK (MFSK)



(b) Multilevel PSK (MPSK)

Bandwidth Efficiency is proportional to BER

Performance

- Bandwidth of modulated signal (B_T)
  - ASK/PSK/FSK
    $$B_T = (1 + r)R$$
  - MPSK
    $$B_T = \left(\frac{1 + r}{L}\right)R = \left(\frac{1 + r}{\log_2 M}\right)R$$
  - MFSK
    $$B_T = \left(\frac{(1 + r)M}{\log_2 M}\right)R$$
    - L = number of bits encoded per signal element
    - M = number of different signal elements
    - r <1; a constant; depends on filtering

BW Efficiency
= R/BT
= Modulation Rate/BT

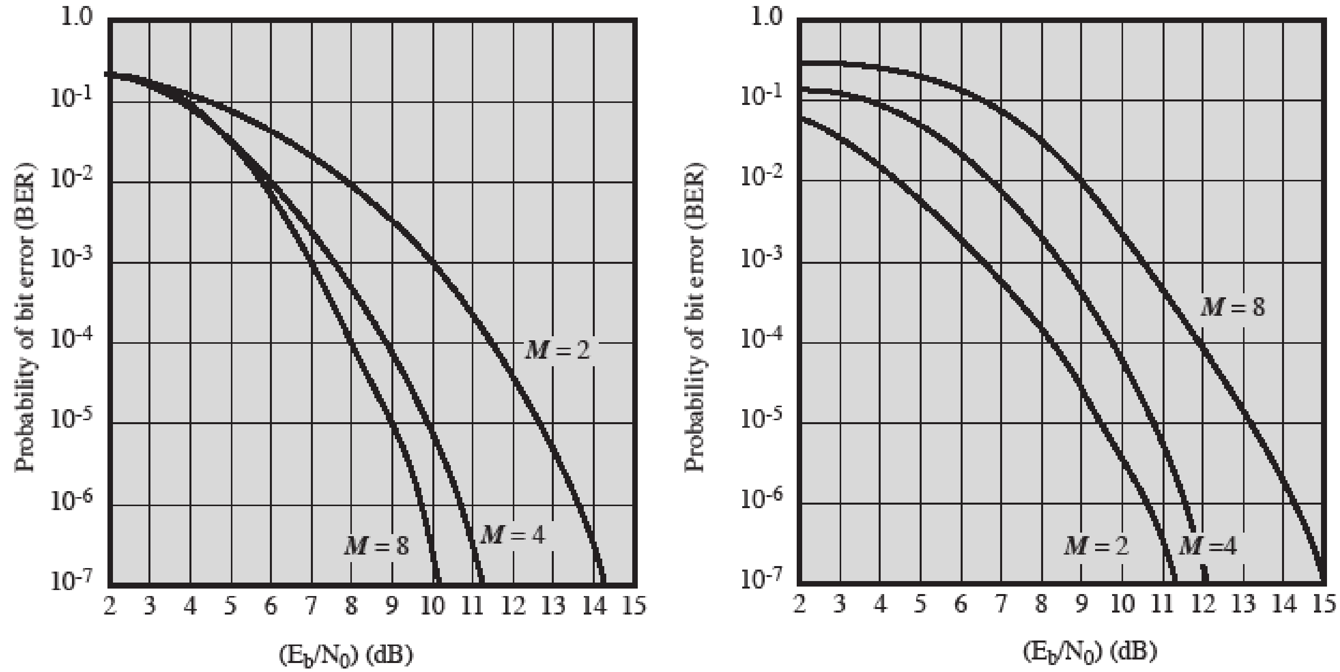
C- Refer to notes!

Remember: Larger number of states → higher data rate
→ Higher potential error!

Example

Find BW_efficiency for PSK if BER 10^-7 with SNR = 12 dB

Eb/No = 11.2 from figure → **Eb/No(dB) = S/N(dB) – R/B_T (dB)**
R/BT = BW_efficiency (dB) = 12 – 11.2 = 0.8
R/BT = 1.2



(a) Multilevel FSK (MFSK)

(b) Multilevel PSK (MPSK)

Reasons for Analog Modulation

Original Data \ Signal Transmitted	Digital	Analog
Digital		ASK FSK/BFSK/MFSK PSK/BPSK/MPSK
Analog	PCM DM	AM PM FM

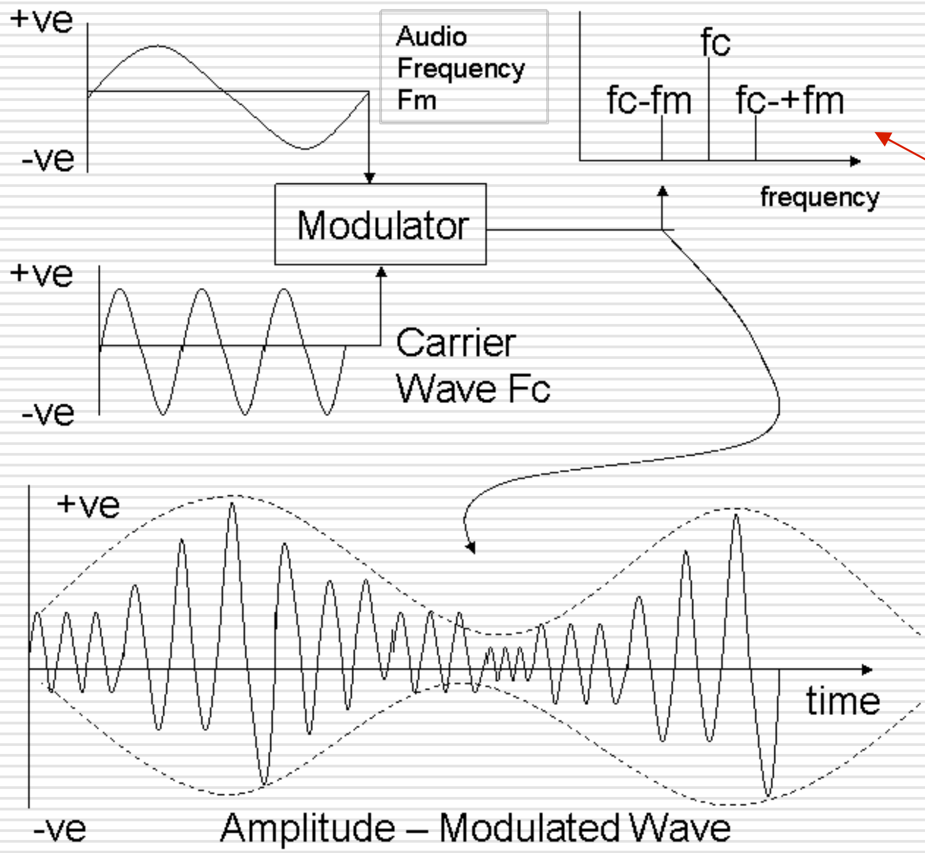
- Modulation of digital signals
  - When only analog transmission facilities are available, digital to analog conversion required
- Modulation of analog signals
  - A higher frequency may be needed for effective transmission
  - Modulation permits frequency division multiplexing

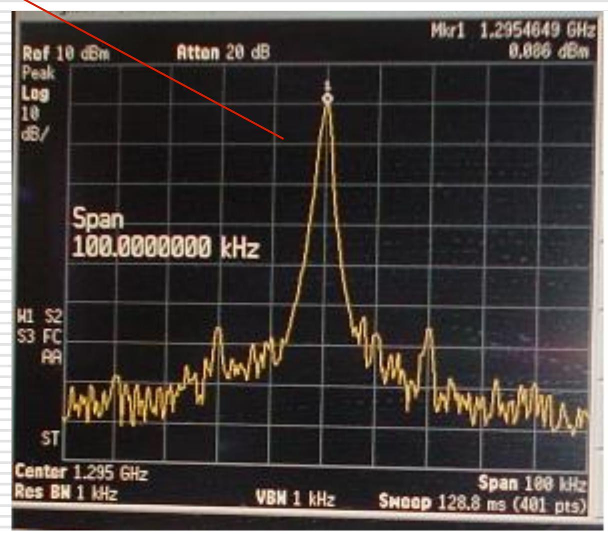
Basic Modulation Techniques

- Analog data to analog signal
  - Amplitude modulation (AM)
  - Angle modulation
    - Frequency modulation (FM)
    - Phase modulation (PM)

http://mason.gmu.edu/~mlyons3/AM_FM/AM_FM_model.html

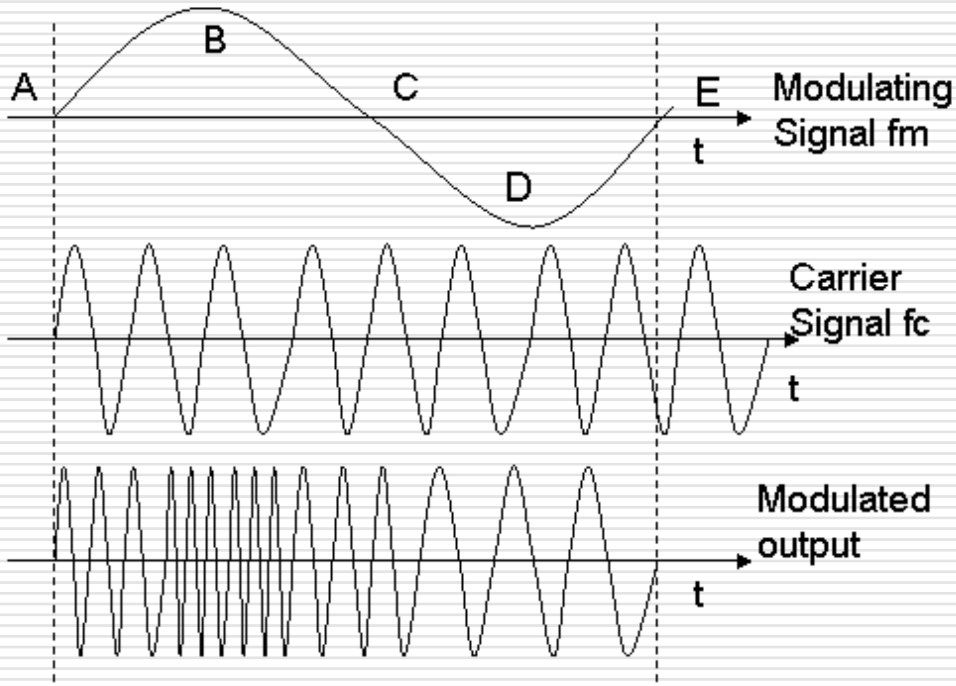
Amplitude modulation





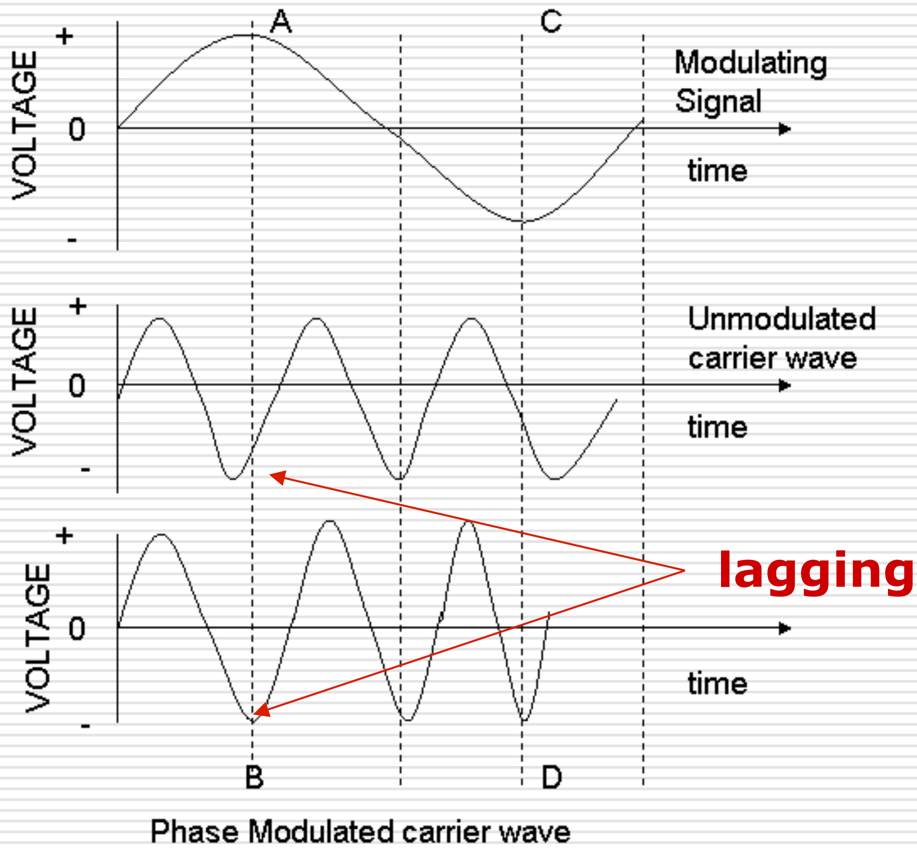
Frequency modulation (FM)

- With frequency modulation, the modulating signal and the carrier are combined in such a way that causes the carrier FREQUENCY (fc) to vary above and below its normal (idling) frequency
- As the voltage of the modulating signal increases in the positive direction from A to B, the frequency of the carrier is increased in proportion to the to the modulating voltage
- The amplitude of the carrier remains constant



Phase modulation (PM)

- The phase of the carrier is changed by the change in amplitude of the modulating signal
- The modulated carrier wave is lagging the carrier wave when the modulating frequency is positive (A and B)
- When the modulating frequency is negative, the modulated carrier wave is leading the carrier wave (C and D)



Varying the phase of the carries linearly in proportion to the modulating signal such that maximum phase shift occurs during positive and negative peaks of the modulating signal.

Comparison

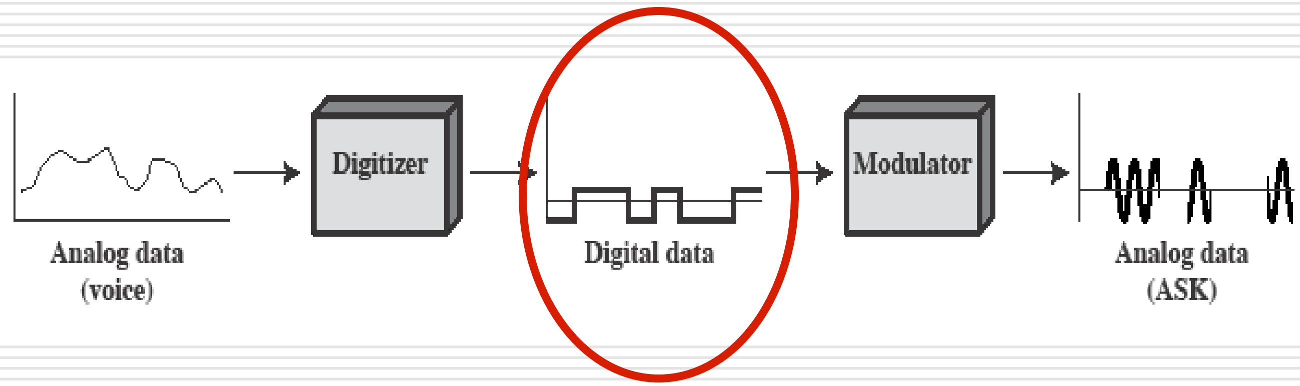
- FM and PM require greater bandwidth than AM
- Applet: http://engweb.info/courses/wdt/lecture07/wdt07-am-fm.html#AM_Applet_

Basic Encoding Techniques

- Analog data to digital signal
  - Pulse code modulation (PCM)
  - Delta modulation (DM)
- Basic process of digitizing analog data

Original Data \ Signal Transmitted	Digital	Analog
Digital		ASK FSK/BFSK/MFSK PSK/BPSK/MPSK
Analog	PCM DM	AM PM FM

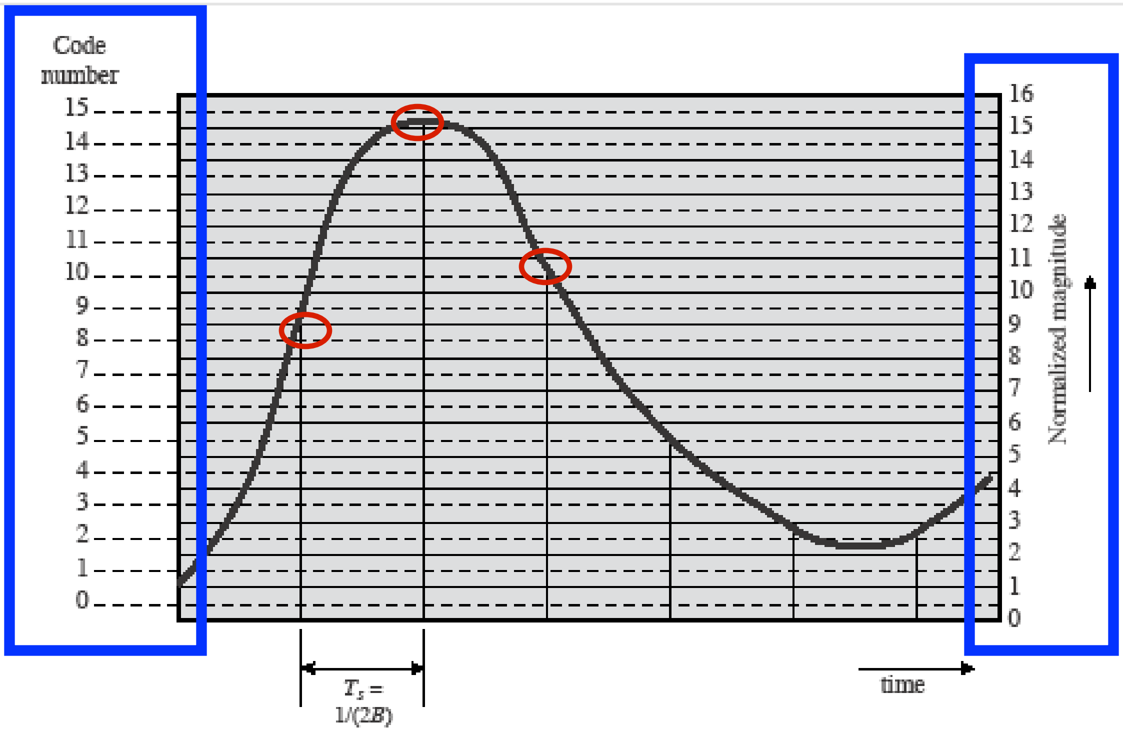
The question is how to represent the digital data



Pulse Code Modulation

- Based on the sampling theorem
- Each analog sample is assigned a binary code
  - Analog samples are referred to as pulse amplitude modulation (PAM) samples
- The digital signal consists of block of *n* bits, where each *n*-bit number is the amplitude of a PCM pulse

Pulse Code Modulation



PAM value	1.1	9.2	15.2	10.8	5.6	2.8	2.7
quantized code number	1	9	15	10	5	2	2
PCM code	0001	1001	1111	1010	0101	0010	0010

1- Sampling frequency (two times fmax)
2- Quantization levels (number of bits available)

http://www.netbook.cs.purdue.edu/animations/convert%20analog%20to%20digital.html

Pulse Code Modulation

- By quantizing the PAM pulse, original signal is only approximated
  - More quantization levels → more accurate signal approximation → more complex system
- Leads to quantizing noise
- Signal-to-noise ratio for quantizing noise

$$\mathrm{SNR}_{\mathrm{dB}} = 20 \log 2^n + 1.76\,\mathrm{dB} = 6.02n + 1.76\,\mathrm{dB}$$

NOTE: each additional bit increases SNR by 6 dB, or a factor of 4

Example:

- Assuming we use 7 bits to reconstruct the voice signal. Bandwidth of voice signal is 4KHz.
  - How may quantization levels can we create?
  - What is the sampling rate for the voice signal?
  - What is the BW of the PCM-encoded digital signal?
  - What is the minimum BW required using the Nyquist criterion?
  - How much the s/N (in dB) will increase if we use 9 bits instead?

Example:

- Assuming we use 7 bits to reconstruct the voice signal. Bandwidth of voice signal is 4KHz.
  - How … create?
  - What … e signal?
  - What … digital signa…
  - What is the minimum BW required using the Nyquist criterion?
  - How much the s/N (in dB) will increase if we use 9 bits instead?

> 2^7 = 128 levels
> Sampling rate: 2B = 8KHz (8000 samples / sec) ← according to the sampling theorem
> Each sample has 7 bits
> PCM BW = 8000 sample/sec x 7 bit/ sample = 56 bit/sec

Delta Modulation

- Analog input is approximated by staircase function
  - Moves up or down by one quantization level (δ) at each sampling interval
- Only the change of information is sent
  - only an increase or decrease of the signal amplitude from the previous sample is sent
  - a no-change condition causes the modulated signal to remain at the same 0 or 1 state of the previous sample

Delta Modulation

- Two important parameters
  - Size of step assigned to each binary digit (δ)
  - Sampling rate
- Accuracy improved by increasing sampling rate
  - However, this increases the data rate
- Advantage of DM over PCM is the simplicity of its implementation

References

- Applets
  - PM and FM Applet:
    - http://cnyack.homestead.com/files/modulation/modfmpm.htm
    - http://williams.comp.ncat.edu/Networks/modulate.htm
    - Very good:
      - http://sem.mosaic-service.com/electron2/frequency_conversion.htm
  - Learn about sampling theorem: http://www.facstaff.bucknell.edu/mastascu/elessonshtml/Signal/SignalNoteNyquistSampling.htm
- Very good basic information about AM, FM: http://cbdd.wsu.edu/kewlcontent/cdoutput/TR502/page21.htm
- Read about The Nyquist Sampling Theorem www.facstaff.bucknell.edu/mastascu/elessonshtml/Signal/SignalNoteNyquistSampling.htm