Digital Baseband Modulation

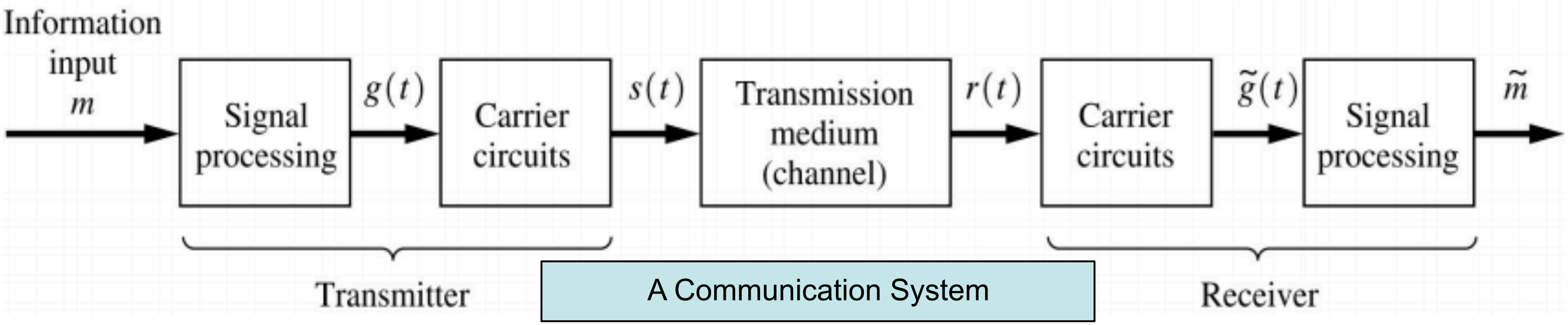
Outline

- Later

Baseband & Bandpass Waveforms

- A baseband waveform has a spectral magnitude that is nonzero for freq in the vicinity of the origin (f=0) and negligible elsewhere.
  - It is a signal whose range of freq is measured from zero to a maximum bandwidth
  - E.g., an audio signal from a microphone, a TTL signal from a digital circuit.
- A bandpass waveform has a spectral magnitude that is nonzero for freq in some band concentrated about a freq $f = \pm f_c$.
  - The spectral magnitude is negligible elsewhere.
  - f_c is called carrier freq.
  - E.g., An AM radio signal that broadcast news over f_c=850 kHz is a bandpass signal

Baseband & Bandpass Waveforms, Modulation



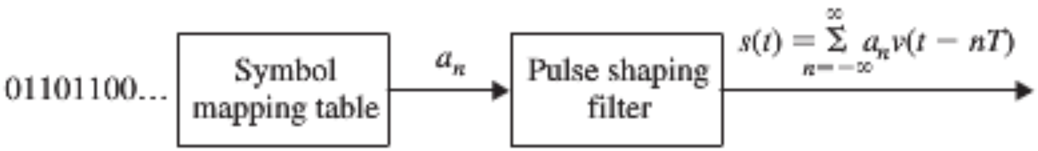
A Communication System

- Modulation is the process of imparting the source information onto a bandpass signal with carrier freq f_c using amplitude or phase perturbation (or both).
  - The bandpass signal is called modulated signal s(t).
  - The baseband signal is called modulating signal m(t).
- Bandpass communication signal is obtained by modulating a baseband analog or digital signal on a carrier.
  - Whereas baseband signal cannot go far, a bandpass signal goes a long distance.

Dig. Baseband Modulators (Line Coders)

- Sequence of bits are modulated into waveforms before transmission
- → Digital transmission system consists of:

- The modulator is based on:



- The symbol mapper takes bits and converts them into symbols a_n) – this is done based on a given table
- Pulse Shaping Filter generates the Gaussian pulse or waveform ready to be transmitted (Baseband signal)

$$s(t) = \sum_{n=-\infty}^{\infty} a_n v(t - nT)$$

Waveform; Sampled at T

Pulse Amplitude Modulation (PAM)

- In the case of pulse amplitude modulation, the symbol mapping table maps each block of k bits into a distinct scalar symbol $a_m \in \mathcal{A}_M$, where $M = 2^k$
- Thus, it is called M-ary or M-level *pulse amplitude modulation (PAM)* scheme. The M transmitted waveforms in an M-ary PAM system are given by

  $$s_m(t) = a_m v(t), m = 1, 2, \ldots\ldots, M$$

  where a_m is an amplitude level from the M-ary symbol set $\mathcal{A}_M$.
  $v(t)$ is an appropriately chosen basic pulse shape
- The modulator outputs a pulse every T seconds, where T is called the **symbol period**
- $D = 1/T$ is called the **symbol or pulse transmission rate**

Example: Binary PAM

- **Antipodal** or **polar** signaling : $\mathscr{A}_2 = \{-1, 1\}$

  $s_1(t) = v(t)$ corresponding to binary 1

  $s_2(t) = -v(t)$ corresponding to binary 0

- **Unipolar** Signaling: $\mathscr{A}_2 = \{0, 1\}$

  $s_1(t) = v(t)$ corresponding to binary 1

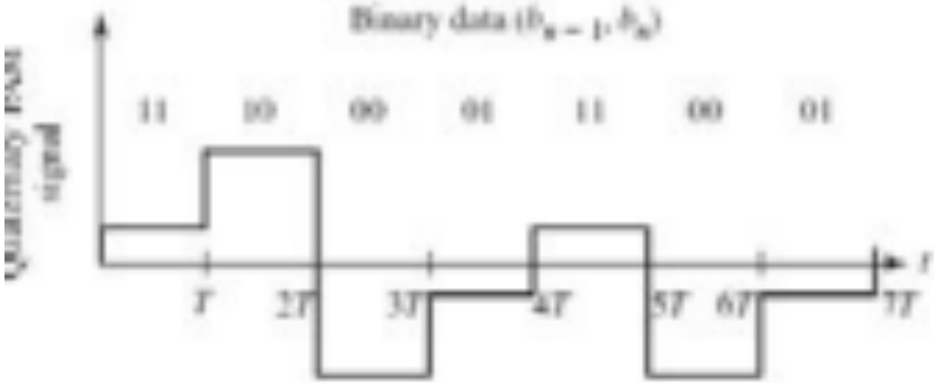
  $s_2(t) = 0$ corresponding to binary 0

Example: Quaternary PAN

Example: *M*-ary Signaling

- $M = 4$

$$(b_{k-1}, b_k) \rightarrow a_k \in \mathcal{A}_4 = \{-3, -1, 1, 3\}$$



b_{k-1}, b_k	a_k
0,0	−3
0,1	− 1
1,1	1
1,0	3

- The modulator transmits 2 bits at a time by employing 4 different amplitude pulses as shown in Figure
  - For example, binary pair (0,0) is sent by using a square pulse of amplitude −3, while the pair (1,1) is transmitted by using a square pulse of amplitude 1

PAM Randomness

- Since the amplitude level is uniquely determined by k bits of random data it represents, the pulse amplitude during the nth symbol interval (a_n) is a discrete random variable
- s(t) is a random process because pulse amplitudes {an} are discrete random variables assuming values from the set AM
- The bit period Tb is the time required to send a single data bit
- Rb = 1/ Tb is the equivalent bit rate of the system

$$s(t) = \sum_{n=-\infty}^{\infty} a_n v(t - nT)$$

PAM

- Since k bits are transmitted during each symbol period T

$$R_b = kD = (\log_2 M)D$$

T= Symbol period

- **Thus the bit rate of the modulation scheme is improved by a factor of $\log_2 M$ by using M-ary signaling**
- The time devoted to transmitting a single bit can now be written as

$$T_b = \frac{1}{R_b} = \frac{1}{kD} = \frac{T}{k} = \frac{T}{\log_2 M}$$

D= Symbol or pulse rate

Example

- Amplitude pulse modulation
- If binary signaling & pulse rate is 9600 find bit rate

- If quaternary signaling & pulse rate is 9600 find bit rate

b_{n-1}, b_n	a_n
0,0	−3
0,1	− 1
1,1	1
1,0	3

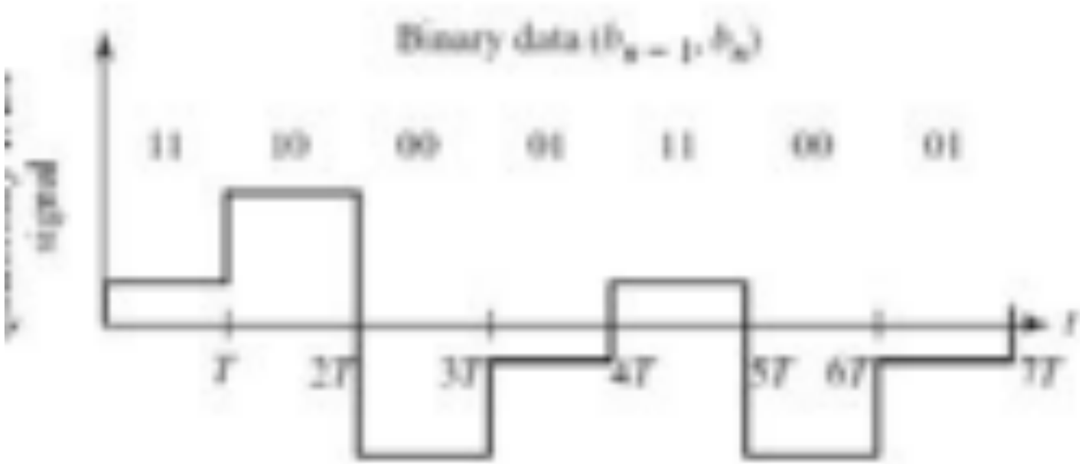
Example

- Amplitude pulse modulation
- If binary signaling & pulse rate is 9600 find bit rate

  M=2→ k=1→ bite rate Rb=1/Tb=k.D = 9600

- If quaternary signaling & pulse rate is 9600 find bit rate

  M=2→ k=1→ bite rate Rb=1/Tb=k.D = 9600



b_{k-1}, b_k	a_k
0,0	−3
0,1	− 1
1,1	1
1,0	3

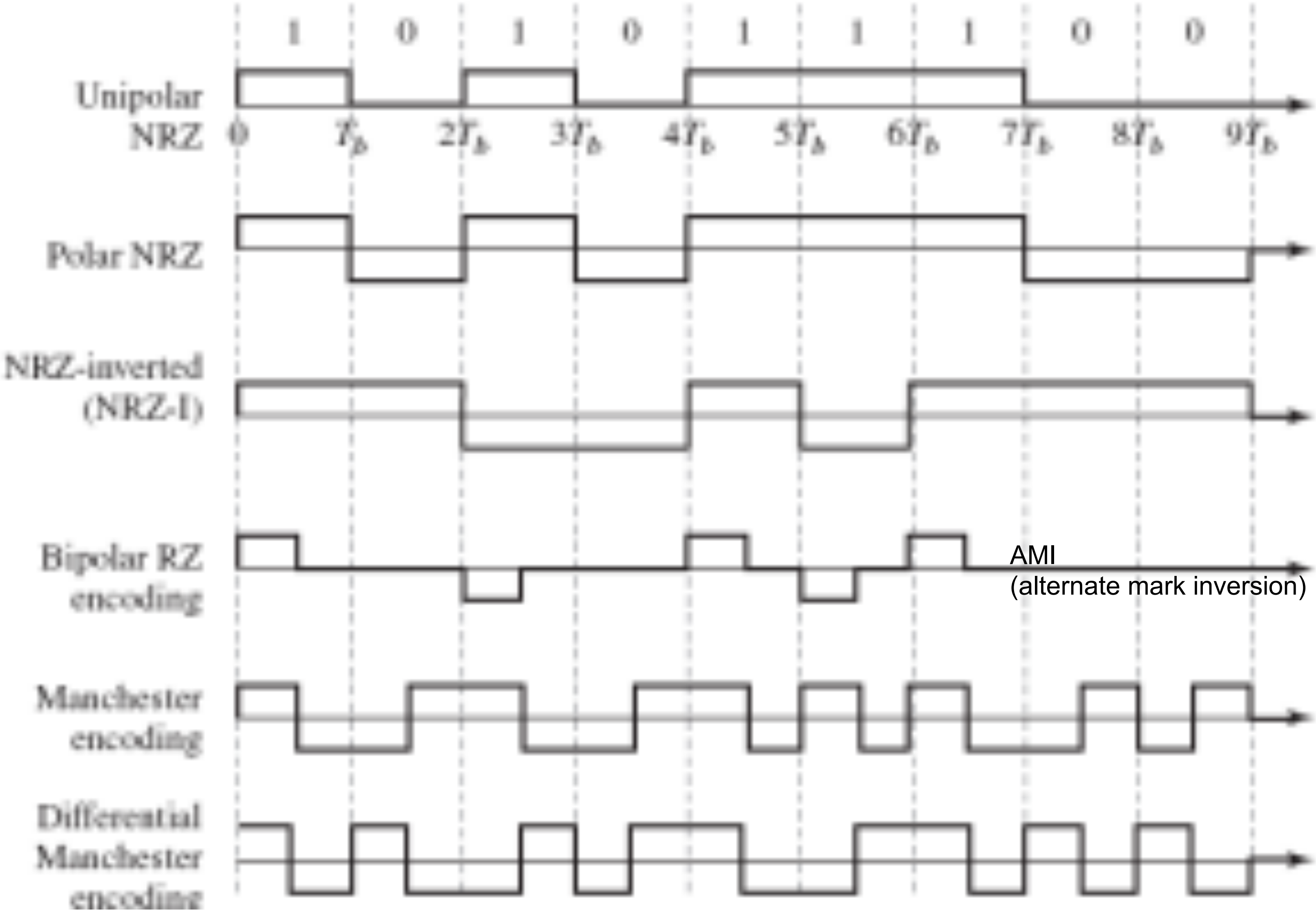
Binary Line Coding Techniques

- Line coding - Mapping of binary information sequence into the digital signal that enters the baseband channel
- Symbol mapping
  - Unipolar - Binary 1 is represented by +A volts pulse and binary 0 by no pulse during a bit period
  - Polar - Binary 1 is represented by +A volts pulse and binary 0 by –A volts pulse. Also called antipodal coding
  - Bipolar (pseudoternary)- Binary 1 is alternately mapped into +A volts and – **A** volts pulses. The binary 0 is represented by no pulse. Also called alternate mark inversion (AMI) coding
- Pulse shape
  - Non-return-to-zero (NRZ). The pulse amplitude is held constant throughout the pulse or bit period

Binary Line Coding Techniques

- Return-to-zero (RZ). The pulse amplitude returns to a zero-volt level for a portion (usually one-half) of the pulse or bit period
- Manchester. A binary 1 is denoted by a transition from a positive pulse to a negative pulse in the middle of the bit period, and a binary 0 by a transition from a negative pulse to a positive pulse
- There is another set of coding schemes that transmit changes between successive data symbols called differential encoding
- A binary 1 causes toggling of the waveform transmitted during the previous symbol interval. No toggling is forced to transmit a binary 0

Binary Line Coding Examples



Symbol Mapping

Symbol Mapping

Symbol Mapping

Which Line Coding?

- How do we know which line coding to choose?
- Depends on a number of factors:
  - How to deal with long stream of 1's and 0's (low frequency content)
  - Spectral characteristics – how much cross-talk or roll-off
  - BW Efficiency – what is the bit rate when BW is limited
  - Error detection capacity
  - Power efficiency - how much power is required to send the data

We look at several key parameters for each line coding:

- Power Spectral Density
- Bandwidth
- Bit rate

Spectra of Linearly Modulated Digital Signals

- Linear modulation in the presence of random pulse (PAM)
- We assume WSS (cyclo-stationary) random process

- The power spectral density (PSD) of a linearly modulated digital signal $s(t) = \sum_{n=-\infty}^{\infty} a_n v(t - nT)$ is given by

$$G_s(f) = \frac{|V(f)|^2}{T} \sum_{\ell=-\infty}^{\infty} R(\ell) e^{-j2\pi \ell f T}$$

$$s(t) = \sum_{n=-\infty}^{\infty} a_n v(t - nT)$$

where $v(t)$ is a basic pulse shape – a square pulse or a Gaussian pulse, and

$$v(t) \xleftrightarrow{\Im} V(f)$$

See notes!

- $R(\ell)$ is the autocorrelation function of the random data $\{a_n\}$ and is given by

$$R(\ell) = E\{a_n a_{n+\ell}\}$$

Spectra of Linearly Modulated Digital Signals

- For M-ary PAM, $\boldsymbol{a}_n$ are real-valued random variables with mean m_a and variance σ_a^2. We further assume that $\boldsymbol{a}_n$ are equiprobable and statistically independent
- For $\ell = 0$, we can

$$R(0) = E\{\boldsymbol{a}_n^2\} = \sigma_a^2 + m_a^2$$

- For $\ell \neq 0$, we can write

$$R(\ell) = E\{\boldsymbol{a}_n \boldsymbol{a}_{n+\ell}\} = E\{\boldsymbol{a}_n\} E\{\boldsymbol{a}_{n+\ell}\} = m_a^2$$

- Therefore,

$$\sum_{\ell=-\infty}^{\infty} R(\ell) e^{-j2\pi \ell f T} = \sigma_a^2 + m_a^2 \sum_{\ell=-\infty}^{\infty} e^{-j2\pi \ell f T}$$

Spectra of Linearly Modulated Digital Signals

$$\sum_{\ell=-\infty}^{\infty} e^{-j2\pi \ell f T} = \frac{1}{T}\sum_{\ell=-\infty}^{\infty} \delta\left(f-\frac{\ell}{T}\right)$$

- Substituting yields

$$G_s(f) = \frac{|V(f)|^2}{T}\left[\sigma_a^2 + m_a^2 \sum_{\ell=-\infty}^{\infty} e^{-j2\pi \ell f T}\right]$$

- Applying Poisson's sum formula, we obtain

1/T = D = symbol rate

$$G_s(f) = |V(f)|^2 D\sigma_a^2 + (Dm_a)^2 \sum_{\ell=-\infty}^{\infty} |V(\ell D)|^2 \delta(f-\ell D) \qquad (*)$$

- The PSD of the digital signal $s(t)$ depends on the statistical properties of the data (via m_a and σ_a^2) and the spectrum of basic pulse shape $V(f)$
  - Observe impulses at harmonics of symbol rate D, unless $m_a = 0$ or $V(f) = 0$ at all values of $f = \ell D$, $\ell = 0, \pm 1, \pm 2 \ldots$

Find the SD for Multi-Level Unipolar NRZ

Multilevel Unipolar NRZ Signalin

$a_n \in \{0, A, 2A, \ldots\ldots\ldots(M-1)A\}$ ← Definition

Remember: $\sigma^2 = \left(\sqrt{\dfrac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+(x_3-\bar{x})^2+\ldots\ldots+(x_n-\bar{x})^2}{n}}\right)^2$

$$m_a = E\{a_n\} = A(M-1)/2, \qquad \sigma_a^2 = Var(a_n) = (M^2-1)\frac{A^2}{12}$$

- Substituting into (*), we get

$$G_s(f) = \frac{(M^2-1)A^2 D}{12}|V(f)|^2 + \frac{(M-1)^2}{4}(DA)^2 \sum_{\ell=-\infty}^{\infty} |V(\ell D)|^2 \delta(f-\ell D)$$

- For a rectangular basic pulse shape,

$$v(t) = \Pi(t/T), \qquad V(f) = T\,\mathrm{sinc}(fT)$$

$$G_s(f) = \frac{(M^2-1)A^2T}{12}\mathrm{sinc}^2(fT) + \frac{(M-1)^2A^2}{4}\sum_{\ell=-\infty}^{\infty}|\mathrm{sinc}(fT)|^2\,\delta(f-\ell D)$$

$$= \frac{(M^2-1)A^2}{12D}\mathrm{sinc}^2(f/D) + \frac{(M-1)^2A^2}{4}\Bigg[\delta(f) + \underbrace{\sum_{\substack{\ell=-\infty\\ \ell\neq 0}}^{\infty}|\mathrm{sinc}(\ell)|^2\,\delta(f-\ell D)}_{0}\Bigg]$$

$$= \frac{(M^2-1)A^2}{12D}\mathrm{sinc}^2(f/D) + \frac{(M-1)^2A^2}{4}\delta(f)$$

http://

Find the SD for Multi-Level Unipolar NRZ

Multilevel Unipolar NRZ Signalin

$$\sigma^2 = \left(\sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+(x_3-\bar{x})^2+\ldots\ldots+(x_n-\bar{x})^2}{n}}\right)^2$$

Remember:

$$a_n \in \{0, A, 2A, \ldots\ldots\ldots(M-1)A\}$$ ← Definition

$$m_a = E\{a_n\} = A(M-1)/2, \qquad \sigma_a^2 = Var(a_n) = (M^2-1)\frac{A^2}{12}$$

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- For a rectangular basic pulse shape,

$$v(t) = \Pi(t/T), \qquad V(f) = T\mathrm{sinc}(fT)$$

- Substituting yields the PSD of M-ary unipolar NRZ signal

$$G_s(f) = \frac{(M^2-1)A^2 T}{12}\ \mathrm{sinc}^2(fT) + \frac{(M-1)^2}{4}A^2\delta(f)$$

DC component

http://en.wikipedia.org/wiki/List_of_mathematical_series

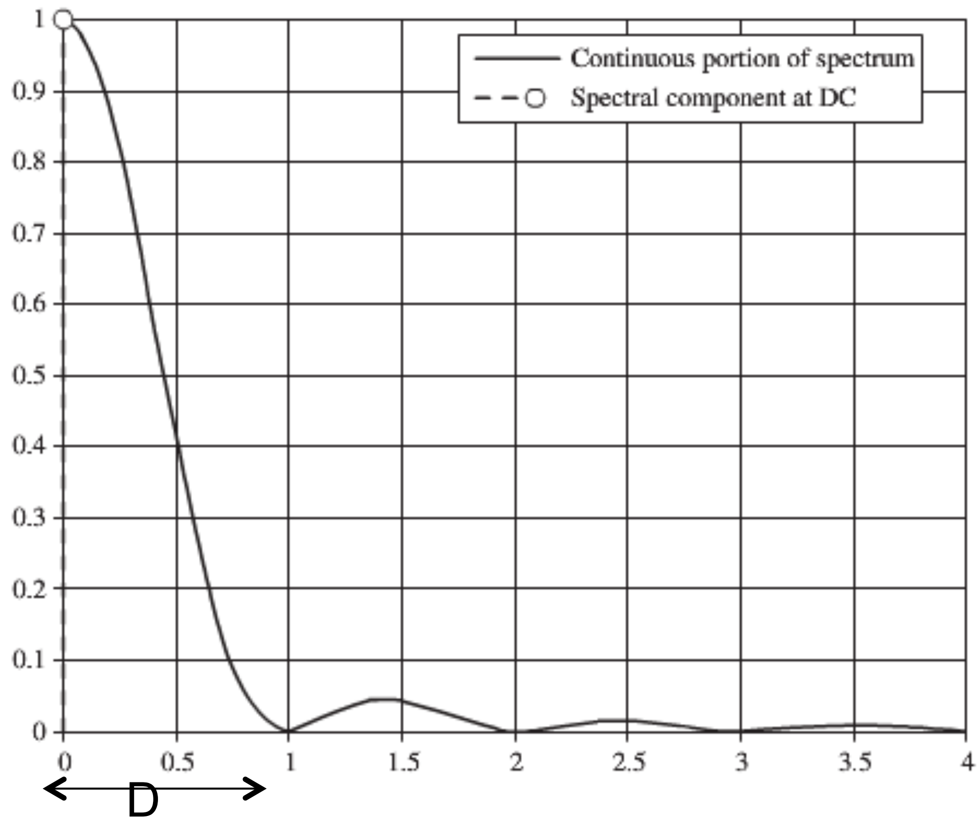
Find the SD for Multi-Level Unipolar NRZ

- Spectral Density:

$$= \frac{(M^2 - 1)A^2}{12D}\,\mathrm{sinc}^2(f/D) + \frac{(M-1)^2A^2}{4}\,\delta(f)$$

- Null-BW

$$B_{null} = D = \frac{R_b}{k} = \frac{R_b}{\log_2 M}$$



PSD for Multilevel Polar NRZ

Remember: $\sigma^2 = \left(\sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+(x_3-\bar{x})^2+\dots\dots+(x_n-\bar{x})^2}{n}}\right)^2$

$a_n \in \{\pm A, \pm 3A, \dots\dots\dots, \pm(M-1)A\}$ ← Definition

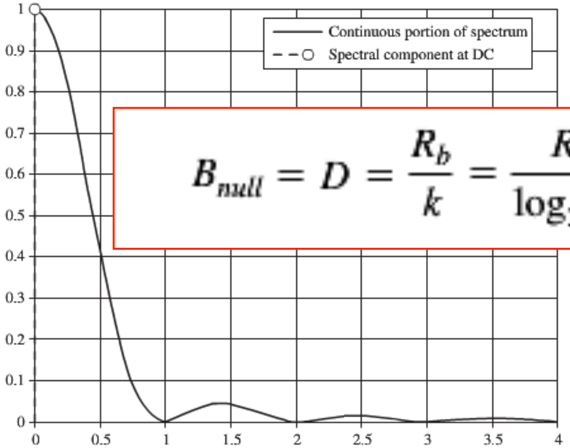
$$m_a = E\{a_n\} = 0, \qquad \sigma_a^2 = Var(a_n) = (M^2-1)\frac{A^2}{3}$$

- Substituting into (*), we get

$$G_s(f) = \frac{(M^2-1)A^2D}{3}|V(f)|^2$$

- For rectangular basic pulse shape,

$$G_s(f) = \frac{(M^2-1)A^2T}{3}\ \mathrm{sinc}^2(fT)$$



$$B_{null} = D = \frac{R_b}{k} = \frac{R_b}{\log_2 M}$$

- The frequency where the first null in the PSD occurs is called **the first null bandwidth (B_{null})**
- We observe $B_{null} = D$ for both unipolar and polar signaling

Example

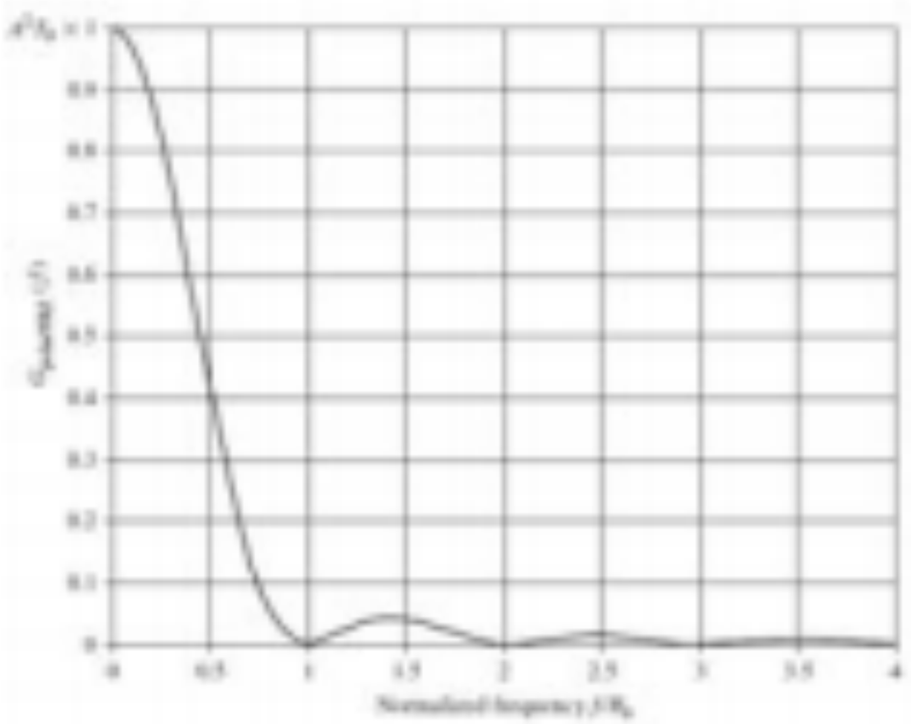
Spectrum of Binary Polar NRZ Signaling

- The PSD is given by

$$G_{\text{polarNRZ}}(f) = \frac{A^2}{R_b}\,\text{sinc}^2\left(\frac{f}{R_b}\right)$$

- f=D Hertz First Null Bandwidth
- PSD contains significant spectral energy at low frequencies

We see an impulse at f=0
→ DC energy
→ Long 1's results in

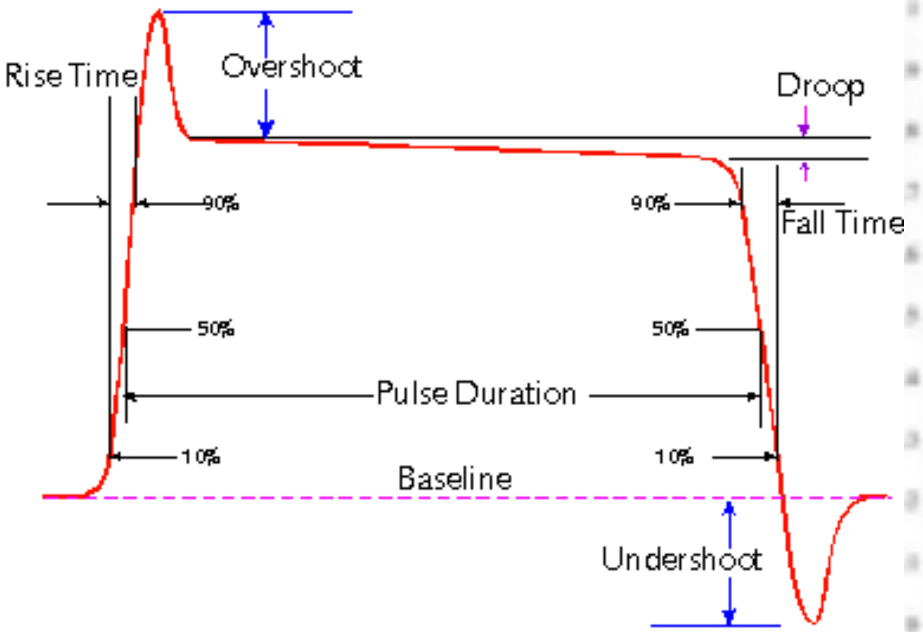


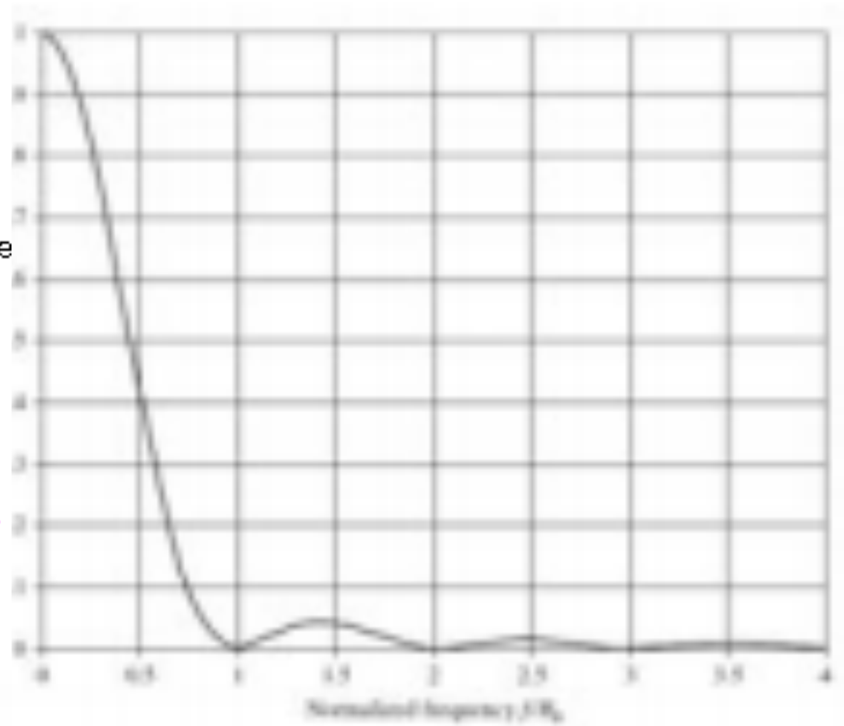
f/Rb (Rb=1)

Example

Spectrum of Binary Polar NRZ Signaling

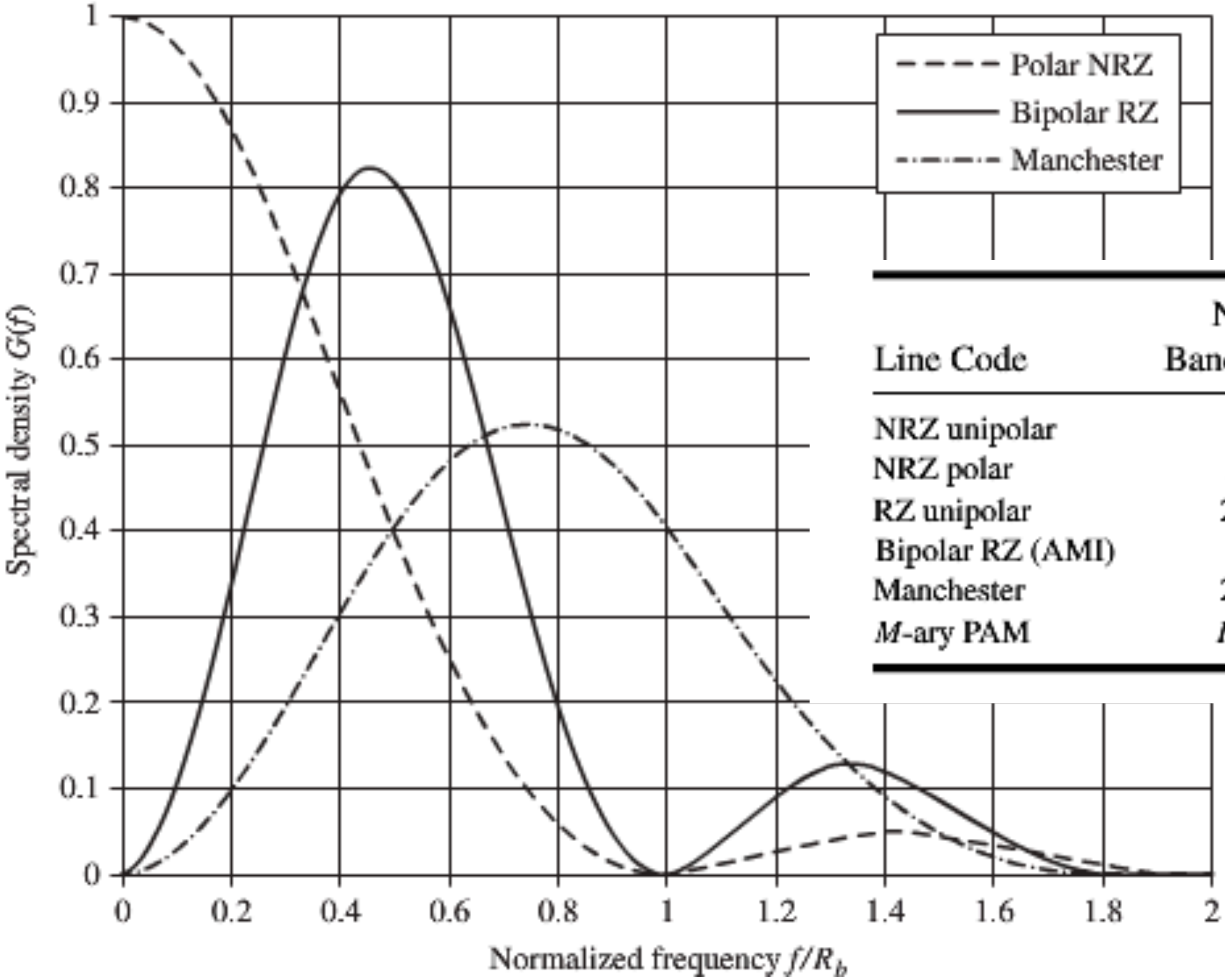
We see an impulse at f=0
→DC energy
→If this signal passes through an SC-coupled circuit
The DC part will get lost → distortion (signal droop)





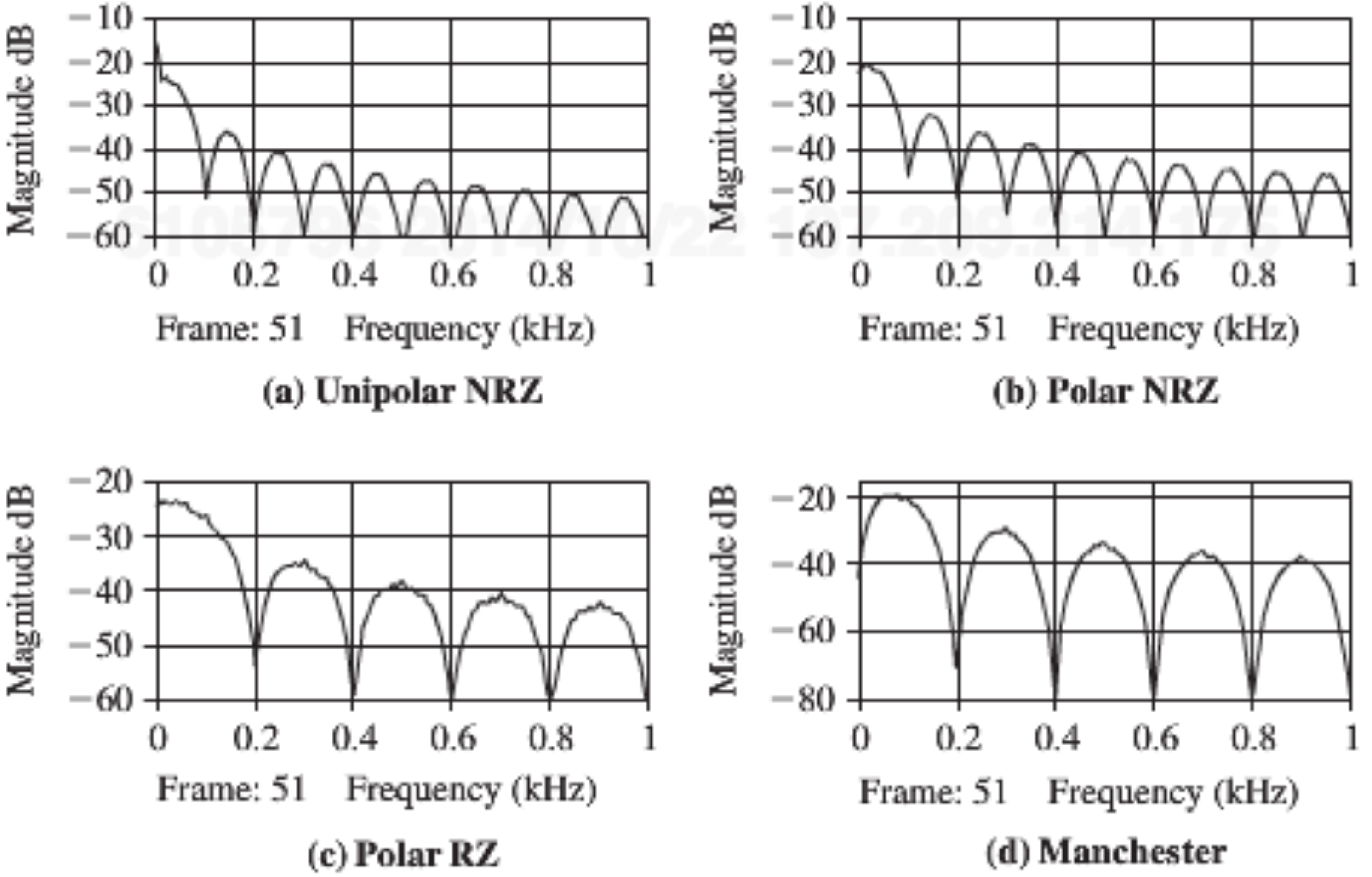
f/Rb (Rb=1)

Comparison of spectra of popular line codes (Rb=1).



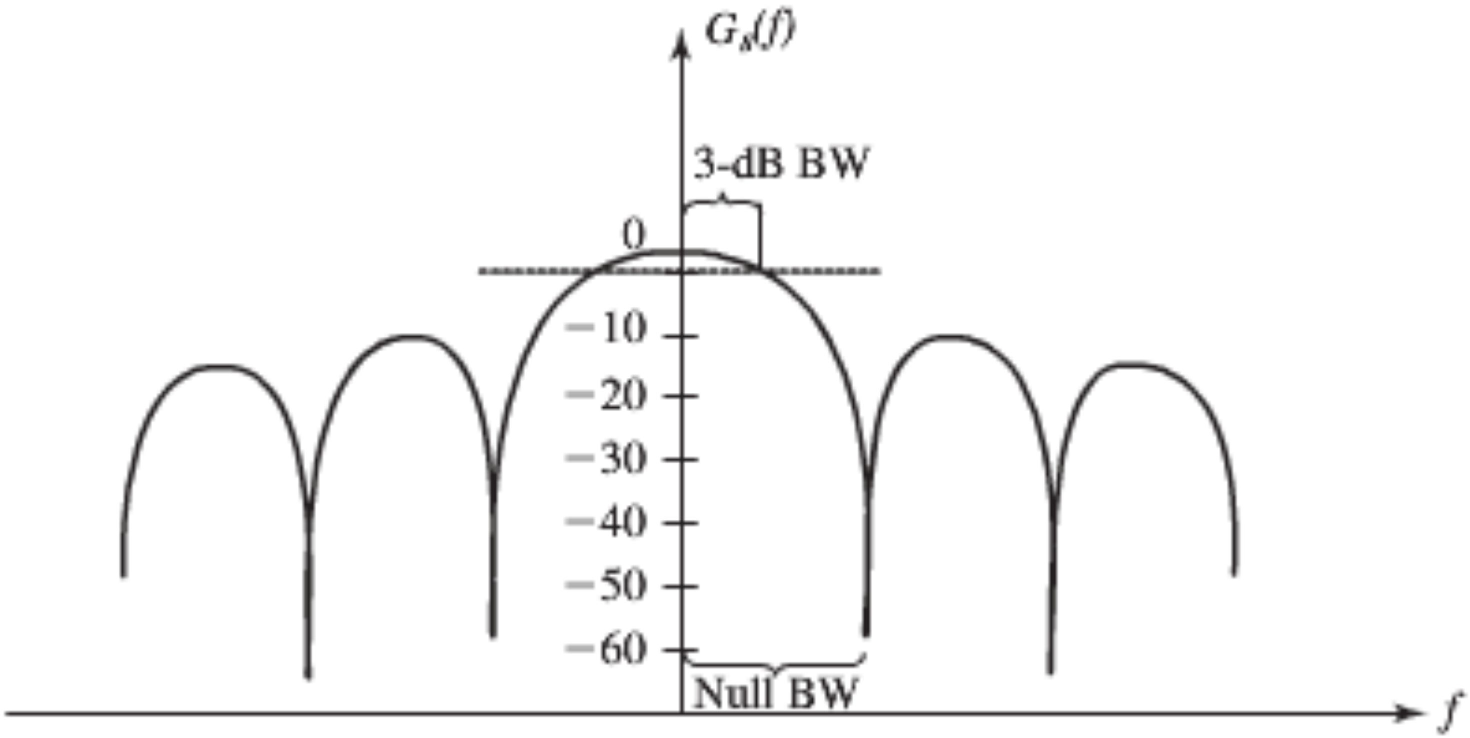
Line Code	Null Bandwidth	Spectral Efficiency η_{null} bits/sec-Hz
NRZ unipolar	R_b	1
NRZ polar	R_b	1
RZ unipolar	$2R_b$	1/2
Bipolar RZ (AMI)	R_b	1
Manchester	$2R_b$	1/2
M-ary PAM	R_b/k	k

Comparison of spectra of popular line codes

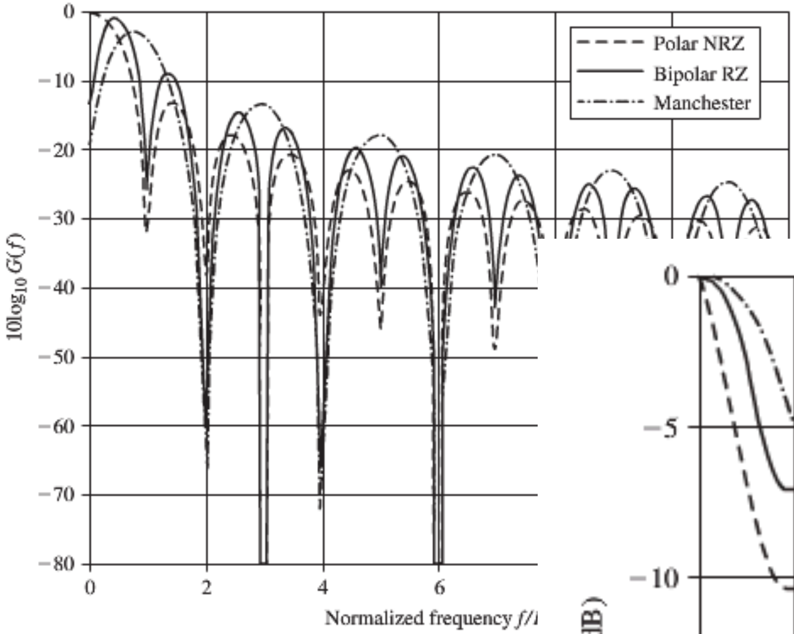


(c)

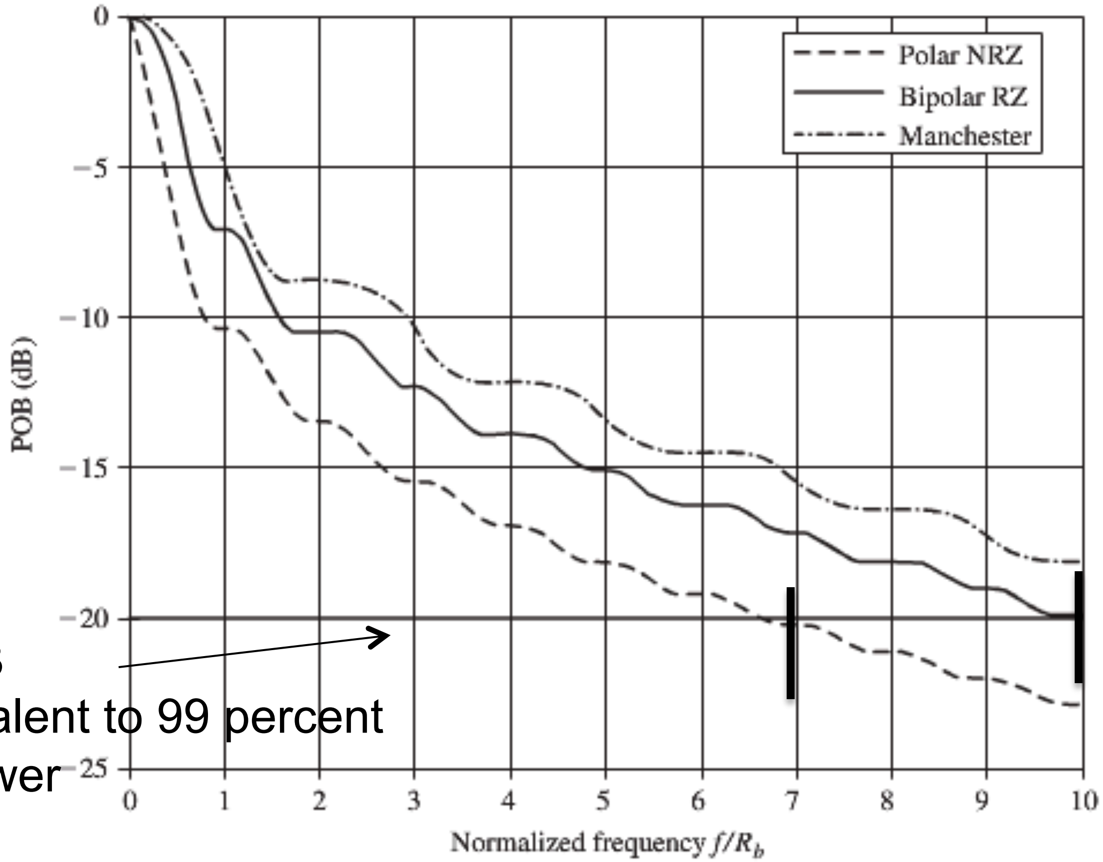
BW



Power Out of Band (POB)



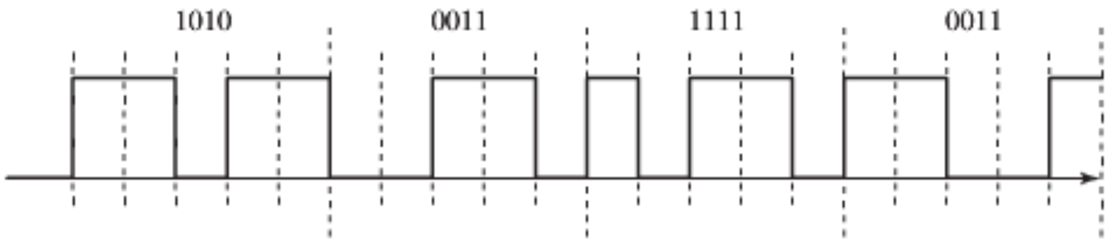
Power out-of-band plots of popular binary line codes.



029dB
Equivalent to 99 percent
Of power

Binary Block Codes (kBnB)

Binary data input	Encoder binary output
0000	11110
0001	01001
0010	10100
0011	10101
0100	01010
0101	01011
0110	01110
0111	01111
1000	10010
1001	10011
1010	10110
1011	10111
1100	11010
1101	11011
1111	11101



Binary Block Codes (HDB3)

- HIGH DENSITY BIPORAL (3-ZEROS)
- Uses NRZ-I (1→ + and next 1→ -)
- Substitute FOUR zeros with 000V followed by B00V

User data stream:	0	0	1	0	1	1	1	1	0	0	0	0	0	0	0	0	1	0	1	0	1	1	1	0	1	0	1	0	0	0	0	1
Bipolar:	0	0	+	0	−	+	−	+	0	0	0	0	0	0	0	0	−	0	+	0	−	+	−	0	+	0	−	0	0	0	0	+
HDB3:	0	0	1	0	1	1	1	1	0	0	0	V	1	0	0	V	1	0	1	0	1	1	1	0	1	0	1	0	0	0	V	1
	0	0	+	0	−	+	−	+	0	0	0	+	−	0	0	−	+	0	−	0	+	−	+	0	−	0	+	0	0	0	+	−

References

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- "M. F. Mesiya, ”Contemporary Communication Systems”, 1st ed./2012, 978-0-07-. 338036-0, McGraw Hill. Chapter 9