Prob. of bit Error in detecting Unipolar NRZ signal (BASK)

prob. that $s_1(t)$, say 1, is sent p

$s_2(t)$, say 0, is sent $1-p$

→ on average BER (bit error rate):

$$BER = P(s_1(t) \text{ sent}) \times P\{\text{error}/ s_1(t) \text{ sent}\} + P(s_2(t) \text{ sent}) \times P\{\text{Error}/ s_2(t) \text{ sent}\}$$

$$= p \cdot P\{\text{error}/ s_1(t) \text{ sent}\} + (1-p)\, P\{\text{error}/ s_2(t) \text{ sent}\} \quad ①$$

An error can happen two ways:

$S_0(t)$ sent $\quad r_0$ (received bit) $> V_T$ (threshold)

$S_1(t)$ sent $\quad r_0$ ( ) $< V_T$ ( )

Thus:

$$P\{\text{error}/ s_1(t) \text{ sent}\} = P\{r_0 < V_T / s_1(t) \text{ sent}\} = \int_{-\infty}^{V_T} f_{r_0}(r_0 / s_1(t) \text{ sent})\, dr_0$$

$$P\{\text{error}/ s_0(t) \text{ sent}\} = P\{r_0 > V_T / s_1(t) \text{ sent}\} = \int_{V_T}^{\infty} f_{r_0}(r_0 / s_2(t) \text{ sent})\, dr_0$$

$$\Rightarrow \int_{-\infty}^{V_T} \frac{1}{\sqrt{2\pi \sigma_0^2}} e^{-(r_0 - S_1)^2/2\sigma_0^2}\, dr_0 = Q\left(\frac{r_0 - V_T}{\sigma_0}\right)$$

$$\int_{V_T}^{\infty} \frac{1}{\sqrt{2\pi \sigma_0^2}} e^{-(r_0 - S_2)^2/2\sigma_0^2}\, dr_0 = Q\left(\frac{V_T - r_0}{\sigma_0}\right)$$

Given
$r_0(1 \text{ sent}) = AT$
$r_0(0 \text{ sent}) = 0$
$V_T = AT/2$
$1-p = q$ if $p=q$

⇒ ① will Give

$$p \cdot Q\left(\frac{r_0 - V_T}{\sigma_0}\right)\Big|_{1 \text{ was sent}} + (1-p)\, Q\left(\frac{V_T - r_0}{\sigma_0}\right)\Big|_{0 \text{ was sent}}$$

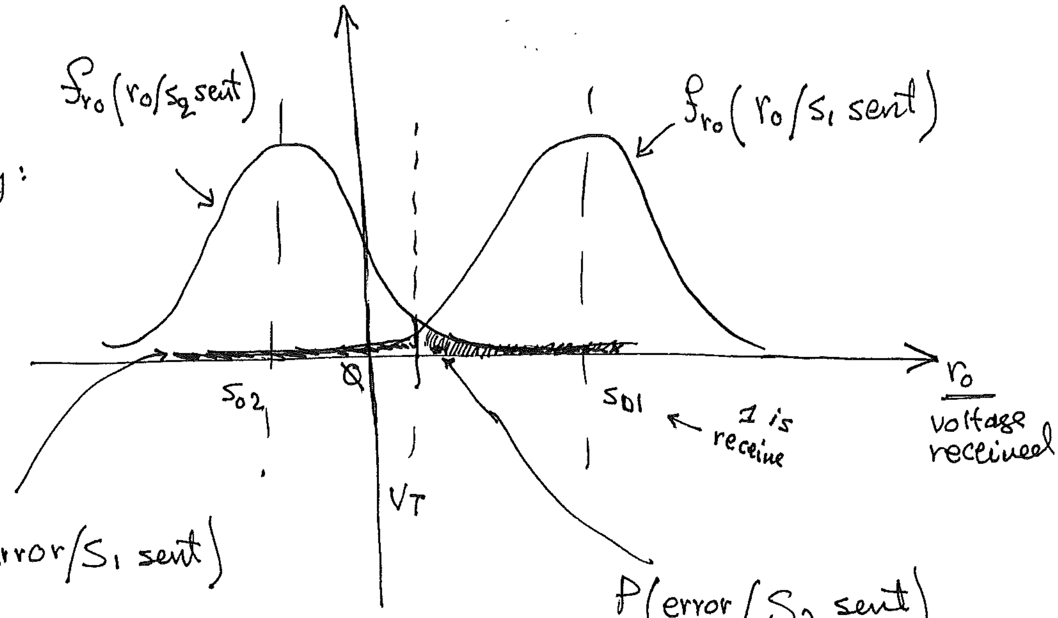
$$= \tfrac{1}{2} Q\left(\frac{AT - AT/2}{\sigma}\right) + (1-p)\, Q\left(\frac{AT/2}{\sigma}\right)$$

$$= \tfrac{1}{2} Q\left(AT/2\sigma\right) + \tfrac{1}{2} Q\left(AT/2\sigma\right) = \boxed{Q\left(AT/2\sigma\right)} = BER$$

Note:
$$1 - Q\left(\frac{x - m_x}{\sigma_x}\right) = Q\left(\frac{m_x - x}{\sigma_x}\right) =$$

$$\left[1 - \int_{x - m_x/\sigma_x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz\right] = \int_{-\infty}^{x - m_x/\sigma} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz$$

Graphiclly:



so the question is where should the trigger point be for max performance (min. BER) - in other words where is the optimum choice of $V_T \equiv V_{opt}$.

$$V_{opt} = \frac{S_{01} + S_{02}}{2} = V_T$$

$$\rightarrow \quad BER = \frac{1}{2} Q\left(\frac{S_{01} - V_T}{\sigma}\right) + \frac{1}{2} Q\left(\frac{V_T - S_{02}}{\sigma}\right)$$

$$= \frac{1}{2} Q\left(\frac{S_{01} - (S_{01}+S_{02})/2}{\sigma}\right) + \frac{1}{2} Q\left(\frac{(S_{01}+S_{02})/2 - S_{02}}{\sigma}\right)$$

$$BER_{opt.} = \boxed{Q\left(\frac{S_{01} - S_{02}}{2\sigma_0}\right)}$$