Question 1:

Let a random process be given by

$$x(t) = A \cos(\omega_0 t + \theta)$$

where A and ω_0 are constants and θ is a random variable. Let

$$f(\theta) = \begin{cases} \frac{2}{\pi}, & 0 \le \theta \le \frac{\pi}{2} \\ 0, & \text{elsewhere} \end{cases}$$

(a) Evaluate $\overline{x(t)}$.
(b) From the result of part (a), what can be said about the stationarity of the process?

(c) Evaluate $\langle x^2(t) \rangle$.

(d) Evaluate $\overline{x^2(t)}$.

(e) Using the results of parts (c) and (d), determine whether the process is ergodic for these averages.

Question 2:

Let $x(t) = A_0 \sin(\omega_0 t + \theta)$ be a random process, where θ is a random variable that is uniformly distributed between 0 and 2π and A_0 and ω_0 are constants.
(a) Find $R_x(\tau)$.
(b) Show that $x(t)$ is wide-sense stationary.
(c) Verify that $R_x(\tau)$ satisfies the appropriate properties.

Question 3:

Let $r(t) = A_0 \cos \omega_0 t + n(t)$, where A_0 and ω_0 are constants. Assume that $n(t)$ is a wide-sense stationary random noise process with a zero mean value and an autocorrelation of $R_n(\tau)$.
(a) Find $\overline{r(t)}$ and determine whether $r(t)$ is wide-sense stationary.
(b) Find $R_r(t_1, t_2)$.