Binary Phase Shift Keying (BPSK)

- In BPSK, the symbol mapping table encodes bits (b_n) 1 and 0 to transmission symbols (a_n) 1 and –1, respectively
- Every T_b seconds the modulator transmits one of the two carrier bursts that corresponds to the information bit being a 1 or 0

Binary 1: $s_1(t) = A_c \cos(2\pi f_c t), \qquad 0 \le t \le T_b$

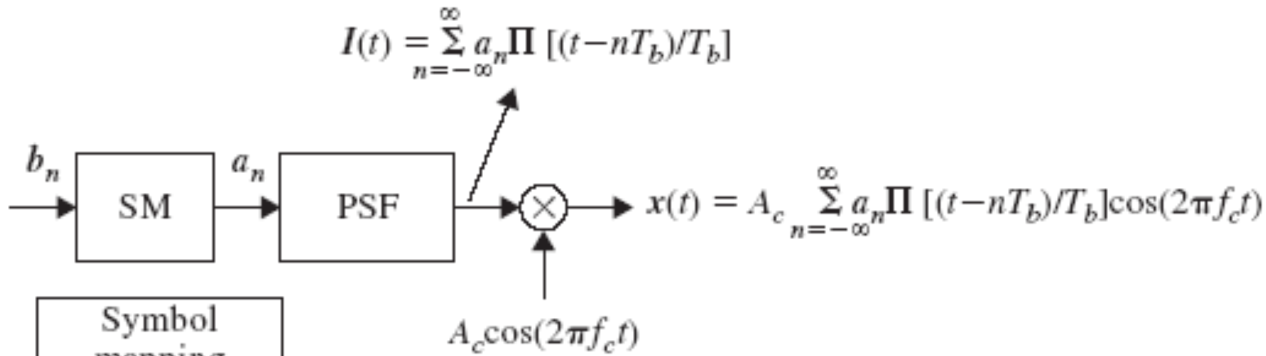
Binary 0: $s_2(t) = A_c \cos(2\pi f_c t + \pi) = -A_c \cos(2\pi f_c t)$

- The resultant BPSK signal can be expressed as

$$\boldsymbol{x}(t) = A_c \underbrace{\sum_{n=-\infty}^{\infty} \boldsymbol{a}_n \Pi\left[\left(t - nT_b\right)/T_b\right]}_{\boldsymbol{I}(t)} \cos\left(2\pi f_c t\right), \quad \boldsymbol{a}_n \in \mathscr{A}_2 = \left\{1 - 1\right\}$$

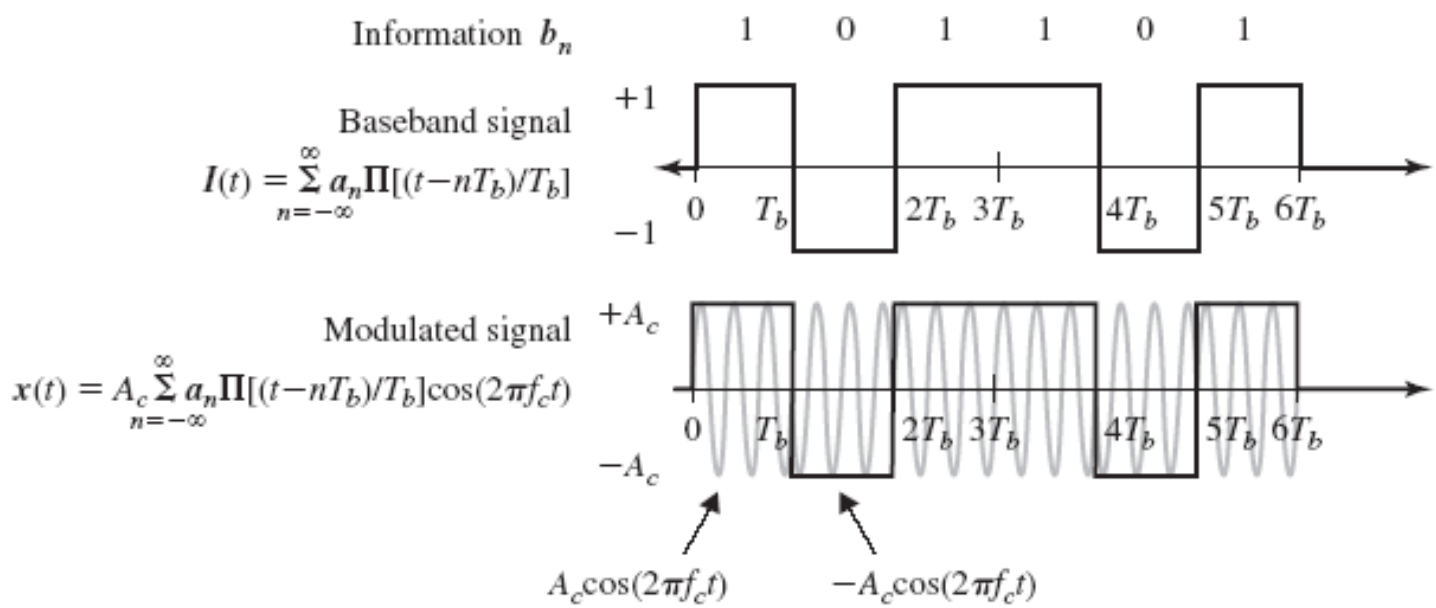
- $\boldsymbol{x}(t)$ contains only the in-phase component $\boldsymbol{I}(t)$; $\boldsymbol{Q}(t)$ is zero

BPSK Modulation

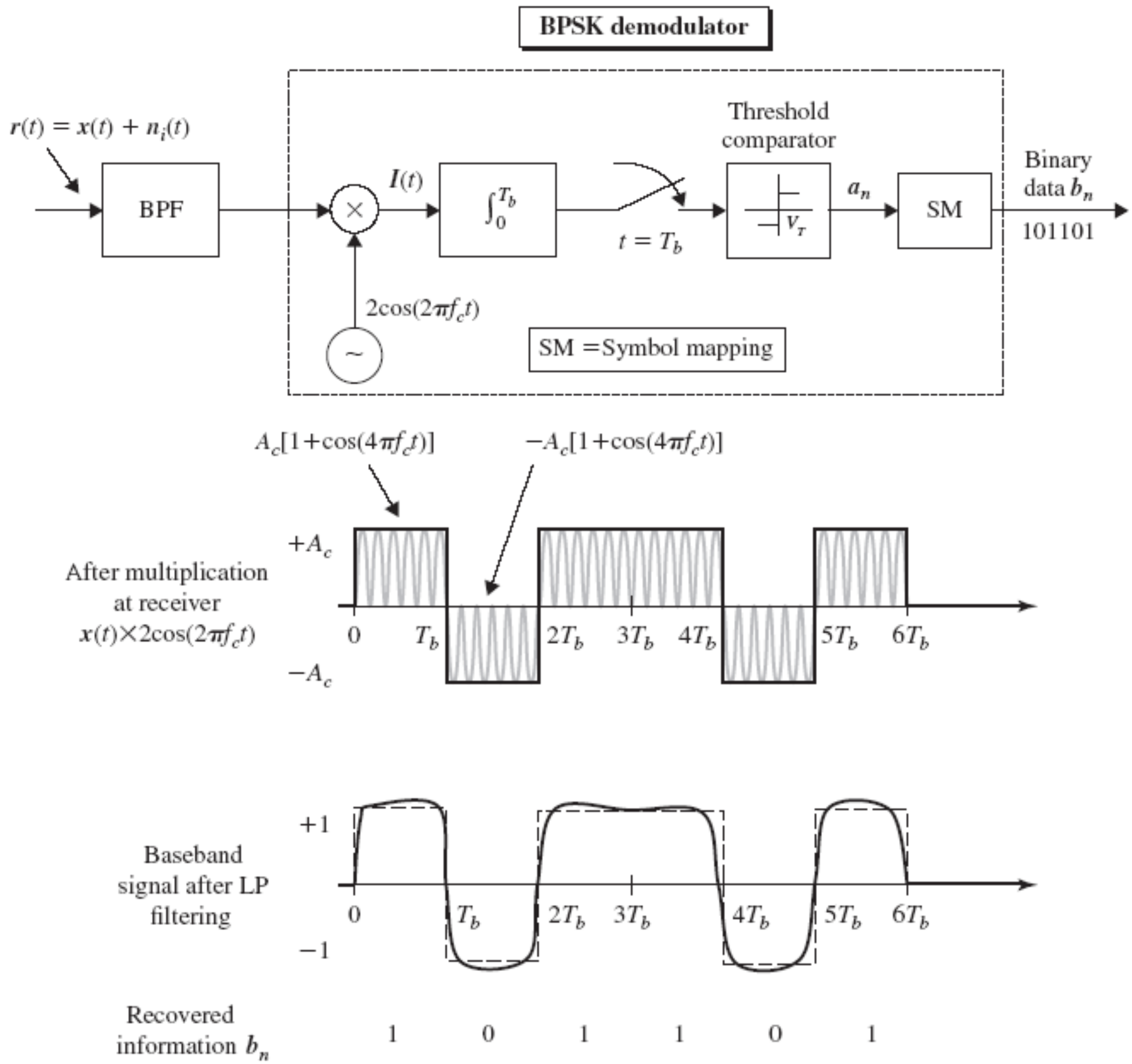


Symbol mapping	
b_n	a_n
1	1
0	−1

SM = Symbol mapping
PSF = Pulse shaping filter



BPSK Coherent Demodulation



Error Performance

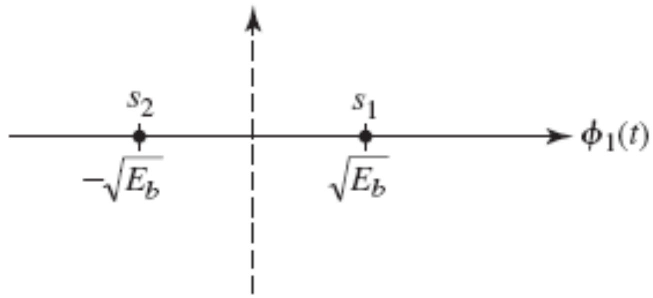
- If we choose the basis function

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t), \qquad 0 \le t \le T_b$$

we can write BPSK waveforms as

$$s_1(t) = A_c \sqrt{\frac{T_b}{2}} \phi_1(t) = \sqrt{E_b} \phi_1(t)$$

$$s_2(t) = -A_c \sqrt{\frac{T_b}{2}} \phi_1(t) = -\sqrt{E_b} \phi_1(t)$$



- BPSK is thus polar signaling with $d_{\min} = 2\sqrt{E_b}$
- The BER performance of BPSK is, therefore, identical to that of polar NRZ signaling

$$BER_{BPSK} = Q\left(\frac{d_{\min}}{\sqrt{2N_o}}\right) = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

Binary Frequency Shift Keying (BFSK)

- In BFSK, information is transmitted by sending carrier bursts of two different frequencies, $f_1 = f_c + \Delta f/2$ and $f_2 = f_c - \Delta f/2$, to transmit binary data. Δf is called the *frequency deviation*

$$\text{Binary 1: } s_1(t) = A_c \cos(2\pi f_c t + \pi \Delta f t + \phi_1), \qquad 0 \le t \le T_b$$

$$\text{Binary 0 : } s_2(t) = A_c \cos(2\pi f_c t - \pi \Delta f t + \phi_2), \qquad 0 \le t \le T_b$$

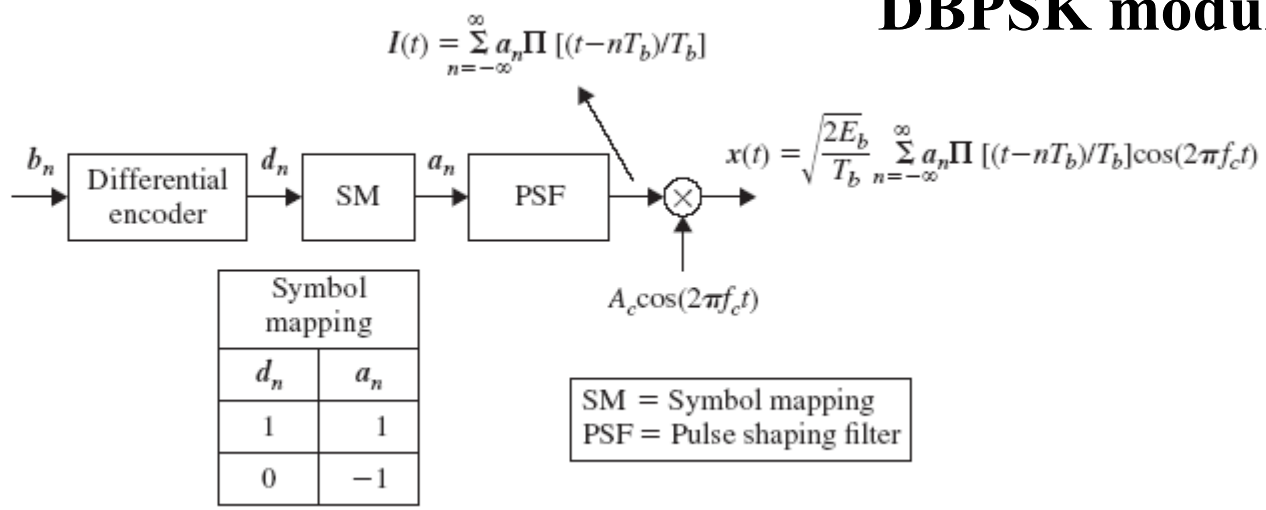
- A simple way to generate a BFSK signal is to use two separate oscillators tuned to frequencies f_1 and f_2 and switch between their outputs in accordance with the amplitude of the random data bit during that bit interval
- ϕ_1 and ϕ_2 are arbitrary phases of two frequency bursts generated by separate oscillators

Other Demodulation Techniques

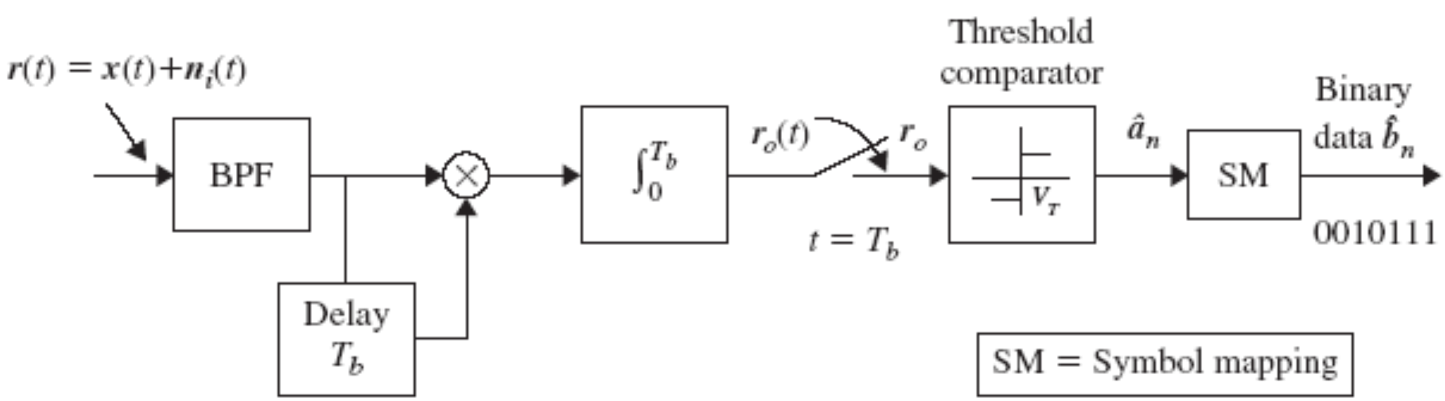
- Coherent demodulation may neither be desirable nor feasible in many practical applications.
  - The propagation delay on some radio channels changes too rapidly to permit accurate tracking of the carrier phase at the demodulator
  - Tracking the incoming signal's carrier phase and synchronizing the demodulator to it requires additional hardware complexity with cost and power efficiency ramifications
- *Differentially Coherent Demodulator* − demodulator uses the carrier phase of the previous symbol period as phase reference for the current period
- *Noncoherent Demodulator* − demodulator does not exploit phase information in the received signal for its demodulation

DBPSK (contd)

DBPSK modulator



DBPSK demodulator



DBPSK (contd)

- The output of the sampler is given by

$$\boldsymbol{r}_o = \begin{cases} E_b + \boldsymbol{n}(T_b), & \boldsymbol{a}_n = \boldsymbol{a}_{n-1} \\ -E_b + \boldsymbol{n}(T_b), & \boldsymbol{a}_n \neq \boldsymbol{a}_{n-1} \end{cases}$$

where $\boldsymbol{n}(t)$ is non-Gaussian noise.

- Since we have polar symmetry, $V_T = 0$ is selected. We can now write the following decision rule for decoding

$$\boldsymbol{r}_o > 0 \Rightarrow \hat{\boldsymbol{a}}_n = \hat{\boldsymbol{a}}_{n-1} \Rightarrow \hat{\boldsymbol{b}}_n = 0$$

$$\boldsymbol{r}_o < 0 \Rightarrow \hat{\boldsymbol{a}}_n \neq \hat{\boldsymbol{a}}_{n-1} \Rightarrow \hat{\boldsymbol{b}}_n = 1$$

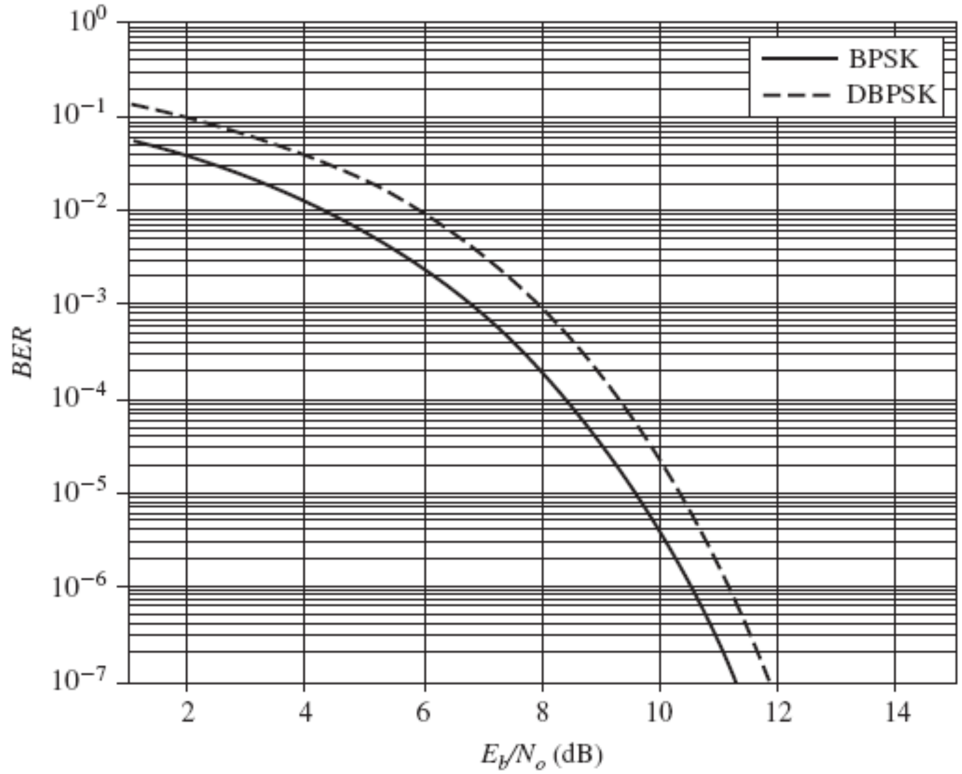
- The probability of bit error for DBPSK scheme is given by

$$BER_{DBPSK} = \frac{1}{2} e^{-\frac{E_b}{N_o}}$$

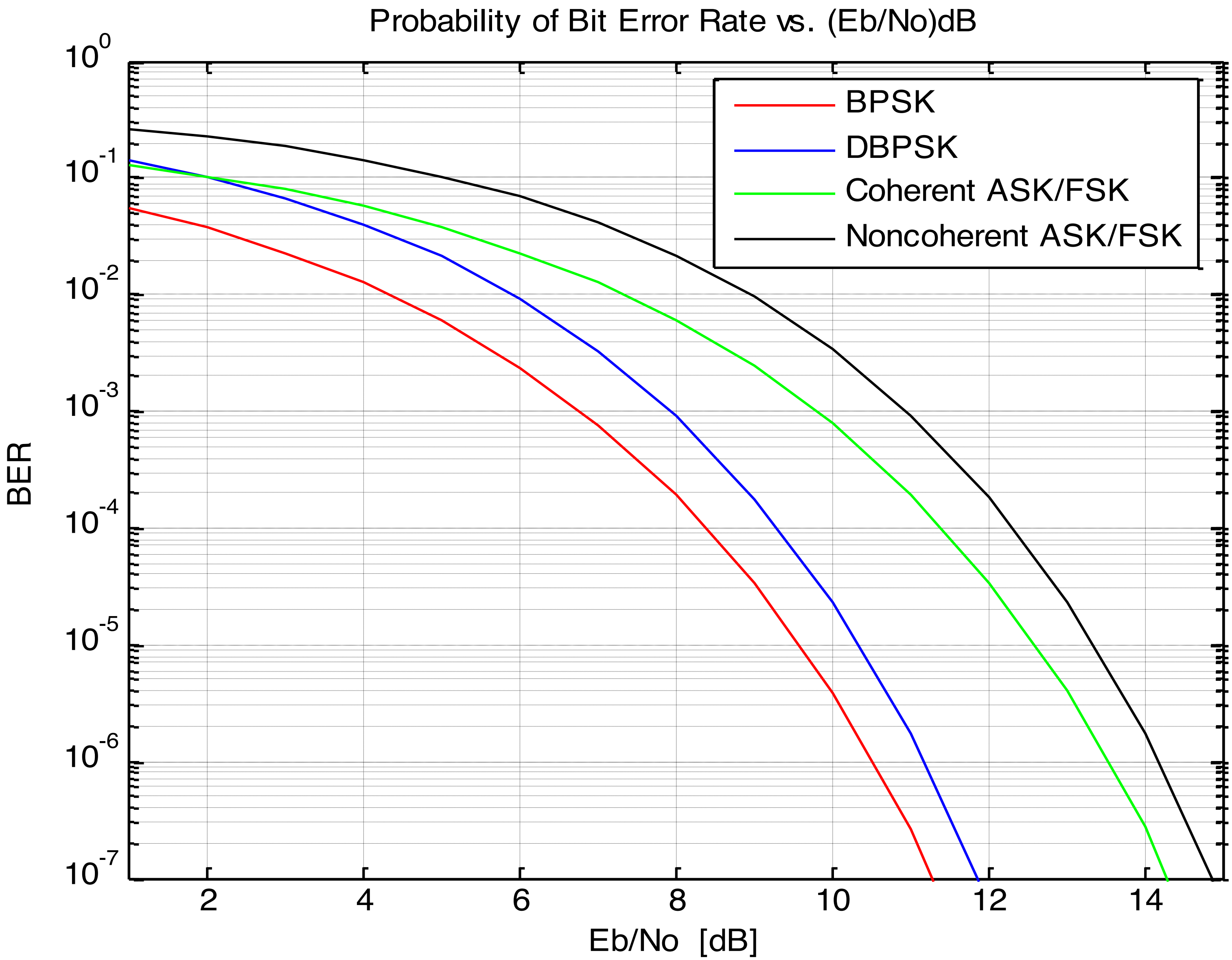
DBPSK (contd)

Table 11.2 Example of Differential Decoding of BPSK

Differentially encoded bits d_n	1	1	0	1	0	1	1	0	0	1
Threshold-comparison sign		+	−	−	−	−	+	−	−	−
Decoded differential bits $\hat{d}_n$	1	1	0	1	0	1	1	0	0	1
Regenerated data bits $\hat{b}_n$		0	1	1	1	1	0	1	0	1



BER Comparison



Quadrature Modulation Schemes

- In BPSK the phase of the carrier burst is shifted 0 or 180 degrees every pulse or symbol interval depending upon the information sequence. Thus each modulated carrier pulse transmits 1 bit of information
- If, on the other hand, the modulation scheme can use phase shifts of 45, 135, 225, or 315 degrees, each modulated carrier pulse transmits 2 bits of information. This technique is called **Quadrature Phase Shift Keying (QPSK)**
  - Using QPSK, we can *double* the data rate over the same channel bandwidth.
- QPSK is one of the modulation methods in the family known as Quadrature modulation schemes which are widely used, including in cellular and cable modem applications

Quadrature Modulation Schemes (contd)

- Suppose an information source generates M-ary symbols at a rate of D symbols/second $\Rightarrow T = 1/D$
  - The symbol stream is split into 2 sequences that consist of odd and even symbols, say, $\boldsymbol{a}_n^I$ and $\boldsymbol{a}_n^Q$, respectively
- Let $\boldsymbol{a}_n^I \in \mathscr{A}_M$ modulate in-phase carrier $A_c\cos(2\pi f_c t)$ every T seconds to produce the signal

$$A_c \sum_{n=-\infty}^{\infty} \boldsymbol{a}_n^I v(t - nT)\cos(2\pi f_c t) = A_c \boldsymbol{I}(t)\cos(2\pi f_c t)$$

- This signal is identical to the BPSK signal if $\boldsymbol{a}_n^I$ is polar binary symbol sequence
- Similarly, let $\boldsymbol{a}_n^Q \in \mathscr{A}_M$ modulate the quadrature carrier $A_c\sin(2\pi f_c t)$ every T seconds to produce the signal

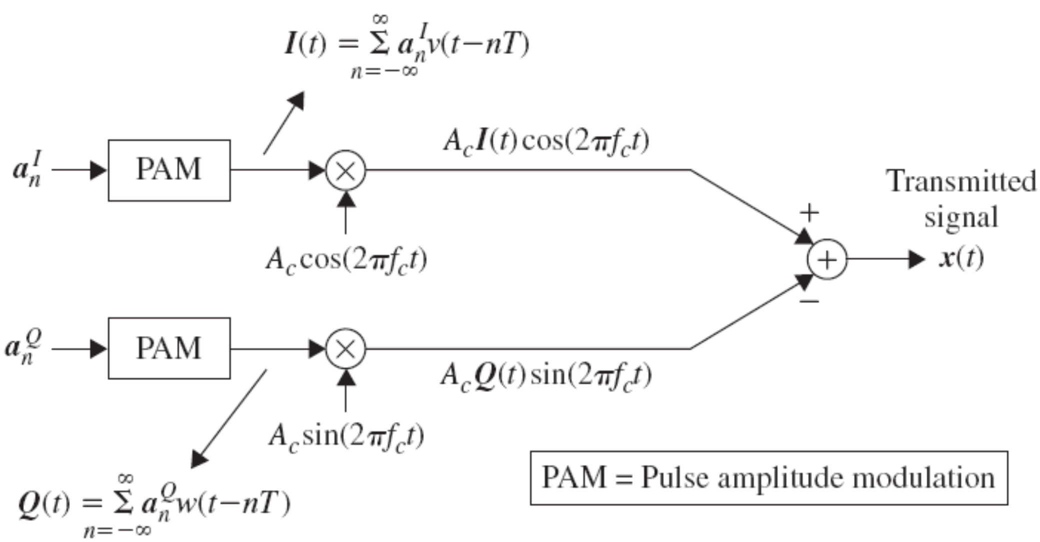
$$A_c \sum_{n=-\infty}^{\infty} \boldsymbol{a}_n^Q w(t - nT)\sin(2\pi f_c t) = A_c \boldsymbol{Q}(t)\sin(2\pi f_c t)$$

Quadrature Modulation Schemes (contd)

- $v(t)$ and $w(t)$ are unit energy pulses of width T seconds. For example $v(t) = w(t) = (1/\sqrt{T})\Pi[(t - nT)/T]$
- Both modulated waveforms will have their power spectrum located within the same frequency band
- The composite modulated signal $\boldsymbol{x}(t)$ is

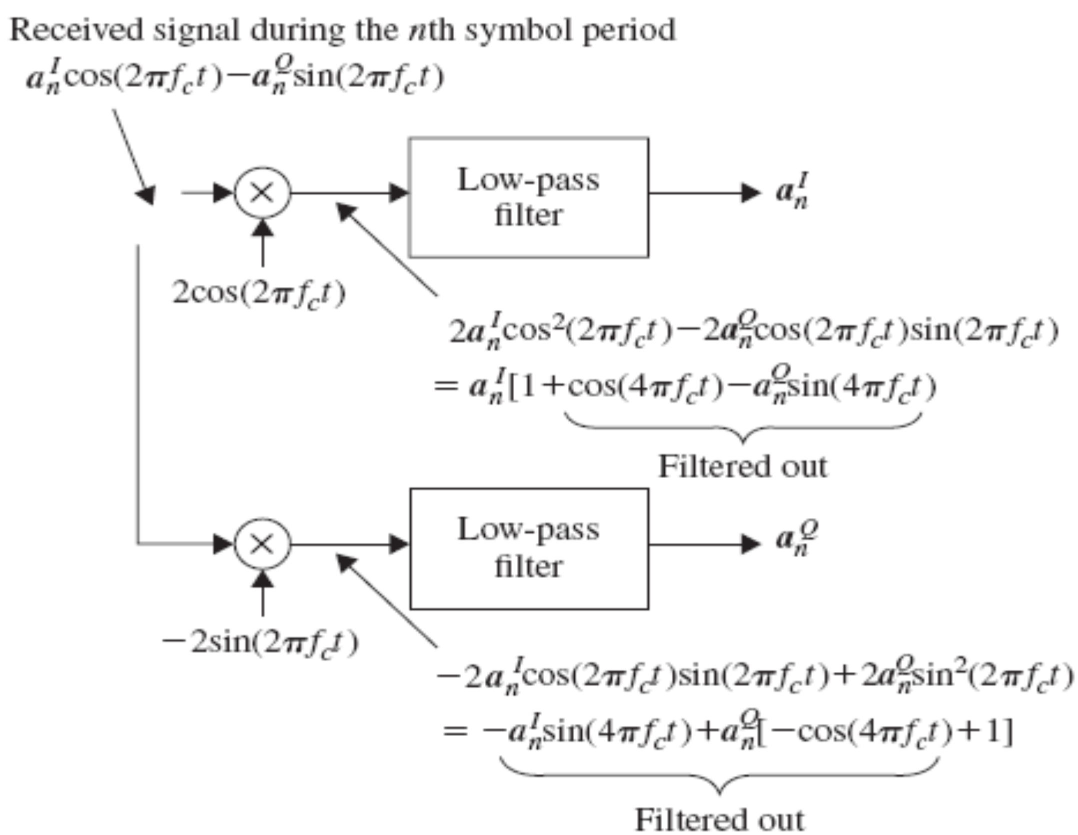
$$\boldsymbol{x}(t) = A_c\left[\boldsymbol{I}(t)\cos\left(2\pi f_c t\right) - \boldsymbol{Q}(t)\sin\left(2\pi f_c t\right)\right]$$

$$= A_c \sum_{n=-\infty}^{\infty}\left[\boldsymbol{a}_n^I v\left(t - nT\right)\cos\left(2\pi f_c t\right) - \boldsymbol{a}_n^Q w\left(t - nT\right)\sin\left(2\pi f_c t\right)\right]$$



Quadrature Modulation Schemes (contd)

- The in-phase and quadrature pulse trains $\boldsymbol{I}(t)$ and $\boldsymbol{Q}(t)$ can be recovered by, respectively, multiplying $\boldsymbol{x}(t)$ with $2\cos(2\pi f_c t)$ and $2\sin(2\pi f_c t)$ and then LP filtering resultant waveforms
- The M-ary symbols $\boldsymbol{a}_n^I$ and $\boldsymbol{a}_n^Q$ are then detected from $\boldsymbol{I}(t)$ and $\boldsymbol{Q}(t)$, respectively, as discussed in Chapter 10



Quaternary Phase Shift Keying (QPSK)

- QPSK is the most common form of phase-shift keying. By using phase shifts of 45, 135, 225, or 315 degrees, each modulated carrier pulse transmits 2 bits of information
- QPSK is a quadrature modulation scheme: each orthogonal carrier is modulated by a statistically independent polar NRZ symbol sequence
- The block diagram of a QPSK modulator is shown in Figure
- Binary data arriving at rate R_b is split by a serial to parallel converter into two data streams, one containing even bits ($\boldsymbol{b}_{2n}$) and other odd bits ($\boldsymbol{b}_{2n+1}$)
- The symbol mapping tables in the upper and lower branches of the modulator encode even and odd bits into polar transmission symbols $\boldsymbol{a}_{2n}$ and $\boldsymbol{a}_{2n+1}$, respectively

QPSK Modulator

- The output of the pulse shaping filter in the upper branch is a binary polar NRZ pulse train $\boldsymbol{I}(t)$ that modulates the in-phase carrier $A_c \cos\left(2\pi f_c t\right)$
- Similarly, a binary polar NRZ pulse train $\boldsymbol{Q}(t)$ generated by the pulse shaping filter in the lower branch modulates the quadrature carrier $A_c \sin\left(2\pi f_c t\right)$
- The QPSK signal $\boldsymbol{x}(t)$ is now obtained by adding the in-phase and quadrature components

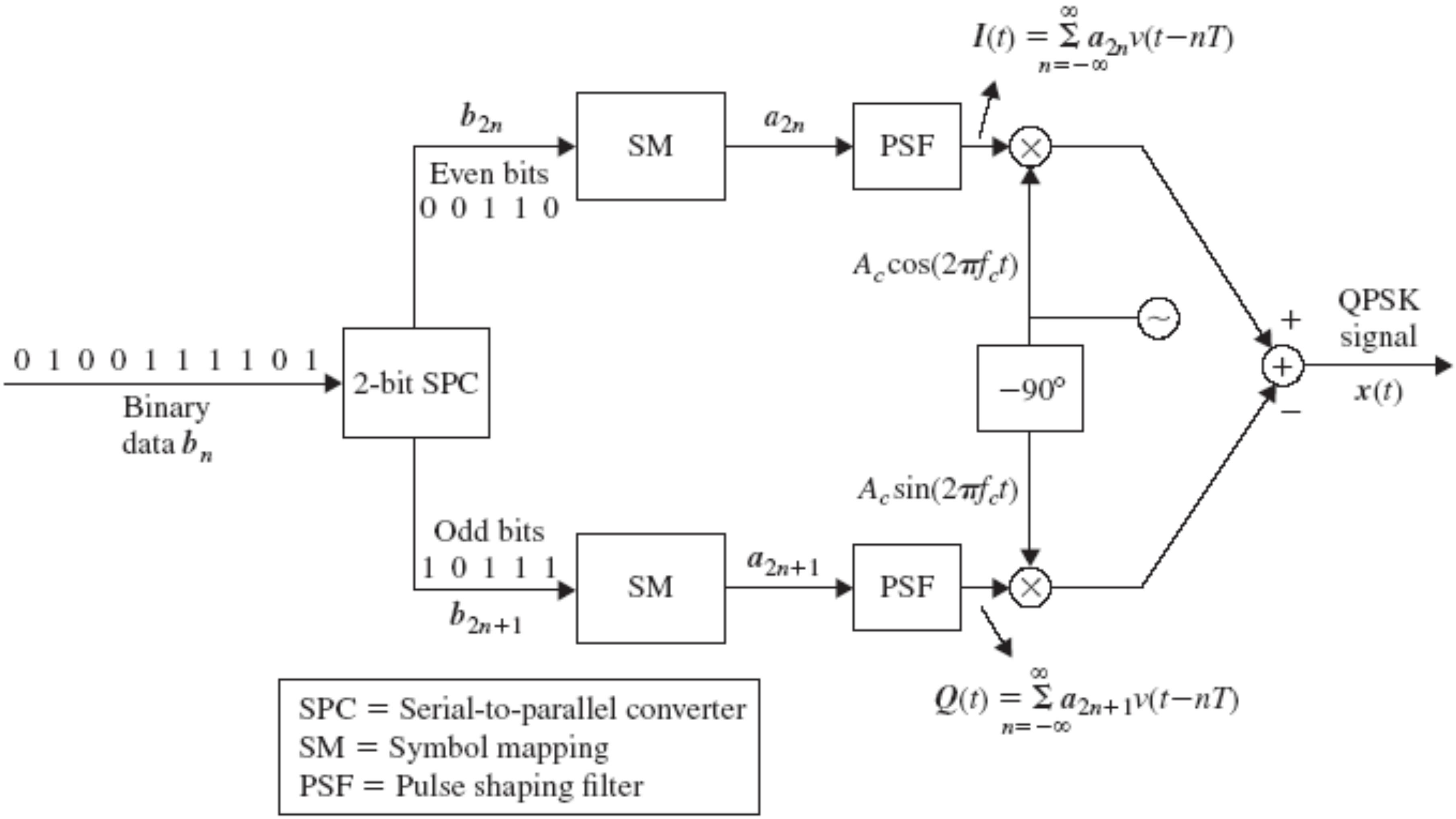
$$\boldsymbol{x}(t) = A_c\left[\boldsymbol{I}(t)\cos\left(2\pi f_c t\right) - \boldsymbol{Q}(t)\sin\left(2\pi f_c t\right)\right]$$

where

$$\boldsymbol{I}(t) = \sum_{n=-\infty}^{\infty} \boldsymbol{a}_{2n} v\left(t - nT\right)$$

$$\boldsymbol{Q}(t) = \sum_{n=-\infty}^{\infty} \boldsymbol{a}_{2n+1} v\left(t - nT\right)$$

QPSK Modulator Block Diagram



QPSK

- The carrier-modulated pulse during the first symbol interval is

$$\boldsymbol{s}(t) = A_c v(t)\left[\boldsymbol{a}_0 \cos\left(2\pi f_c t\right) - \boldsymbol{a}_1 \sin\left(2\pi f_c t\right)\right], \qquad 0 \le t \le T$$

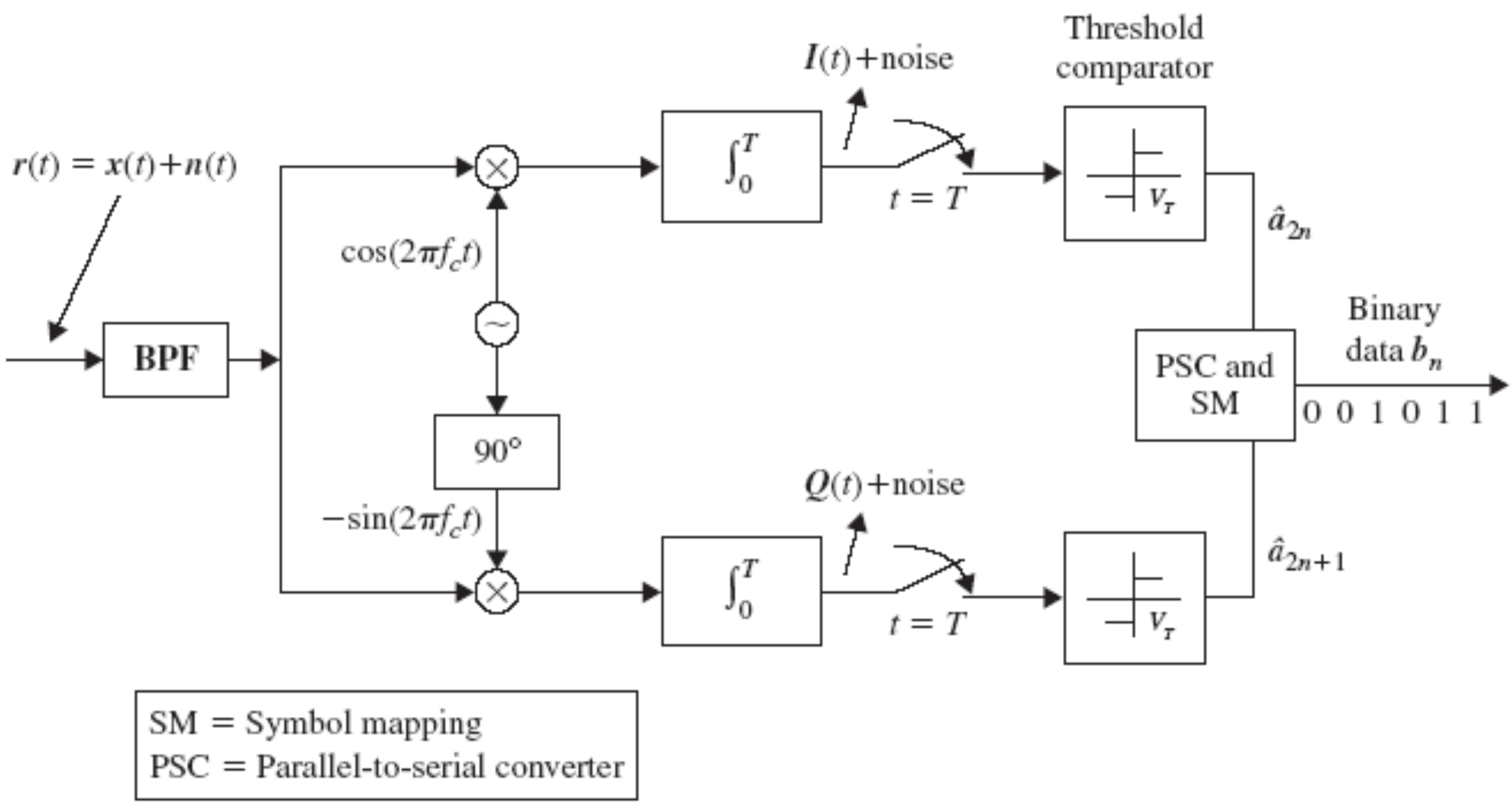
$$= A_c v(t) \cos\left(2\pi f_c t + \boldsymbol{\psi}_0\right), \qquad \boldsymbol{\psi}_0 = \tan^{-1} \frac{\boldsymbol{a}_1}{\boldsymbol{a}_0}$$

\` where phase of the transmitted carrier burst $\boldsymbol{\psi}_0$ is a discrete random variable assuming one of the four possible values $\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$ depending on the binary pair $(\boldsymbol{a}_0, \boldsymbol{a}_1)$

Odd and Even Bits (b_0, b_1)	(a_0, a_1)	Transmitted Carrier Burst $s_i(t)$, $0 \le t \le T$
11	$(1/\sqrt{2}, 1/\sqrt{2})$	$s_1(t) = A_c v(t) \cos(2\pi f_c t + \pi/4)$
01	$(-1/\sqrt{2}, 1/\sqrt{2})$	$s_2(t) = A_c v(t) \cos(2\pi f_c t + 3\pi/4)$
00	$(-1/\sqrt{2}, -1/\sqrt{2})$	$s_3(t) = A_c v(t) \cos(2\pi f_c t + 5\pi/4)$
10	$(1/\sqrt{2}, -1/\sqrt{2})$	$s_4(t) = A_c v(t) \cos(2\pi f_c t + 7\pi/4)$

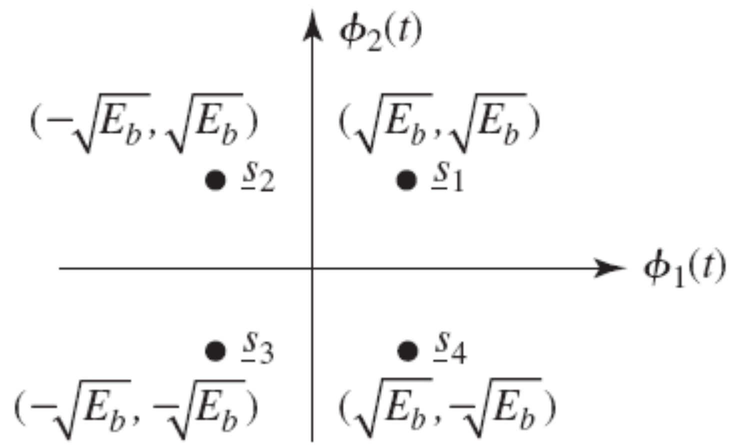
QPSK Demodulator Block Diagram

- The coherent demodulation of the QPSK signal is shown in Figure



Error Performance

- By choosing the basis functions



$$\phi_1(t) = \sqrt{2}v(t)\cos(2\pi f_c t)$$

$$\phi_2(t) = \sqrt{2}v(t)\sin(2\pi f_c t)$$

it is possible express all four possible carrier bursts in table as vectors in the plane spanned by ϕ_1 and ϕ_2

$$\underline{s} = (a_{2n}\sqrt{E_s}, a_{2n+1}\sqrt{E_s}) = (\pm\sqrt{\frac{E_s}{2}}, \pm\sqrt{\frac{E_s}{2}}) = (\pm\sqrt{E_b}, \pm\sqrt{E_b})$$

- The nearest neighbor estimate for the BER of for QPSK is (K = 4, M =4)

$$BER_{QPSK} = \frac{2K}{M\log_2 M} Q\left(\frac{d_{\min}}{\sqrt{2N_o}}\right) = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

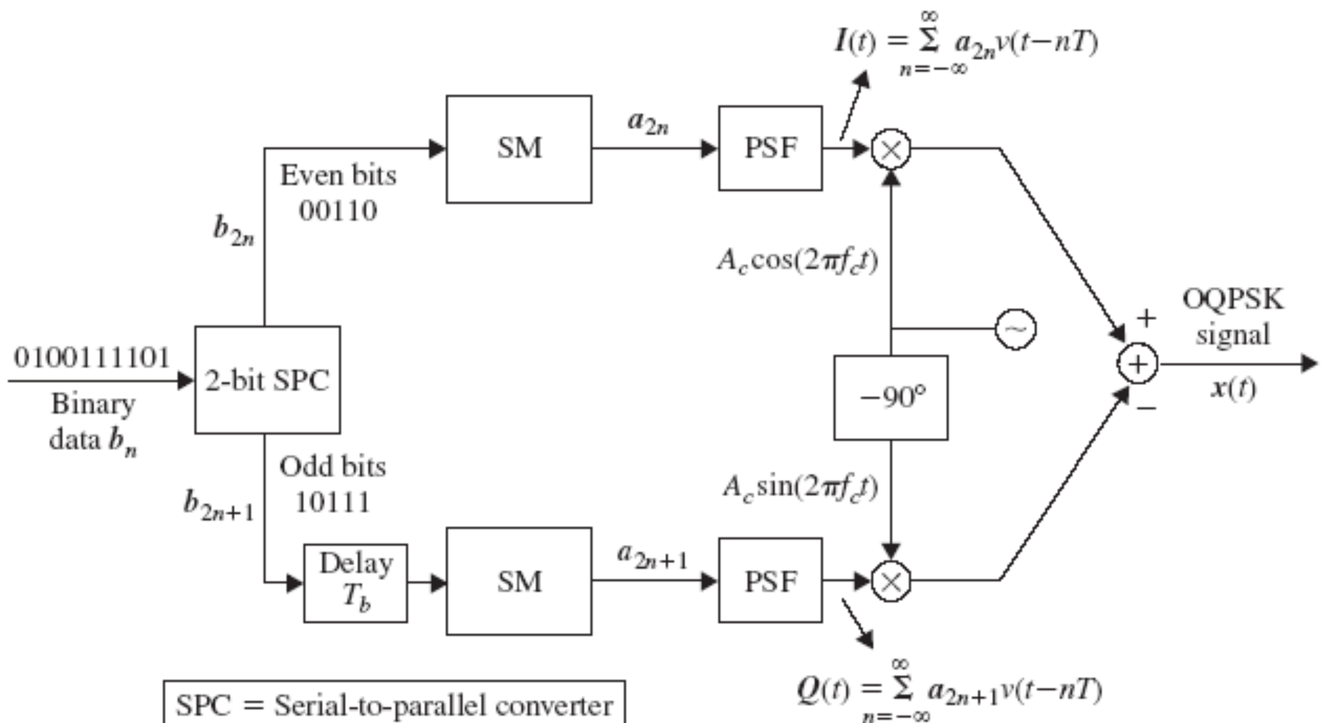
$$E_b = E_s / 2$$
$$d_{\min} = 2\sqrt{E_b}$$

It is exact value

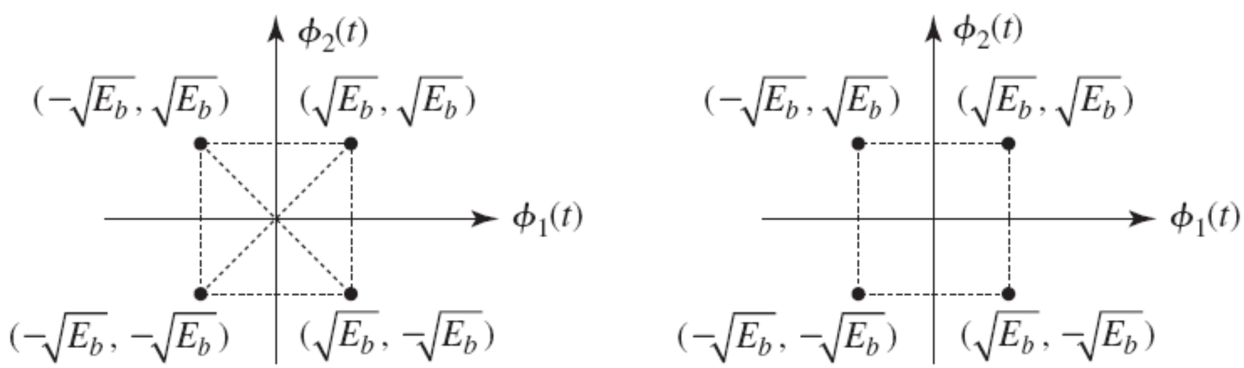
Offset QPSK (OQPSK)

- OQPSK is a minor but important variation on QPSK
- In QPSK, there is no constraint on allowed phase transitions (0, 90 or 180 degrees as shown by dotted lines) as shown in Figure
  - $\boldsymbol{I}(t)$ and $\boldsymbol{Q}(t)$ in QPSK can switch signs simultaneously (e.g. if 11 is followed by 00) ⇒ the phase $\psi(t)$ changes by 180^o
- Constant envelope nature of the QPSK signal destroyed with the filtered pulses - the waveform can't change instantaneously from one peak to another when 180^o phase transitions occur
- However, Class-C amplifiers are highly nonlinear and restore the filtered sidelobes causing adjacent channel interference, when amplifying a waveform with envelope variation
- In OQPSK, either $\boldsymbol{a}_{2n}$ or $\boldsymbol{a}_{2n+1}$ can change but not both because of a single bit delay in the quadrature path ⇒ $\pm 90^o$ phase transitions only to adjacent neighbors. Less envelope variation

OQPSK Modulator



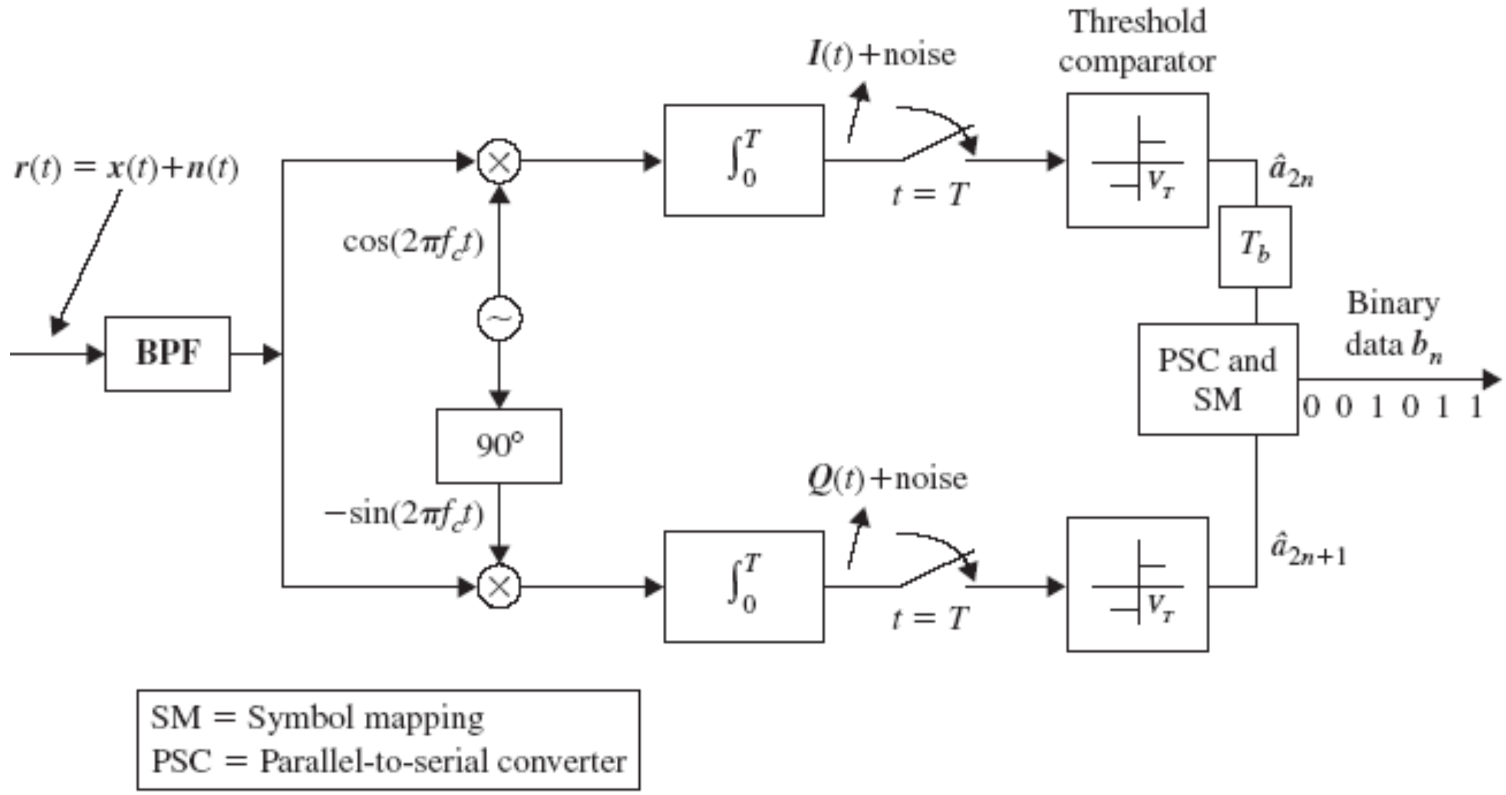
SPC = Serial-to-parallel converter
SM = Symbol mapping
PSF = Pulse shaping filter



(a) QPSK

(b) OQPSK

OQPSK Demodulator



- The OQPSK demodulator is identical to that of QPSK demodulator except for a single bit delay in the inphase path
- Since OQPSK constellation is identical to that of QPSK, its BER performance is identical to that of QPSK

M-ary Phase Shift Keying

- In *M*-ary PSK, *M* different phase shifts of the carrier are used to convey the information. The $M = 2^k$ signal waveforms, each representing *k* information bits, are represented as

$$s_i(t) = A_c v(t)\cos[2\pi f_c t + \psi_i + \varphi], \quad 0 \le t \le T \quad i = 1,......,M$$

where

$$\varphi = 0 \text{ or } \frac{\pi}{M} = \text{Fixed phase offset}$$

$$\psi_i = \frac{2\pi(i-1)}{M} = M \text{ possible phases of the carrier}$$

$v(t)$= unit energy pulse

All *M*-ary PSK waveforms have equal energy E_s

- *M*-ary PSK signal

Phase of the carrier during the *n*th symbol interval

$$\boldsymbol{x}(t) = \sqrt{\frac{2E_s}{T}} \sum_{n=-\infty}^{\infty} v(t - nT)\cos[2\pi f_c t + \boldsymbol{\psi}_n + \varphi]$$

M-ary PSK (contd)

- By choosing the same basis functions as for QPSK, it is possible to express all waveforms in the M-PSK signal set as vectors in the plane spanned by ϕ_1 and ϕ_2 as

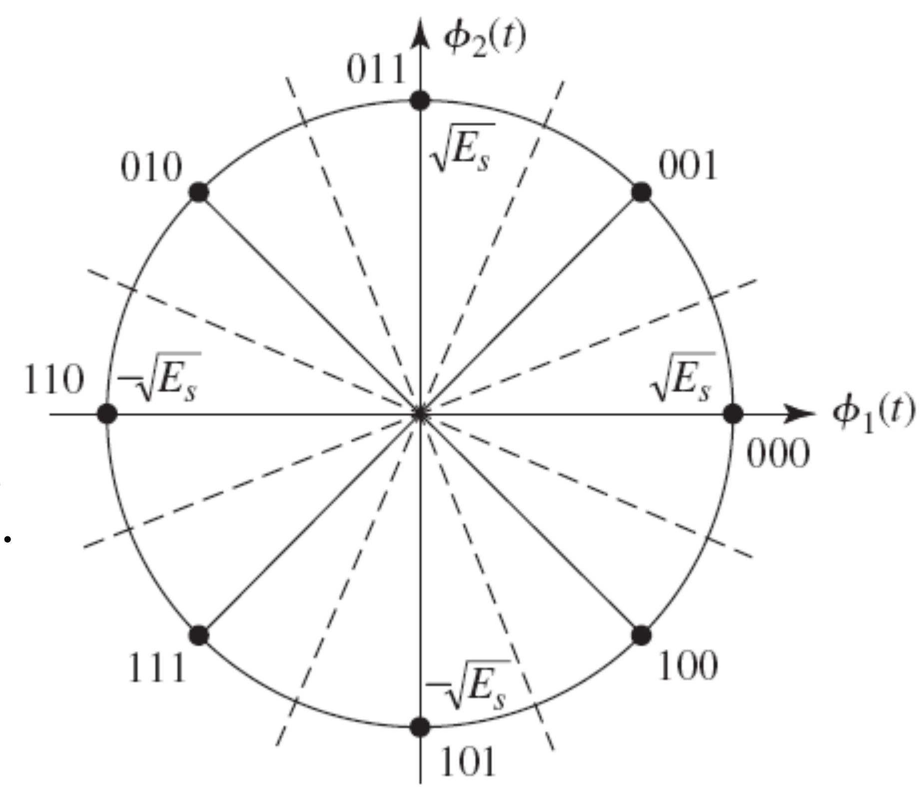
$$\underline{s} = (a_n^I \sqrt{E_s}, a_n^Q \sqrt{E_s})$$

where

$$a_n^I = \cos(\psi_n + \varphi)$$

$$a_n^Q = \sin(\psi_n + \varphi)$$

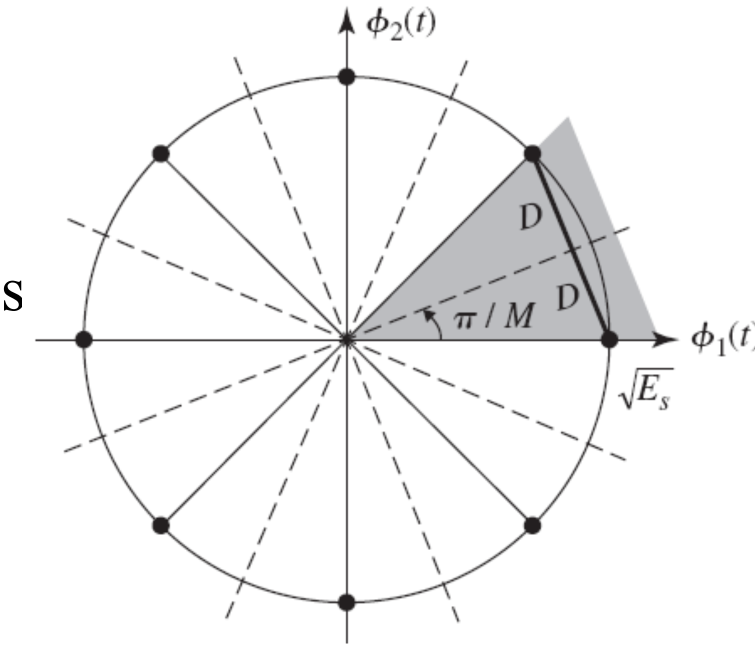
- The signal vectors lie around a circle of radius $\sqrt{E_s}$. The constellation for 8-PSK ($M = 8$) is shown in Figure



M-ary PSK (contd)

- As illustrated in Figure, the minimum distance between two adjacent signal points is

$$d_{min} = 2D = 2\sqrt{E_s}\sin\left(\frac{\pi}{M}\right)$$



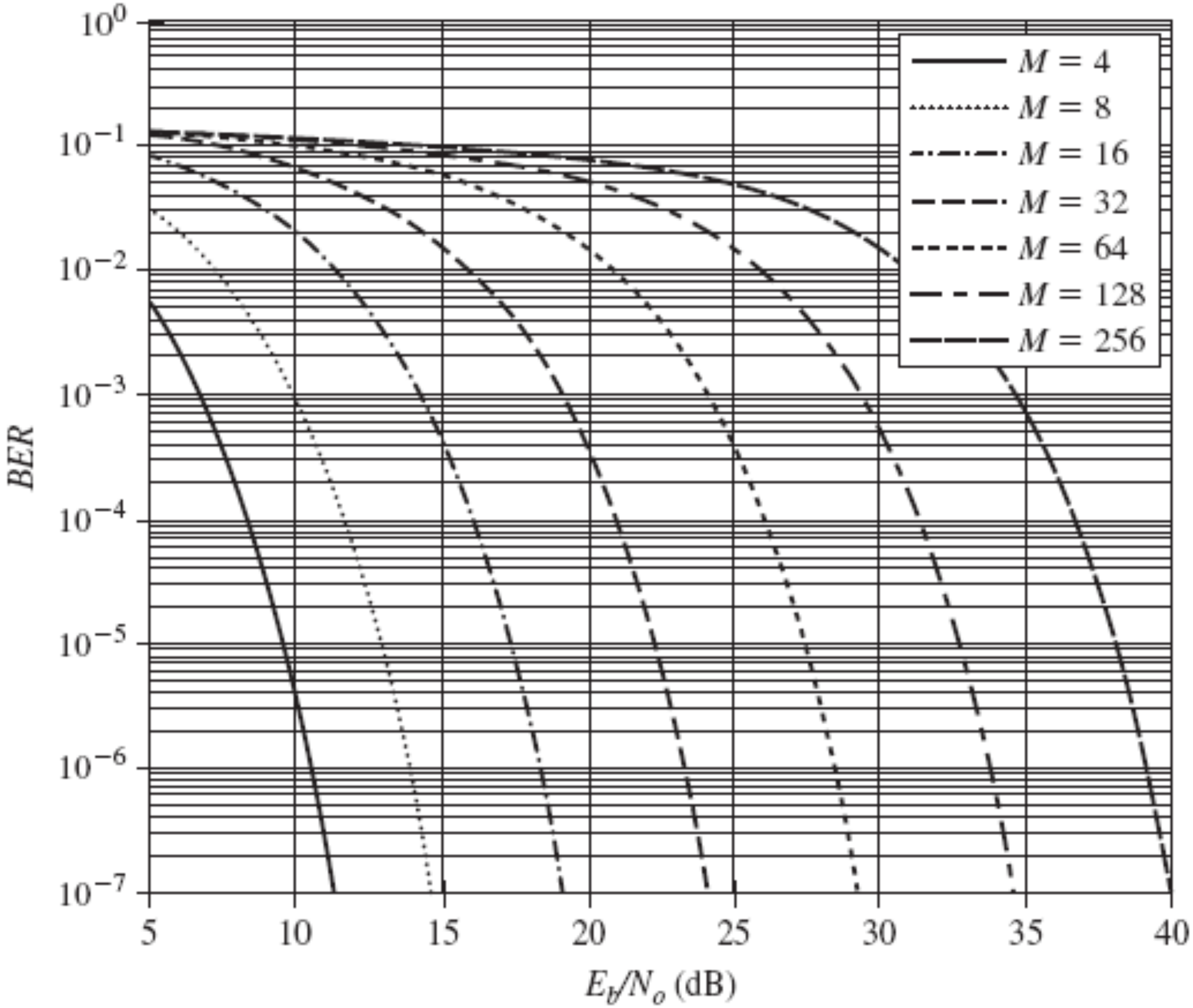
- The nearest-neighbor estimate of P_e is

$$P_e \approx 2Q\left[\sqrt{\frac{2E_s}{N_o}\sin^2\left(\frac{\pi}{M}\right)}\right]$$

$$= 2Q\left[\sqrt{\frac{2E_b \log_2 M}{N_o}\sin^2\left(\frac{\pi}{M}\right)}\right]$$

$$BER_{MPSK} \approx \frac{1}{\log_2 M} 2Q\left[\sqrt{\frac{2E_b \log_2 M}{N_o}\sin^2\left(\frac{\pi}{M}\right)}\right]$$

M-PSK BER Performance



Digital Carrier Modulation Schemes

Binary BP Signaling	Null-to-Null RF Bandwidth (Hz)	Abs-Abs Bandwidth(Hz)	BER with Coherent Detection	BER with Noncoherent Detection
ASK	$2R_b$	$R_b(1+\alpha)$	$Q(\sqrt{E_b/N_o})$	$0.5e^{-E_b/2N_o}$
BPSK	$2R_b$	$R_b(1+\alpha)$	$Q(\sqrt{2E_b/N_o})$	Requires coherent detection
Sunde's FSK	$3R_b$		$Q(\sqrt{E_b/N_o})$	$0.5e^{-E_b/2N_o}$
DBPSK	$2R_b$	$R_b(1+\alpha)$		$0.5e^{-E_b/N_o}$
***M*-ary BP Signaling**				
QPSK/OQPSK	R_b	$R_b(1+\alpha)/2$	$Q(\sqrt{2E_b/N_o})$	Requires coherent detection
MSK	$1.5R_b$	$3R_b(1+\alpha)/4$	$Q(\sqrt{2E_b/N_o})$	Requires coherent detection
M-PSK (M>4)	$2R_b/\log_2 M$	$R_b(1+\alpha)/\log_2 M$	$\frac{2}{\log_2 M}Q(\sqrt{2\log_2 M\sin^2(\pi/M)E_b/N_o})$	Requires coherent detection
M-DPSK (M>4)	$2R_b/\log_2 M$	$R_b(1+\alpha)/\log_2 M$		$\frac{2}{\log_2 M}Q(\sqrt{4\log_2 M\sin^2(\pi/2M)E_b/N_o})$
M-QAM (Square constellation)	$2R_b/\log_2 M$	$R_b(1+\alpha)/\log_2 M$	$\frac{4}{\log_2 M}(1-\frac{1}{\sqrt{M}})Q(\sqrt{\frac{3\log_2 M}{M-1}E_b/N_o})$	Requires coherent detection
M-FSK Coherent Noncoherent	$(M+3)R_b/2\log_2 M$ $2M R_b/\log_2 M$		$\frac{M-1}{\log_2 M}Q(\sqrt{(\log_2 M)E_b/N_o})$	$\frac{M-1}{2\log_2 M}0.5e^{-(\log_2 M)E_b/2N_o}$