Question 1:
A Gaussian RV with mean and variance can be defined in MATLAB as follow:

```
x = -1:0.01:3.5; // define the range of the RV

for (i = 1:1:length(x))
  f(i) = (1/(sqrt(2*pi)*sigma))*exp(-((x(i)-m)^2)/(2*sigma^2));
  F(i) = 1-Q((x(i)-m)/sigma);
end;
```

Let’s assume the mean is unity and variance of 0.2.

a) Using subplot(211) plot the PDF and CDF for this RV.
b) What happens to PDF if the variance is doubled?
c) What happens to CDF if the variance is doubled?
d) Assuming the mean is unity and variance of 0.2. Show the MATLAB code to find P(1.4 < x =< 1.6). What is the value of P?

Question 2:
The cumulative distribution function (CDF) for the Gaussian distribution is

$$F(a) = Q\left(\frac{m-a}{\sigma}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{m-a}{\sqrt{2}\,\sigma}\right)$$

where the Q function is defined by

$$Q(z) \triangleq \frac{1}{\sqrt{2\pi}} \int_z^{\infty} e^{-\lambda^2/2}\, d\lambda$$

and the complementary error function (erfc) is defined as

$$\mathrm{erfc}(z) \triangleq \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-\lambda^2}\, d\lambda$$

The Q(z) function can be calculated directly using MATLAB (shown above). A relatively simple closed-form upper bound for Q(z), z >0, is

$$Q(z) < \frac{1}{\sqrt{2\pi}\, z} e^{-z^2/2}, \quad z > 0$$

Write a MATLAB code to show both the Q(z) function and its simple closed-form upper bound for z=[0 5]. Make sure both plots are on the same semi log plot. HINT:

```
x = 0:0.1:5;
y1 = zeros(length(x),1);
m = 1;
for (i = 1:1:length(x))
  y1(i) =  f(x) // this is the upper bound function.
  if (y1(i) > 1)
    m = i;
  end;
end;
plot(x(m+1:length(x)),y1(m+1:length(x))); //  plot using semi log
plot
```

Question 3:

A binomial random variable x_k has values of k, where

$$k = 0,1,\ldots,n; \quad P(k) = \binom{n}{k} p^k q^{n-k}; \quad q = 1 - p$$

Assume that $n = 160$ and $p = 0.1$.

(a) Plot $P(k)$.

(b) Compare the plot of part (a) with a plot $P(k)$ using the Gaussian approximation

$$\binom{n}{k} p^k q^{n-k} \approx \frac{1}{\sqrt{2\pi}\sigma} e^{-(k-m)^2/2\sigma^2}$$

which is valid when $npq \gg 1$ and $|k - np|$ is in the neighborhood of $\sqrt{npq}$, where $\sigma = \sqrt{npq}$ and $m = np$.

(c) Also plot the Poisson approximation

$$\binom{n}{k} p^k q^{n-k} \approx \frac{\lambda^k}{k!} e^{-\lambda}$$

where $\lambda = np$, n *is* large, and p is small.

HINT: You need to plot three functions on the same plot, subplot(311): P(k), Gaussian Approximation, and Poison Approximation. Make sure you choose the same ranges for x and y for all plots.